Physics 2211ABC Spring 2018 **Test 2**

Test form **822**

Name Solutions

Recitation Section (see cover page):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

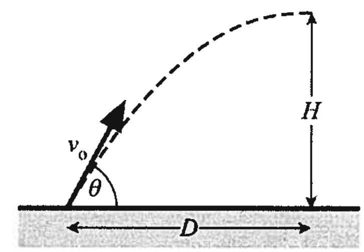
4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

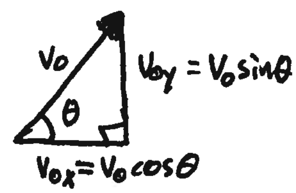
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

[I[*(20 points)* A cannon launches a projectile from ground level with speed v_0, at some angle θ above the horizontal. Find expressions for the maximum height H reached by the projectile, and also the horizontal distance D that the cannonball travels as it reaches that maximum height (see figure). In each case, your answer should involve only v_0, θ, and g.

Finally, calculate the ratio H / D. Express your answer in terms of θ **only**.



⓪ decompose initial velocity



① vertical motion: $\vec{a}_y = \langle -g \rangle = \text{constant}$

Max height occurs when $\vec{V}_y \rightarrow 0$

so $\Delta \vec{V}_y = \langle -g \rangle \Delta t$

$\langle 0 \rangle - \langle +V_0 \sin\theta \rangle = \langle -g \rangle \Delta t_{up}$

$\rightarrow \boxed{\Delta t_{up} = \frac{V_0 \sin\theta}{g}}$ (where Δt_{up} is the specific time required to reach maximum height)

then $\Delta \vec{y} = \vec{V}_{0y} \Delta t + \frac{1}{2} \langle -g \rangle \Delta t^2$

$\langle +H \rangle = \langle +V_0 \sin\theta \rangle \Delta t_{up} + \frac{1}{2} \langle -g \rangle \Delta t_{up}^2 = V_0 \sin\theta \left(\frac{V_0 \sin\theta}{g} \right) - \frac{1}{2} g \left(\frac{V_0 \sin\theta}{g} \right)^2$

$\rightarrow \boxed{H = \frac{V_0^2 \sin^2\theta}{2g}}$

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② horizontal motion: $\vec{V}_x = \text{constant}$

so $\Delta \vec{x} = \vec{V}_x \Delta t$

$\langle +D \rangle = \langle +V_0 \cos\theta \rangle \Delta t_{up}$

$D = V_0 \cos\theta \left(\frac{V_0 \sin\theta}{g} \right)$

$\boxed{D = \frac{V_0^2 \sin\theta \cos\theta}{g}}$

③ ratio $\frac{H}{D}$ is:

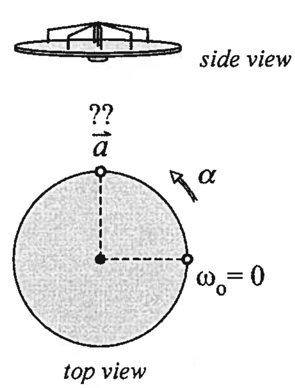
$$\frac{H}{D} = \frac{\frac{\cancel{V_0^2}\, \cancel{\sin\theta} \sin\theta}{2\cancel{g}}}{\frac{\cancel{V_0^2}\, \cancel{\sin\theta} \cos\theta}{\cancel{g}}} \Rightarrow \boxed{\frac{H}{D} = \frac{1}{2} \tan\theta}$$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* A playground merry-go-round of radius R is initially at rest. It is pushed by several children, giving it a constant *angular* acceleration α. What is the acceleration vector $\vec{a}$ (magnitude and direction) of a point on the rim of the merry-go-round, after it has rotated through **one-quarter** of a revolution?

Express the magnitude of $\vec{a}$ in terms of α and R. Express the direction of $\vec{a}$ as an angle in degrees, measured from the radially inward direction.

