Physics 2211ABC Test form 811 Name Solutions

Spring 2018

Test 1 Recitation Section (see cover page):________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

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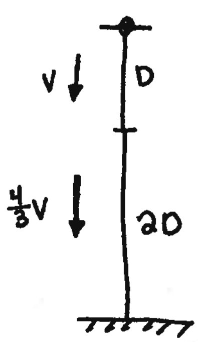
[1] *(20 points)* A skydiver is falling with her primary chute deployed. She falls a distance D at a constant speed v, when suddenly her primary chute fails. She immediately deploys her emergency chute, and falls the remaining distance to the ground, $2D$, at a speed $4/3\ v$. What is her average speed during the descent? Express your answer as a multiple of v. (You may assume that the emergency chute is deployed after a negligible time delay.)



$$\text{avg. speed} = \frac{\text{total distance}}{\text{total time}} = \frac{3D}{\Delta t_1 + \Delta t_2}$$

$\Rightarrow$ we need time elapsed for each portion of descent

$\rightarrow$ each is at constant velocity, so $\vec{\Delta y} = \vec{v}\Delta t$

$\longrightarrow \Delta t = \frac{|\vec{\Delta y}|}{|\vec{v}|}$ for each portion

so $\Delta t_1 = \frac{D}{v}$

and $\Delta t_2 = \frac{2D}{4/3\,v} = \frac{6}{4}\frac{D}{v} = \frac{3}{2}\frac{D}{v}$

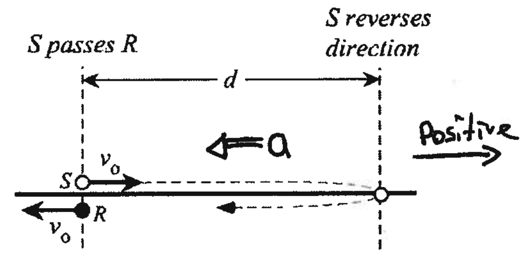
$\rightarrow$ total descent time is $\Delta t_1 + \Delta t_2 = \frac{D}{v} + \frac{3}{2}\frac{D}{v} = \boxed{\frac{5}{2}\frac{D}{v}}$

avg. speed is thus $\dfrac{3D}{\frac{5}{2}\frac{D}{v}} = 3\cancel{D} \cdot \frac{2v}{5\cancel{D}}$

$$\boxed{v_{av} = \frac{6}{5}v}$$

The following problem will be hand-graded. <u>*Show all your work for this problem*</u>*. Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* Spiff the Spaceman is in his saucer, traveling through space at speed v_0, when he suddenly passes his nemesis, Roger the Raider, who is traveling in the opposite direction in his rocketship, also at speed v_0. Spiff engages his inertial thruster, giving him a backward acceleration of constant magnitude a, in order to pursue Roger. Spiff travels a total distance d <u>in the wrong direction</u> (i.e. forward) while coming to a stop and reversing course, maintaining his acceleration as he starts to overtake Roger—who continues to travel at constant speed v_0.



How far will Spiff be from his starting point (where he originally passed Roger), when he finally overtakes Roger? Express your answer as a multiple of the stopping distance d.

Hint: Start by finding a relationship between v_0, a, and d.

Constant acceleration equations:

$$\boxed{\begin{aligned} \vec{\Delta V} &= \vec{a}\,\Delta t \\ \vec{\Delta x} &= \vec{V}_i \Delta t + \tfrac{1}{2}\vec{a}\,\Delta t^2 \\ V_f^2 &= V_i^2 + 2\vec{a}\,\vec{\Delta x} \end{aligned}}$$

Consider stopping process, and use 3rd equation at right:

$V_f = 0 \quad V_i = V_0$

$\vec{a} = \langle -a \rangle$ and $\vec{\Delta x} = \langle +d \rangle$

direction ← → magnitude

$$0 = V_0^2 + 2\langle -a\rangle\langle +d\rangle$$

$$\Rightarrow \boxed{a = V_0^2/2d}$$

Now, let D be the distance of <u>both</u> ships from starting point, when S catches R

- R has moved at constant velocity: $\vec{\Delta x}_R = \vec{V}_R \Delta t \rightarrow \boxed{\langle -D\rangle = \langle -V_0\rangle \Delta t_{catch}}$
- S has moved with constant acceleration: $\vec{\Delta X}_s = \vec{V}_{si}\,\Delta t + \tfrac{1}{2}\vec{a}_s \Delta t^2$

$$\rightarrow \boxed{\langle -D\rangle = \langle +V_0\rangle \Delta t_{catch} + \tfrac{1}{2}\left\langle -V_0^2/2d\right\rangle \Delta t_{catch}^2}$$

