Physics 2211ABC Test form 833 Name Solutions
Spring 2018
Test 3 Recitation Section (see cover page):_________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

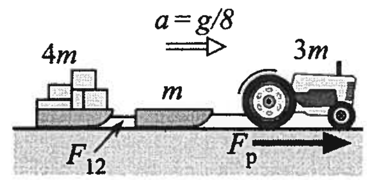
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

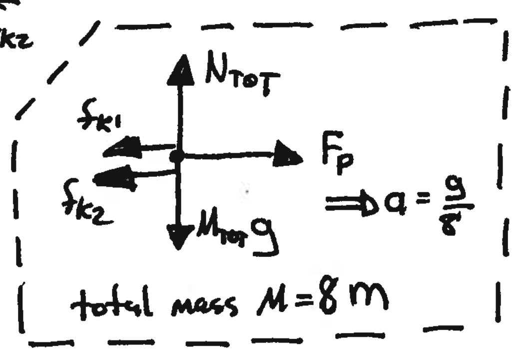
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

[I] *(20 points)* A tractor pulls two identical sleds along rough ground. The first sled is unloaded (mass m), while the second sled is piled with cargo (mass $4m$). The tractor has a mass $3m$ and its wheels provide a $\boxed{\text{net propulsive force } F_p = 2mg}$ resulting in an acceleration $a = g/8$ for all three objects. Determine the magnitude of the coupling force F_{12} between the two sleds. Express your answer as a multiple of mg.



*Hint: You have not been given a coefficient of friction, but there **is** a way to deduce the two friction forces on the two sleds.*

① Tractor provides "net propulsive force F_p" — we don't worry about friction, for tractor
Sleds will have kinetic friction forces $\vec{f}_{k1}$ and $\vec{f}_{k2}$



② Consider "tractor + both sleds" as a single system:

$\Sigma F_x = M_{TOT}\vec{a}_x$

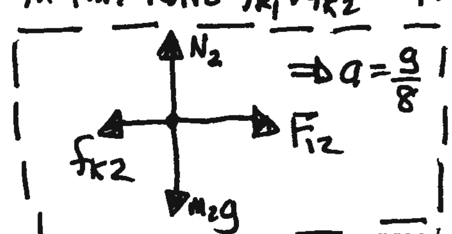
$\langle +F_p\rangle + \langle -f_{k1}\rangle + \langle -f_{k2}\rangle = (8m)\langle +\frac{g}{8}\rangle$

$2mg - f_{k1} - f_{k2} = mg$

$\Rightarrow \boxed{f_{k1} + f_{k2} = mg}$

③ Relative sled masses are $\Rightarrow m : 4m = 1:4$ so normal forces are in this same ratio $\Rightarrow$ so friction forces are in this ratio $f_{k1} : f_{k2} = 1:4$

or $\boxed{f_{k1} = \frac{1}{5}mg}$ and $\boxed{f_{k2} = \frac{4}{5}mg}$



④ Horizontal forces on sled 2: $\Sigma F_x = m_2\vec{a}_x$

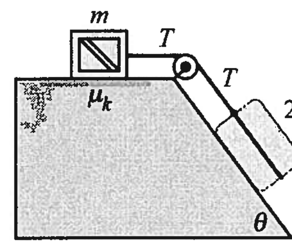
$\langle +F_{12}\rangle + \langle -f_{k2}\rangle = (4m)\langle +g/8\rangle = mg/2$

$F_{12} = f_{k2} + mg/2 = \frac{8}{10}mg + \frac{5}{10}mg$ so $\boxed{F_{12} = \frac{13}{10}mg}$

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

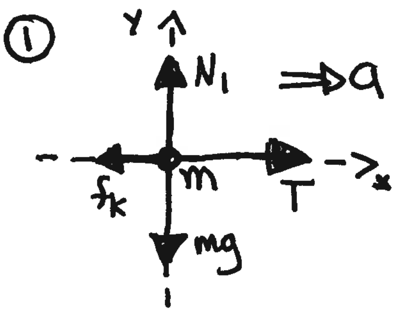
[II] *(20 points)* A crate of mass rests on a level surface. It is connected, via an ideal cord, to a large block of ice (mass $2m$) lying on a ramp that is inclined at an angle θ = 53.1° below the horizonal. The level surface is rough, with coefficient of kinetic friction μ_k = 0.25. There is negligible friction bewteen the iceblock and the ramp. When released the objects accelerate to the right/down the ramp.

