Physics 2211ABC   Test form 842   Name Solutions
Spring 2018
Test 4   Recitation Section (see cover page):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

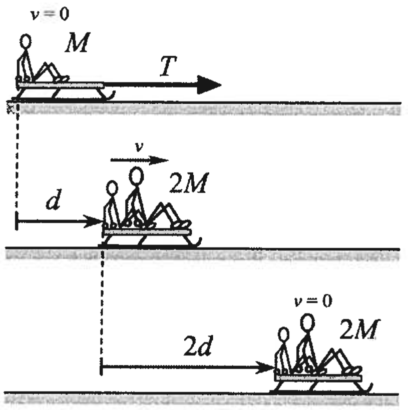
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

[I] *(20 points)* You are pulling your little brother on a sled (combined mass M). The sled rests on snowy ground, with coefficient of kinetic friction μ_k between the sled's runners and the ground. Starting from rest, you pull the tow-rope with a horizontal tension force T. After pulling for a distance d you let go of the rope and jump onto the sled yourself, raising the total mass from M to $2M$. You slide an additional distance $2d$ before the sled finally comes to a stop.

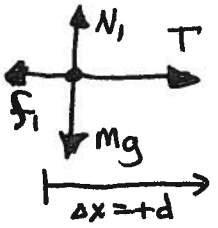


(*i*) Identify all forces acting on the sled throughout the motion, from start to finish. Write out an expression for the work done by each such force.

(*ii*) Use the Work-Energy principle to determine the tension in the rope while you were pulling. Express your answer as a multiple of Mg.

① Vertical forces (normal, gravitational) $\boxed{\text{do zero work}}$ during a horizontal displacement

② Work while pulling



$\boxed{W_T = \vec{T}\cdot\vec{\Delta x} = +Td}$

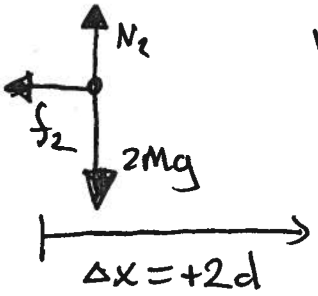
$W_{f_1} = \vec{f}_1\cdot\vec{\Delta x} = -\mu_k N_1 d$

$\boxed{W_{f_1} = -\mu_k Mgd}$

Work KE theorem: sled ends up with speed V

$\Delta K = W_{TOT} \rightarrow \boxed{\tfrac{1}{2}Mv^2 - 0 = Td - \mu_k Mgd}$ Ⓐ

③ work while coasting to a stop: you (also moving with speed V) jump onto sled: M→2M



$W_{f_2} = \vec{f}_2\cdot\vec{\Delta x} = -\mu_k N_2(2d)$

$= -\mu_k(2Mg)(2d)$

$\boxed{W_{f2} = -4\mu_k Mgd}$

$\Delta K = W_{Tot} \rightarrow 0 - \tfrac{1}{2}(2M)V^2 = -4\mu_k Mgd$

$\boxed{+\tfrac{1}{2}MV^2 = +2\mu_k Mgd}$ Ⓑ

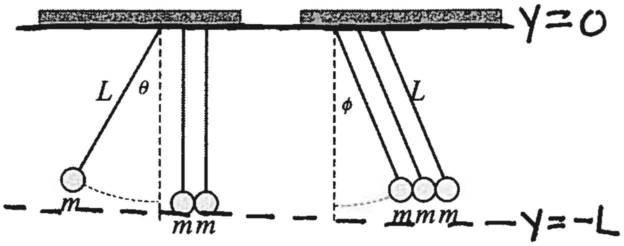
Now: Plug Ⓑ into Ⓐ, to eliminate unknown V

$2\mu_k Mg\cancel{d} = T\cancel{d} - \mu_k Mg\cancel{d}$

$\boxed{T = 3\mu_k Mg}$

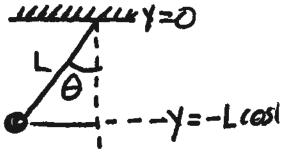
The following problem will be hand-graded. <u>*Show all your work for this problem*</u>*. Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* A "totally inelastic Newton's Cradle" apparatus is constructed from three sticky spheres of mass m, each suspended from the ceiling by a cord of length L. One sphere is pulled back such that its cord makes an angle θ relative to the vertical. When it swings to the bottom of its arc, it sticks to the other two, and they all rise together, coming to a stop with each of their cords making an angle ϕ relative to the vertical.