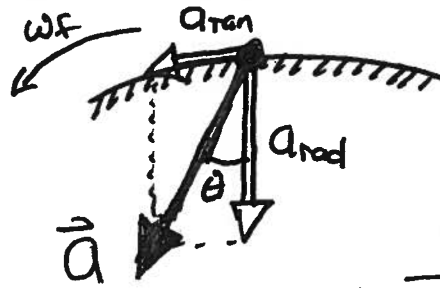
Hint: start by finding the angular speed of the merry-go-round after ¼ revolution.



(0) $\vec{a}$ = acceleration vector in 2D

→ it has two components: here, we will choose to decompose as radial and tangential components

[Why? because problem asks for direction angle relative to radial direction!]



(1) per hint, find angular speed after 1/4 revolution $\left(\Delta\theta = \frac{1}{4}(2\pi) = \frac{\pi}{2}\right)$, starting from rest $(\omega_i = 0)$, with constant angular acceleration α

⇒ "speed equation" for angular motion: $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

$$\omega_f^2 = 0 + 2\alpha\left(\frac{\pi}{2}\right)$$

$$\boxed{\omega_f^2 = \pi\alpha}$$

(2) find radial and tangential acceleration components

$\boxed{a_{Tan} = \alpha R}$, no further calculation needed

$a_{rad} = \omega_f^2 R$ → substitute for ω_f^2: $\boxed{a_{rad} = (\pi\alpha)R}$

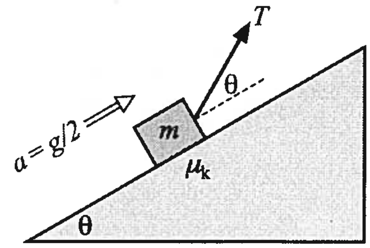
(3) find magnitude, direction angle for $\vec{a}$

$$|\vec{a}| = \sqrt{a_{tan}^2 + a_{rad}^2} = \boxed{\alpha R\sqrt{1+\pi^2} \approx 3.30\alpha R}$$

$$\theta = \tan^{-1}\left(\frac{a_{tan}}{a_{rad}}\right) = \tan^{-1}\left(\frac{\alpha R}{\pi\alpha R}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = \boxed{17.7^\circ \text{ from radial}}$$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* A block of mass m is being pulled up a rough inclined plane by a cable, as shown at right. The plane is inclined at an angle $\theta = 30°$ above the horizontal, and the cable is inclined at the same angle $\theta = 30°$ **above the incline itself.** The coefficient of kinetic friction between the block and the surface is $\mu_k = 1/4$. The block is observed to accelerate up the incline with $|\vec{a}| = ½\, g$ (where g is the acceleration due to gravity).



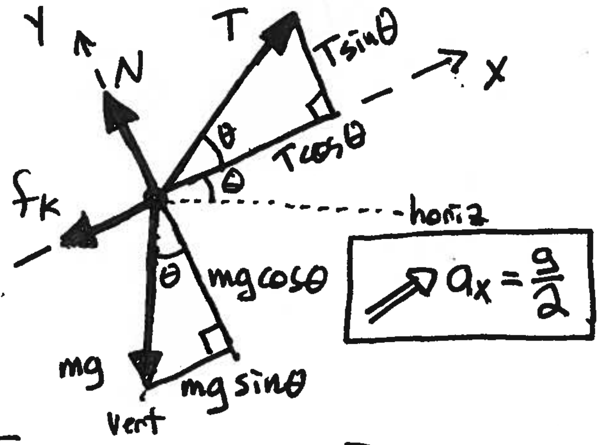
$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

(*i*) Draw a free body diagram for the block. The quality and clarity of your diagram will be graded as part of your work!

(*ii*) Determine the tension in the cable. Express your answer as a numerical multiple of the block's true weight, *mg*.