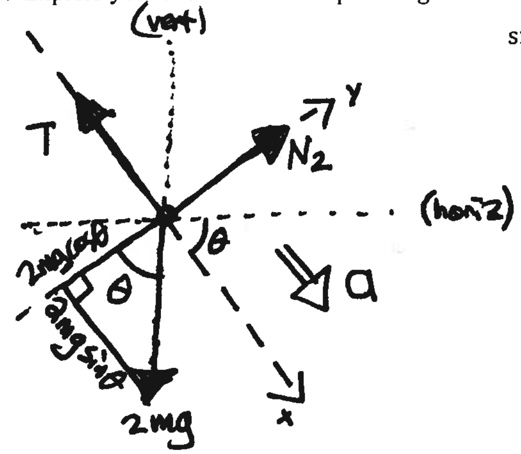
Draw free body diagrams for both objects, and use the 2nd and 3rd laws to determine the tension T in the cord. Express your answer as a multiple of mg.



$\sin(53.1°) = \frac{4}{5}$ $\cos(53.1°) = \frac{3}{5}$

①





• ideal cord has zero mass: T = the same in all parts of cord
• ideal cord does not stretch: both objects have the same "a"

② Block m: $\Sigma \vec{F}_y = m\vec{a}_y = 0 \rightarrow \langle +N_1 \rangle + \langle -mg \rangle = 0 \rightarrow N_1 = mg$ so $f_k = \mu_k N_1$ $\boxed{f_k = \mu_k mg}$

$\Sigma \vec{F}_x = m\vec{a}_x \rightarrow \langle +T \rangle + \langle -f_k \rangle = m\langle +a \rangle$

$\boxed{A}$ $\boxed{T - \mu_k mg = ma}$

③ Block 2m: with x-axis <u>along</u> incline, we have:

$\Sigma \vec{F}_x = m\vec{a}_x \longrightarrow \langle +2mg\sin\theta \rangle + \langle -T \rangle = (2m)\langle +a \rangle$

$\boxed{B}$ $\boxed{2mg\sin\theta - T = 2ma}$

④ Equations $\boxed{A}$ and $\boxed{B}$ give us two equations in unknowns T, A

→ to find T, substitute $\boxed{A}$ into $\boxed{B}$:

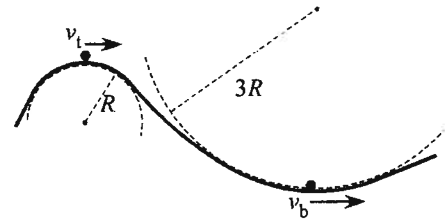
$$2mg\sin\theta - T = 2(ma) = 2\left[T - \mu_k mg\right]$$

$$\Rightarrow 3T = 2\mu_k mg + 2mg\sin\theta$$

$$T = \frac{2}{3}mg(\mu_k + \sin\theta) = mg\left[\frac{2}{3}\left(\frac{1}{4} + \frac{4}{5}\right)\right]$$

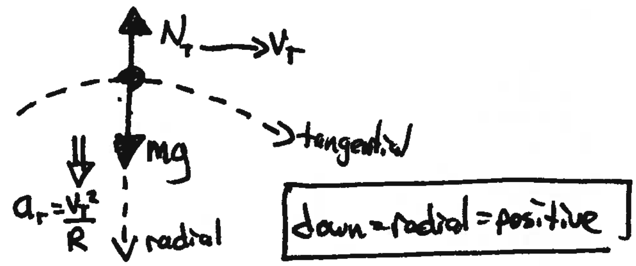
$$= mg \cdot \frac{2}{3}\left(\frac{5+16}{20}\right) = \boxed{\frac{17}{10}mg}$$

The following problem will be hand-graded. <u>*Show all your work for this problem*</u>*. Make no marks and leave no space on your answer card for it.*



[III] *(20 points)* A roller-coaster passes over the top of a steep hill and then immediately down through a shallow dip. The hill has a radius of curvature R at its highest point, while the dip has a radius of curvature $3R$ at its lowest point. The car passes over the hill with a speed v_t such that passengers just *barely* lose contact with their seats at the very top. The car then passes through the dip, reaching a speed $v_b = \frac{3}{2}v_t$ at the very bottom of the dip.

What will be the apparent weight of a passenger, as the car passes through the bottom of the dip? Express your answer as a multiple of the true gravitational force, mg, acting on the passenger.