Find an expression for the final angle ϕ in terms of the initial angle θ. Your answer should include no physical parameters other than θ.

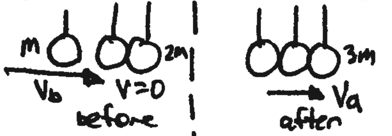
Define ceiling to be $Y=0$ → initial position of m is $Y_i = -L\cos\theta$



① Energy conserved from initial to before impact

$$K_i + U_i = K_b + U_b \longrightarrow 0 + (-\cancel{m}gL\cos\theta) = \tfrac{1}{2}\cancel{m}V_b^2 + (-\cancel{m}gL)$$

→ just before impact, $\boxed{V_b = \sqrt{2gL(1-\cos\theta)}}$

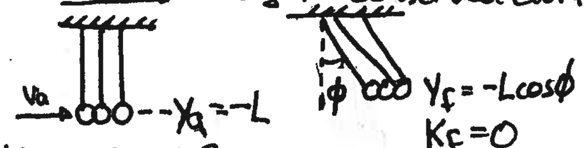


② Impact at bottom conserves momentum : before → after

$$\vec{P}_a = \vec{P}_b \rightarrow \langle +3mV_a \rangle = \langle +mV_b \rangle + \langle 0 \rangle$$

$$\rightarrow \boxed{V_a = \tfrac{1}{3}V_b = \tfrac{1}{3}\sqrt{2gL(1-\cos\theta)}}$$

③ Energy is again conserved during upswing



$K_f = 0$

$$K_a = \tfrac{1}{2}(3m)V_a^2$$

$$= \tfrac{1}{2}(3m)\left(\tfrac{2}{9}gL(1-\cos\theta)\right)$$

$$= (3m)\tfrac{gL}{9}(1-\cos\theta)$$

$$K_a + U_a = \overset{0}{\cancel{K_f}} + U_f$$

$$(\cancel{3m})\frac{gL}{9}(1-\cos\theta) + (-\cancel{3m}gL) = 0 + (-\cancel{3m}gL\cos\phi)$$

$$\frac{\cancel{gL}}{9}(1-\cos\theta) = \cancel{gL}(1-\cos\phi)$$

$$1-\cos\theta = 9 - 9\cos\phi$$

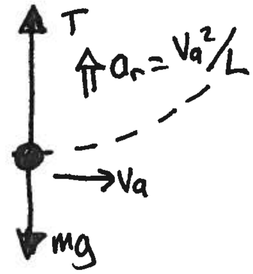
$$9\cos\phi = 8+\cos\theta$$

$$\boxed{\cos\phi = \frac{8+\cos\theta}{9}}$$

Extra credit! *(+4 points)* Find an expression for the tension in each of the cords, immediately *after* the first sphere collides and sticks to the other two. (Hint: all three tensions are the same, so you only need to calculate <u>one</u> value.) Express your answer in terms of the weight of one sphere, *mg*, and the angle θ.