All labels above are <u>magnitudes</u> of the forces/components

- Kinetic friction: $f_K = \mu_K N$
- 2nd Law — <u>two</u> equations:

① $\sum \vec{F}_y = m\vec{a}_y = 0$

$\langle +N \rangle + \langle +T\sin\theta \rangle + \langle -mg\cos\theta \rangle = 0$

$\rightarrow \boxed{N + T\sin\theta = mg\cos\theta}$

② $\sum \vec{F}_x = m\vec{a}_x \neq$ zero!

$\langle +T\cos\theta \rangle + \langle -f_K \rangle + \langle -mg\sin\theta \rangle = m\langle +\frac{g}{2} \rangle$

$\boxed{T\cos\theta - \mu_K N = mg(\sin\theta + ½)}$

From the 2nd Law equations, we have <u>two</u> equations in unknown forces T and N:

① $N + T\sin\theta = mg\cos\theta$

② $-\mu_K N + T\cos\theta = mg(\sin\theta + ½)$

Solve by adding $\mu_K \cdot (1) + (2)$

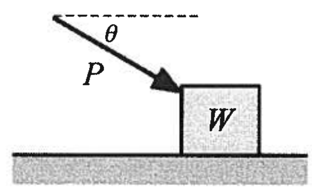
$\underbrace{\mu_K(N + T\sin\theta) - \mu_K N + T\cos\theta = \mu_K(mg\cos\theta) + mg(\sin\theta + ½)}$ — N drops out!

$T(\mu_K \sin\theta + \cos\theta) = mg[\mu_K\cos\theta + \sin\theta + ½]$

$$\boxed{T = mg\,\frac{\mu_K\cos\theta + \sin\theta + \frac{1}{2}}{\mu_K\sin\theta + \cos\theta}} = mg\,\frac{\frac{1}{4}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{2}}{\frac{1}{4}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}} = \boxed{1.23\,mg}$$

Question value 8 points

(1) In the figure at right, a block of weight $W = 25$ N is placed on a rough surface, and pushed with a force of magnitude $P = 15$ N, directed at an angle 30° below the horizontal. It is observed that the block <u>does not move</u>. What can we say about the coefficient of static friction?



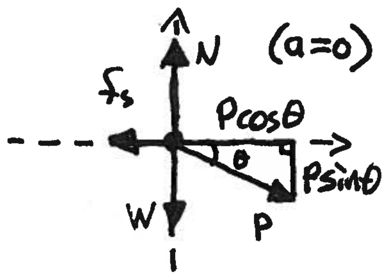
(a) μ_s must be $= 0.300$

(b) μ_s must be ≥ 0.600

(c) μ_s must be ≤ 0.400

(d) μ_s must be ≥ 0.400

(e) μ_s must be ≤ 0.300



(a=0)

① $\langle +N \rangle + \langle -P\sin\theta \rangle + \langle -W \rangle = 0$

$N = P\sin\theta + W$

$= 7.5N + 25N$

$\boxed{N = 32.5N}$

② $\langle +P\cos\theta \rangle + \langle -f_s \rangle = 0$

$f_s = P\cos\theta$

$\boxed{f_s = 13.0N}$

③ compare <u>actual</u> static friction to <u>upper limit</u> on friction

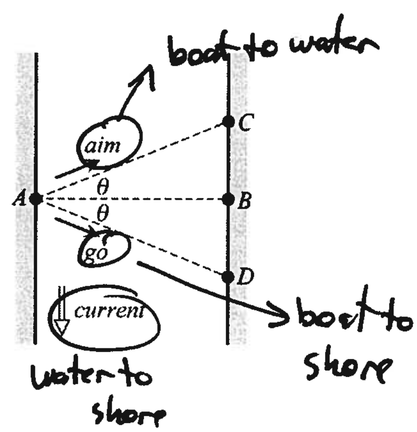
$f_s \leq \mu_s N$

$\mu_s \geq \frac{f_s}{N} = \frac{13.0N}{32.5N} = 0.3997$

so $\boxed{\mu_s \geq 0.400}$

Question value 8 points

(2) A kayaker can paddle with a sustained speed v through still water. She wishes to ferry across a river, directly to the other side (from A to B). She aims *upsteam* at an angle θ from straight across (toward C), but she finds that she actually ends up drifting *downstream* at the same angle θ from straight across (ending up on the far side at D). What is the speed of the river current, v_{water}?