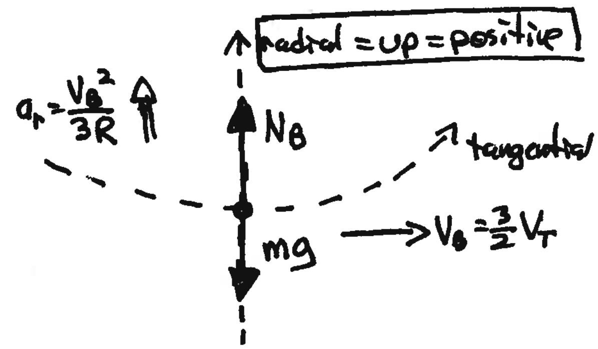
① Circular motion at top : $\Sigma \vec{F}_r = m\vec{a}_r$

$$\langle -N_T \rangle + \langle +mg \rangle = m\left\langle +\frac{V_T^2}{R} \right\rangle$$

"barely lose contact" $\Rightarrow$ $N_T \to 0$ at top

so $mg = mV_T^2/R \longrightarrow \boxed{V_T^2 = gR}$

② speed at bottom: $V_B = \frac{3}{2}V_T$ so $V_B^2 = \left(\frac{9}{4}\right)V_T^2 \Rightarrow \boxed{V_B^2 = \frac{9}{4}gR}$



③ Circular motion at bottom : $\Sigma \vec{F}_r = m\vec{a}_r$

$$\langle +N_B \rangle + \langle -mg \rangle = m\left\langle +\frac{V_B^2}{3R} \right\rangle$$

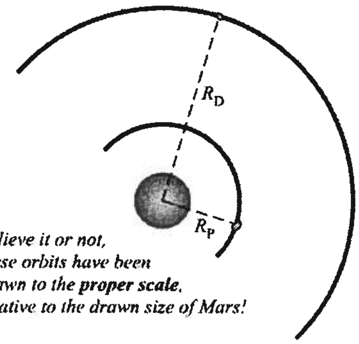
$$N_B - mg = m\,\frac{\frac{9}{4}gR}{3R} = \frac{3}{4}mg$$

so $\boxed{N_B = \frac{7}{4}mg}$ $\Rightarrow$ normal force pushing up on passengers creates their perception of "weight"

so $\boxed{W_{app} = N_B = \frac{7}{4}mg}$

The next two questions involve the following situation:

You are one of the first human colonists on Mars, looking up at the moons Phobos and Deimos as they go through their orbital phases. Reviewing the moons' astronomical data, you see that their orbital radii are very nearly in the ratio $R_D = 2.5\,R_P$. Assume that their orbits are exactly circular.



*Believe it or not, these orbits have been drawn to the **proper scale**, relative to the drawn size of Mars!*

Question value 4 points

(1) What are the relative speeds of Phobos and Deimos in their orbits?

(a) $v_D = 1.5\,v_P$

(b) $v_D = 0.40\,v_P$

(c) $\boxed{v_D = 0.63\,v_P}$

(d) $v_D = 2.5\,v_P$

(e) $v_D = 0.16\,v_P$

circular orbits: gravitational force provides radial acceleration

$$\sum \vec{F}_r = m\vec{a}_r$$

$$\left\langle +G\frac{Mm}{R^2}\right\rangle = m\left\langle +\frac{V_{orb}^2}{R}\right\rangle \rightarrow V_{orb}^2 = \frac{GM}{R} \rightarrow V_{orb} = \sqrt{\frac{GM}{R}}$$

so, compare D to P:

$$\frac{V_D}{V_P} = \frac{\sqrt{GM/R_D}}{\sqrt{GM/R_P}} = \sqrt{\frac{R_P}{R_D}} \qquad \boxed{V_D = \left(\sqrt{\frac{R_P}{R_D}}\right)V_P} = \sqrt{\frac{1}{2.5}}\,V_P = \sqrt{\frac{2}{5}}\,V_P$$

so $\boxed{V_D = 0.632\,V_P}$

Question value 4 points

(2) What are the relative periods of Phobos' and Deimos' orbits? (Recall that the *period* of an orbit is the time required to complete exactly one orbit.)