At the instant they start their upswing, each mass is following a circular path of radius L, moving with a speed $V_a = \tfrac{1}{3}\sqrt{2gL(1-\cos\theta)}$

radial accel $a_r = \dfrac{V_a^2}{L} = \dfrac{2g\cancel{L}(1-\cos\theta)}{9\cancel{L}} = \dfrac{2g(1-\cos\theta)}{9}$



$$\Sigma F_r = ma_r$$

$$\langle +T \rangle + \langle -mg \rangle = m\langle +a_r \rangle$$

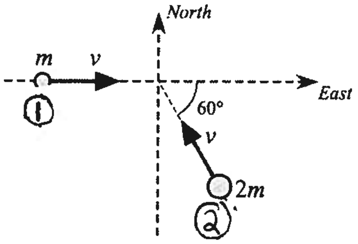
$$T = mg + ma_r$$

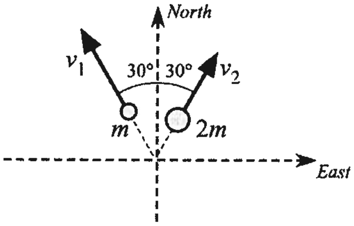
$$\boxed{T = \left(\frac{11-2\cos\theta}{9}\right)mg}$$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* Two bumper-cars are traveling with the velocities shown at top right when they have a collision. Immediately after the collision (as shown at bottom right), the first car (mass m) is seen to be traveling 30° west of north, while the second car (mass $2m$) is seen to be traveling 30° east of north.

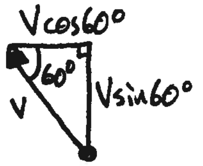
Determine the speed of each car immediately after the collision. Express each answer in terms of v.





Collision: momentum is conserved in 2D

$$\vec{P}_{1i} = \langle +mV\rangle\hat{i} + \langle 0\rangle\hat{j}$$



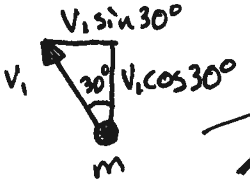
$$\vec{P}_{2i} = 2m\left[\langle -V\cos60^\circ\rangle\hat{i} + \langle +V\sin60^\circ\rangle\hat{j}\right]$$

$$= 2m\left[\langle -\tfrac{1}{2}V\rangle\hat{i} + \langle +\tfrac{\sqrt{3}}{2}V\rangle\hat{j}\right.$$

$$\vec{P}_{2i} = \langle -mV\rangle\hat{i} + \langle +\sqrt{3}mV\rangle\hat{j}$$

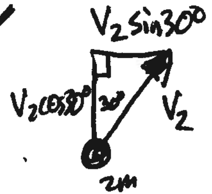
Hence, $\vec{P}_i = \vec{P}_{1i} + \vec{P}_{2i} = \langle +mV\rangle\hat{i} + \left[\langle -mV\rangle\hat{i} + \langle +\sqrt{3}mV\rangle\hat{j}\right] \longrightarrow \boxed{\vec{P}_i = \langle 0\rangle\hat{i} + \langle \sqrt{3}mV\rangle\hat{j}}$

Final momenta are:



$$\vec{P}_{1f} = \langle -mV_1\sin30^\circ\rangle\hat{i} + \langle +mV_1\cos30^\circ\rangle\hat{j}$$

$$= \langle -\tfrac{1}{2}mV_1\rangle\hat{i} + \langle +\tfrac{\sqrt{3}}{2}mV_1\rangle\hat{j}$$



$$\vec{P}_{2f} = \langle +2mV_2\sin30^\circ\rangle\hat{i} + \langle +2mV_2\cos30^\circ\rangle\hat{j}$$

$$= \langle +mV_2\rangle\hat{i} + \langle +\sqrt{3}mV_2\rangle\hat{j}$$

Now, write separate equations preserving $\vec{P}_x$ and $\vec{P}_y$:

$$\vec{P}_{xf} = \vec{P}_{xi} \longrightarrow \langle -\tfrac{1}{2}\cancel{m}V_1\rangle + \langle +\cancel{m}V_2\rangle = \langle 0\rangle \longrightarrow \boxed{V_1 = 2V_2}$$

$$\vec{P}_{yf} = \vec{P}_{yi} \longrightarrow \langle +\tfrac{\sqrt{3}}{2}\cancel{m}V_1\rangle + \langle +\sqrt{3}\cancel{m}V_2\rangle = \langle +\sqrt{3}\cancel{m}V\rangle$$