(a) $v_{water} = 2\,v\sin\theta$

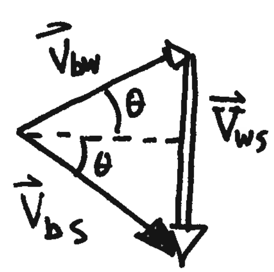
(b) $v_{water} = (v/2)\cos\theta$

(c) $v_{water} = v\sin\theta$

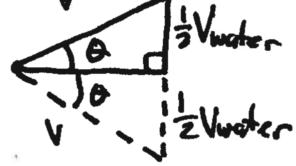
(d) $v_{water} = 2\,v\tan\theta$

(e) $v_{water} = v\cos\theta$

$\vec{V}_{boat\,to\,shore} = \vec{V}_{boat\,to\,water} + \vec{V}_{water\,to\,shore}$



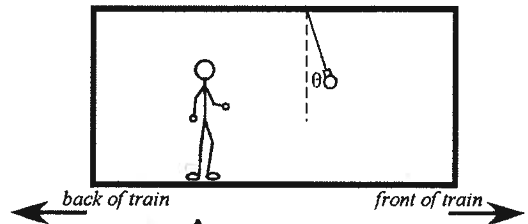
Note that geometry involves an isoceles triangle ($\vec{V}_{bw}$ and $\vec{V}_{ws}$ are mirror images)



$$\frac{opp}{hyp} = \sin\theta = \frac{\frac{1}{2}V_{water}}{V}$$

$$\boxed{V_{water} = 2V\sin\theta}$$

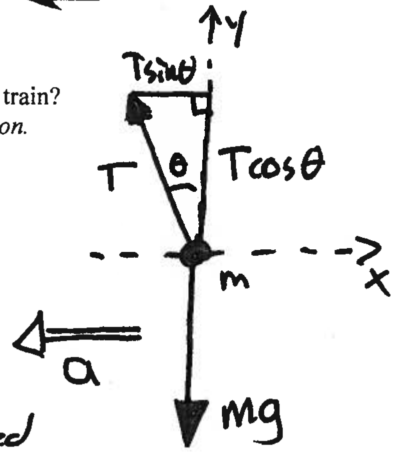
The next two questions involve the following situation:
You are on a train in a windowless boxcar, facing toward what you know to be the front of the train. A single lightbulb hangs from the ceiling. You observe that the cord does not hang vertically, but instead hangs tilted toward the front of the train, making an angle $\theta = 18°$ from the vertical.



Question value 4 points

(3) Which of the following statements might describe the motion of the train?
Hint: draw a free body diagram before you try to answer this question.

(a) The train is moving backward and losing speed.
(b) The train is moving backward at constant speed.
(c) The train is moving forward and gaining speed.
(d) The train is moving backward and gaining speed.
(e) The train is moving forward at constant speed.



From diagram, there is an unbalanced force toward rear of train → by 2nd Law, the bulb (and hence, train) must be accelerating to the rear

⇒ only one situation described above has this acceleration

Question value 4 points

(4) What is the magnitude of the train's acceleration?