(a) $T_D = 2.5\,T_P$

(b) $\boxed{T_D = 4.0\,T_P}$

(c) $T_D = 1.6\,T_P$

(d) $T_D = 6.3\,T_P$

(e) $T_D = 1.4\,T_P$

To find period, note that orbital speed, orbital radius, and period are related by: $V_{orb} = \frac{2\pi R}{T}$

so $V_{orb}^2 = \frac{4\pi^2 R^2}{T^2}$ then, from above we have

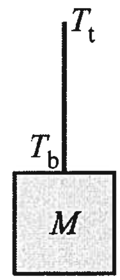
$$\frac{4\pi^2 R^2}{T^2} = \frac{GM}{R} \Rightarrow T^2 = \frac{4\pi^2}{GM}R^3$$

Comparing Deimos to Phobos:

$$\frac{T_D^2}{T_P^2} = \frac{\cancel{\frac{4\pi^2}{GM}}R_D^3}{\cancel{\frac{4\pi^2}{GM}}R_P^3} \rightarrow \boxed{T_D = T_P\left(\frac{R_D}{R_P}\right)^{1.5}} = T_P(3.95) \approx \boxed{4.0\,T_P}$$

Question value 8 points

(3) The mass M in the figure hangs in equilibrium from a rope. Let the tension at the top of the rope be T_t and the tension at the bottom be T_b. Under what condition will the two tension values be identical?



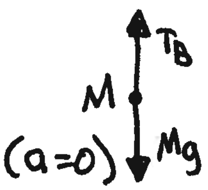
(a) $T_t = T_b$ always.

(b) $T_t = T_b$ only if the rope doesn't stretch.

(c) $T_t = T_b$ only if the rope is massless.

(d) $T_t = T_b$ only if the rope is massless *and* doesn't stretch.

(e) $T_t = T_b$ only if the rope is thin compared to its length.



block: Tension T_B supports weight of block

$\langle +T_B \rangle + \langle -Mg \rangle = 0$

cord: Tension T_T supports cord
→ by 3rd Law, block pulls down on cord with tension force T

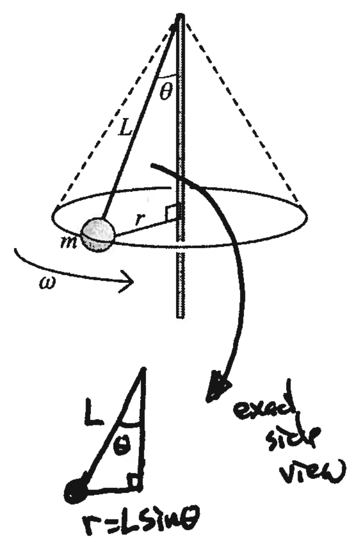
$\langle +T_T \rangle + \langle -T_B \rangle + \langle -mg \rangle = 0$

⇓

$\boxed{T_T = T_B \text{ only if } m=0 \text{ for cord}}$

Question value 8 points

(4) A tetherball consists of a small volleyball of mass m, attached to a vertical pole by a cord of fixed length L. When swung around in a circle of radius r with angular speed ω, the cord will trace out a cone with interior angle θ (shown). Which of the statements below best characterizes the relationship between the angular speed of the ball and the angle of the cone?



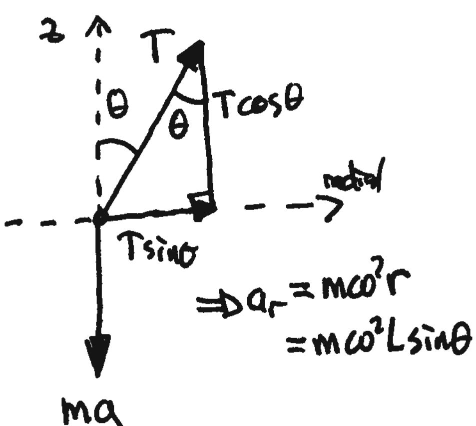
(a) The angular speed will be fixed by the values of m and L, while the cone angle can independently have any value we want.

(b) As the angular speed increases, the cone angle must get larger.

(c) As the angular speed increases, the cone angle must get smaller.

(d) The cone angle will be fixed by the values of m and L, while the angular speed can independently have any value we want.

(e) The angular speed and cone angle can independently have any value we want, with no mathematical relationship between them.