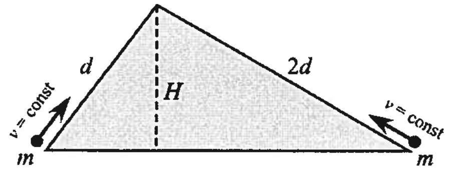
$$\tfrac{\cancel{\sqrt{3}}}{\cancel{2}}(\cancel{2}V_2) + \cancel{\sqrt{3}}V_2 = \cancel{\sqrt{3}}V \longrightarrow \boxed{2V_2 = V}$$

So, we have: $\boxed{V_2 = \tfrac{1}{2}V}$ and $V_1 = 2V_2 = 2(\tfrac{1}{2}V)$

$$\boxed{V_1 = V}$$

Question value 8 points

(1) A tow-rope pulls a skier of mass m up a hill at constant speed v, along a steep slope of height H and length d. On the other side of the hill, another tow-rope pulls a skier of the same mass m up a shallow slope of the same height H but length $2d$, at the same speed v. Compare the average power provided to the skiers by the tow ropes, during their ascents.



(a) $P_{steep} = \frac{1}{4}P_{shallow}$

(b) $P_{steep} = 4P_{shallow}$

[✓ (c) $P_{steep} = 2P_{shallow}$]

(d) $P_{steep} = P_{shallow}$

(e) $P_{steep} = \frac{1}{2}P_{shallow}$

Assume friction is negligible (skis on snow)

- constant speed implies $\Delta K = 0$
- letting "system" = skier + Earth, $E_{sys} = K + U_g$

$\Rightarrow$ Energy principle: $\Delta E_{sys} = W_{ext}$

$\Delta U_g = W_{Tow\text{-}rope}$

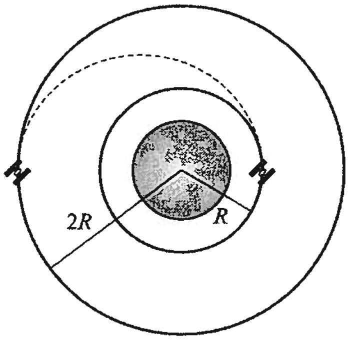
$\Rightarrow W_{TowRope} = \Delta U_g = \boxed{+mgH}$ in both cases!

but: if distance d at speed V requires time Δt_1,
then distance 2d at same speed requires time $\Delta t_2 = 2\Delta t_1$

$$\Rightarrow P_{steep} = \frac{W_{TR}}{\Delta t_1} = \frac{mgH}{\Delta t_1} \text{ and } P_{shallow} = \frac{W_{TR}}{\Delta t_2} = \frac{mgH}{2\Delta t_1} \rightarrow \boxed{P_{steep} = 2\,P_{shallow}}$$

Question value 8 points

(2) A ground-to-orbit shuttle places a communications satellite into a circular orbit of radius R. In that orbit the satellite has total mechanical energy E_0. A booster rocket attached to the satellite ignites, lifting it to a higher circulat orbit of radius $2R$. Which of the statements below best characterizes the energy changes within the system consisting of Earth + satellite



(a) Negative work is done by the booster, The potential energy of the system increases and the kinetic energy increases.

[✓ (b) Postive work is done by the booster, The potential energy of the system increases and the kinetic energy decreases.]

(c) Zero net work is done by the booster, The potential energy of the system decreases and the kinetic energy increases.

(d) Postive work is done by the booster, The potential energy of the system increases and the kinetic energy increases.

(e) Negative work is done by the booster, The potential energy of the system decreases and the kinetic energy increases.