(a) zero
(b) 9.32 m/s^2
(c) 3.18 m/s^2
(d) 3.03 m/s^2
(e) 6.17 m/s^2

Now, apply 2nd Law:

$$\sum \vec{F}_y = 0$$

$$\langle +T\cos\theta \rangle + \langle -mg \rangle = 0$$

$$T = \frac{mg}{\cos\theta}$$

$$\sum \vec{F}_x = m\vec{a}_x$$

$$\langle -T\sin\theta \rangle = m\langle -a \rangle$$

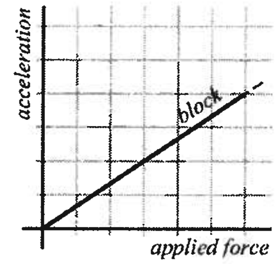
$$-\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = -\cancel{m}a$$

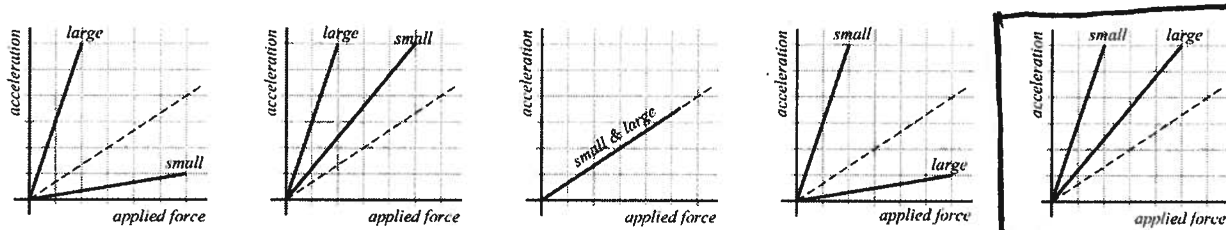
mass drops out!

$$\boxed{a = g\tan\theta = 3.18\ \mathrm{m/s^2}}$$

Question value 8 points

(5) A block is subjected to an applied force F of varying magnitude, and the magnitude of the block's resulting acceleration is plotted as a function of F, as shown at right. The block is then broken apart into two unequal fragments, large and small, and each fragment is *separately* subjected to the same series of force measurements that was used on the whole block. Which of the graphs below *best* represents the a–vs–F graphs for the two fragments?





(a) (b) (c) (d) (e)

Plot $|\vec{a}|$ as a function of $|\Sigma\vec{F}|$

$a = \frac{\Sigma F}{m}$ → "slope $= \frac{1}{m}$"

→ large mass means small slope
small mass means large slope

masses related by
$m_{small} < M_{large} < M_{TOTAL}$

so $\boxed{\text{slope(small)} > \text{slope(large)} > \text{slope(whole block)}}$

Question value 8 points

(6) A block is placed on the floor of a stationary elevator. When given a shove with initial speed v_o, it slides a distance D before stopping. With the same elevator is in motion, the same block is given a shove with the same initial speed v_o, and the block is observed to slide a distance $2D$ before stopping. Which of the following statements might be an accurate description of the elevator's motion?

(a) It can either be ascending or descending, but it must be moving with decreasing speed.

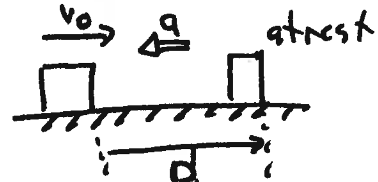
(b) It is descending at increasing speed.

(c) This situation is not possible; the block will always have the same stopping distance D, no matter how the elevator is moving.

(d) It is descending at constant speed.

(e) It is ascending at increasing speed.

1



Kinematics of "stopping distance"

$V_f^2 = V_i^2 + 2\vec{a}\vec{\Delta x}$

$0 = V_0^2 + 2(-a)(+d)$

$a = V_0^2/2d$

so: small stopping distance
= large stopping accel

and $\boxed{\text{large stopping distance = small stopping accel}}$

2 When elevator is moving:
D → larger
accel → smaller
friction force → smaller, by 2nd law
normal force → smaller ($f_k = \mu_k N$)

so, we must have $N < mg$
→ vertical motion must be:
$\boxed{\text{accelerating downward}}$