$\Rightarrow a_r = m\omega^2 r$
$= m\omega^2 L\sin\theta$

Free Body Diagram

① horizontal component of tension provides radial force

$$\sum F_r = ma_r \rightarrow T\sin\theta = m\omega^2 r = m\omega^2 L\sin\theta$$

$$T = m\omega^2 L$$

Conclusion: $\boxed{\text{large } \omega \text{ requires large tension}}$

② Vertical forces are in equilibrium:

$$\langle +T\cos\theta \rangle + \langle -Mg \rangle = 0$$

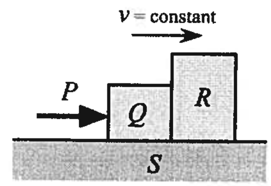
$$T\cos\theta = mg = \text{fixed}$$

$\boxed{\text{large T: small } \cos\theta \text{: large } \theta}$

$\boxed{\text{as } \omega \text{ increases, } \theta \text{ must increase}}$

Question value 8 points

(5) Blocks Q and R are placed side-by-side as shown, and an external push P causes them to move with constant speed along rough surface S as shown. Which pair of forces listed below constitutes a valid third-law pair?



mis-match

~~(a)~~ The normal force by Q forward on R, and the friction force by S backward on R.

~~(b)~~ The external push P forward on Q, and the friction force by surface S backward on Q.

mis-match

~~(c)~~ The normal force by surface S upward on R, and the gravitational force by the whole Earth downward on R.

mis-match

~~(d)~~ The friction force by S forward on Q, and the friction force by S backward on R. – Three objects involved

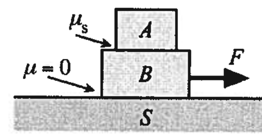
(e) The normal force by Q forward on R, and the normal force by R backward on Q.

3rd law requires
① ONE interaction type
② Two objects, both exerting forces on each other (same force type on each!)
③ Force vectors are opposite and equal

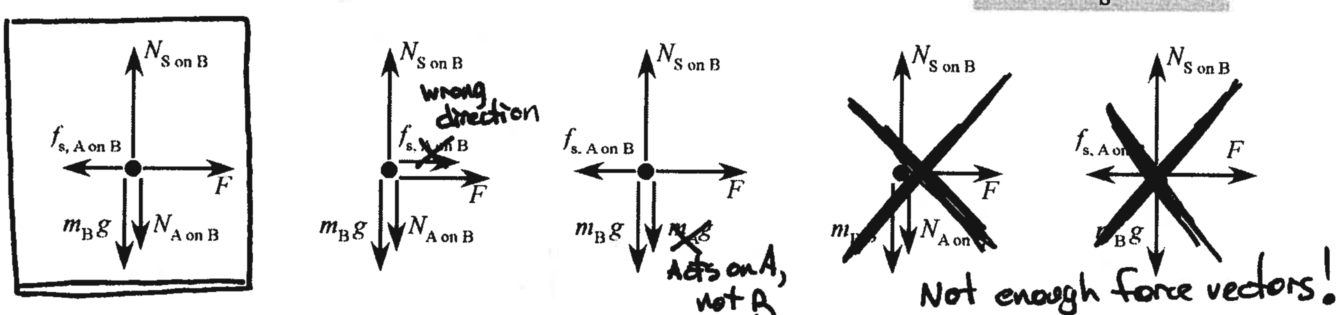
only answer "e" above violates none of these rules

Question value 8 points

(6) Block A rests on block B, which sits on frictionless surface S. Block B is pulled by a horizontal force F as shown. Which free body diagram below correctly depicts the forces acting on block b only?

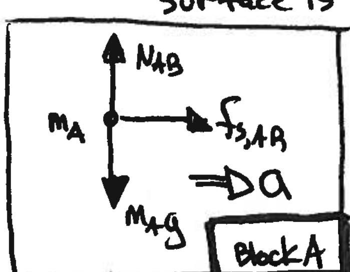


⇒ implicit accel



(a) (b) (c) (d) (e)

Surface is frictionless, so there will be acceleration

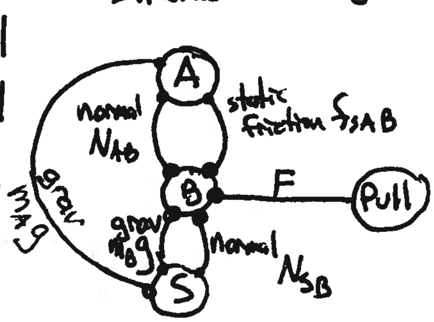


So, consider A: there must be a forward-directed force on A – only possibility is friction with B

so $f_{s,AB}$ = forward on A and backward on B

→ vertical: N_{AB} is up on A so N_{AB} is down on B

Interaction Diagram



We see Five interactions leading to forces on block B