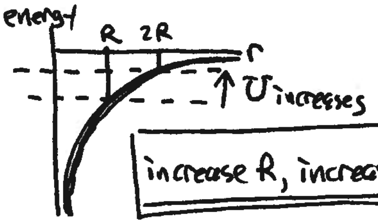
① $V_{orb} = \sqrt{\frac{GM}{R}}$

so $K_{orb} = \frac{1}{2}mV_{orb}^2$

$= \frac{GMm}{2R}$

$\boxed{\text{increase } R\text{, decrease } K_{orb}}$

② $U_g = -\frac{GMm}{R}$



$\boxed{\text{increase } R\text{, increase } U}$

(large neg → small neg is an increase)

③ $E_{sys} = K_{orb} + U = \frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{2R}$

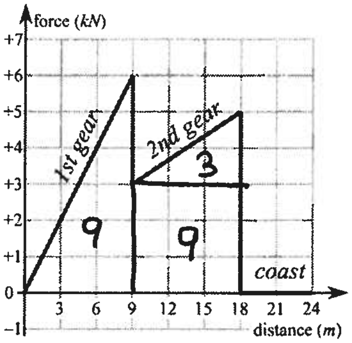
just as with U, increase R, increase E_{sys}

so ΔE_{sys} = positive

$\rightarrow \boxed{W_{booster} = \text{positive}}$

Question value 4 points

(3) Starting from rest, a 2500 kg automobile experiences a net propulsive force given by the graph at right. What will be the speed of the car when it starts to coast?



- (a) 25 m/s
- [✓ (b) 7.1 m/s]
- (c) 10 m/s
- (d) 5.0 m/s
- (e) 13 m/s

Force-vs-distance:

area gives work done by force

$\Rightarrow$ use energy principle

$$\Delta K = W_{Force}$$

$$\tfrac{1}{2}mv_f^2 - 0 = (\text{area under curve})$$

$$\tfrac{1}{2}mv_f^2 = 63{,}000\,J$$

Total area = 9+9+3 = 21 squares

each square has "height" $10^3 N$ and "width" 3m

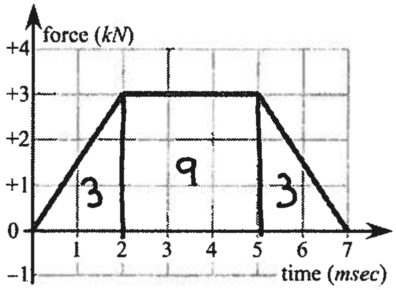
$\rightarrow$ 1 square = 3×10^3 Nm

= 3 kilojoules

$$\boxed{v_f = \sqrt{\frac{2W_{force}}{m}} = 7.099\,m/s}$$

Question value 4 points

(4) A 2.5 kg medicine ball strikes the ground travelling straight down at a speed of 3.5 m/s. The graph at right diplays the net vertical force acting on the ball while in contact with the ground. With what speed will the ball rebound upward?



- (a) 0 m/s (i.e. no rebound at all)
- (b) 3.5 m/s
- (c) 4.9 m/s
- [✓ (d) 2.5 m/s]
- (e) 0.5 m/s

Force-vs-time:

area gives impulse

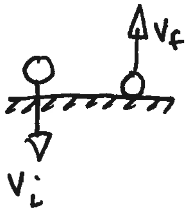
$\Rightarrow$ use impulse-momentum principle : $\vec{\Delta p} = \vec{J} = \int \vec{F}dt$ $= (\text{area})$

Total area = 3+9+3 = 15 squares

$\rightarrow$ each square has "height" $10^3 N$ and "width" 10^{-3} sec

$\rightarrow$ each square represents 1.0 N·s = 1.0 kgm/s

$\vec{J} = \langle +15\ kgm/s \rangle$



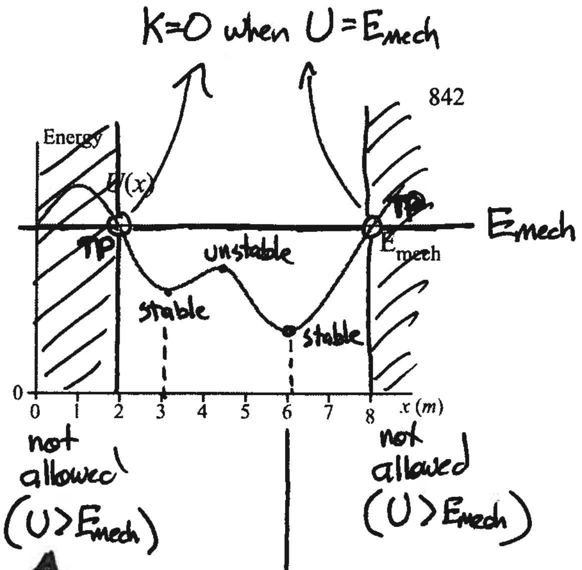
$$\vec{\Delta p} = m\vec{\Delta v} = m\langle +v_f\rangle - m\langle -v_i\rangle = \vec{J}$$

$$\text{so } v_f = \frac{J}{m} - v_i = (+6.0\,m/s) - (3.5\,m/s)$$

$$\boxed{v_f = 2.5\,m/s}$$

Question value 8 points

(5) A particle is subject to a conservative force, resuling in the potential energy curve at right. The total mechanical energy of the particle is indicated by the horizontal line. Which of the following statements about the graph is $\boxed{\textbf{not true?}}$



(a) $\boxed{\text{Starting from rest at } x = 2\text{m, the particle can move either to the right or left.}}$

(b) The particle's speed is maximum at $x = 6$m. true

(c) The particle is at rest when it is at $x = 2$m. true

(d) The particle has a turning point at $x = 8$m. true

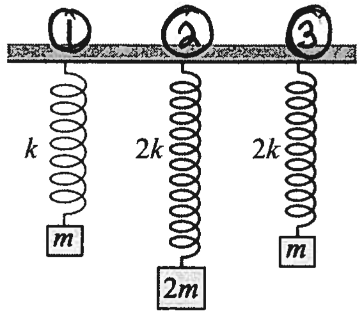
(e) There are two points of stable equilibrium for this potential energy function. true

If at rest at $x=2$m ($K=0$ and $U = E_{mech}$), moving to the left would mean U increases and hence that K must decrease ... but K can never be less than zero!

U=min implies K=max

Question value 8 points

(6) Three different blocks are suspended from the ceiling by three different springs. Each of the blocks is in equilibrium. Rank, from greatest to least, the stored potential energy in each spring. [Hint: do not assume the springs have the same unstretched lengths.]



(a) $U_3 > U_1 > U_2$

(b) $U_2 > U_3 > U_1$

(c) $\boxed{U_2 > U_1 > U_3}$

(d) $U_1 > U_3 = U_2$

(e) $U_3 = U_2 > U_1$

① Masses are in equilibrium → find stretch Δs for each spring:

$$|\vec{F}_s| = k|\vec{\Delta s}| = k_i d_i$$

$$|\vec{F}_g| = m_i g$$

Equilibrium: $k_i d_i = m_i g$

$$\boxed{d_i = \frac{m_i g}{k_i}}$$

② For each spring, stored PE is $U_i = \frac{1}{2}k_i d_i^2 = \frac{1}{2}k_i \frac{m_i^2 g^2}{k_i^2} = \boxed{\frac{m_i^2 g^2}{2k_i}}$

Comparing:

$$U_1 = \frac{m^2 g^2}{2k}$$

$$U_2 = \frac{(2m)^2 g^2}{2(2k)} = \frac{4m^2g^2}{4k} = 2\left(\frac{m^2g^2}{2k}\right) = 2U_1$$

$$U_3 = \frac{m^2g^2}{2(2k)} = \frac{m^2g^2}{4k} = \frac{1}{2}\left(\frac{m^2g^2}{2k}\right) = \frac{1}{2}U_1$$

→ $\boxed{U_2 > U_1 > U_3}$