① solve for Δt_c

We have two equations in unknowns Δt_c and D: $\quad \Delta t_c = \dfrac{D}{V_0}$

② substitute into 2nd equation

now, solve for D!

$$\boxed{-D = \cancel{V_0}\left(\frac{D}{\cancel{V_0}}\right) - \frac{\cancel{V_0^2}}{4d}\left(\frac{D}{\cancel{V_0}}\right)^2}$$

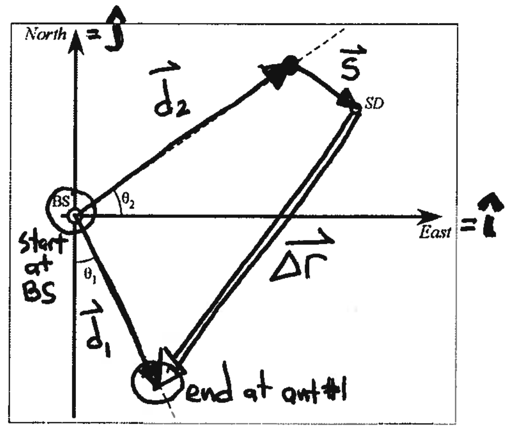
$$-D = +D - \frac{D^2}{4d}$$

$$-2\cancel{D} = -\frac{D^{\cancel{2}}}{4d} \rightarrow 8d = D \Rightarrow \boxed{D = 8d}$$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* You are a Lunar colonist. You set out from Burdell Station with a partner, to repair a pair of relay antennas. You decide to split up, so your partner travels a distance d_1 = 12.0 km in a direction θ_1 = 27.0° east of south to the 1st antenna, while you travel a distance d_2 = 15.0 km in a direction θ_2 = 35.0° north of east to the 2nd antenna. You perform your repair quickly, but your partner needs some spare fiberoptic cable. You travel to a supply depot that is 4.00 km due southeast of your location to pick up the cable. What displacement (magnitude and direction) will take you directly from the depot to your partner?

Hint: your grader will probably find it easier to credit your work if you solve symbolically, and save numerical calculations for the final steps.

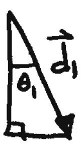


let $\vec{d}_1$ = partner's displacement to 1st antenna
$\vec{d}_2$ = your " " 2nd "
$\vec{s}$ = " " antenna to supply depot
$\vec{\Delta r}$ = " " depot to antenna #1

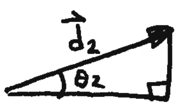
These displacements must satisfy
$\vec{d}_1 = \vec{d}_2 + \vec{s} + \vec{\Delta r}$
(see diagram above

so $\boxed{\vec{\Delta r} = \vec{d}_1 - \vec{d}_2 - \vec{s}}$
→ decompose each vector

here



$\vec{d}_1 = \langle +d_1 \sin\theta_1 \rangle \hat{i} + \langle -d_1 \cos\theta_1 \rangle \hat{j}$



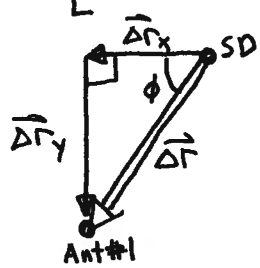
$\vec{d}_2 = \langle +d_2 \cos\theta_2 \rangle \hat{i} + \langle +d_2 \sin\theta_2 \rangle \hat{j}$

$\vec{s} = \langle +s \cos 45° \rangle \hat{i} + \langle -s \sin 45° \rangle \hat{j}$

$$\vec{\Delta r} = \vec{d}_1 - \vec{d}_2 - \vec{s} = \langle +d_1 \sin\theta_1 \rangle \hat{i} + \langle -d_1 \cos\theta_1 \rangle \hat{j} - \left[\langle +d_2 \cos\theta_2 \rangle \hat{i} + \langle +d_2 \sin\theta_2 \rangle \hat{j} \right] - \left[\langle +s \cos 45° \rangle \hat{i} + \langle -s \sin 45° \rangle \hat{j} \right]$$

$$\vec{\Delta r} = \left[+5.448\text{ km}\,\hat{i} - 10.692\text{ km}\,\hat{j} \right] - \left[+12.287\text{ km}\,\hat{i} + 8.604\text{ km}\,\hat{j} \right] - \left[+2.828\text{ km}\,\hat{i} - 2.828\text{ km}\,\hat{j} \right]$$

$$= \left[\langle -9.67\text{ km} \rangle \hat{i} + \langle -16.47\text{ km} \rangle \hat{j} \right]$$



distance $= \sqrt{(\Delta r_x)^2 + (\Delta r_y)^2} = \boxed{19.1\text{ km}}$

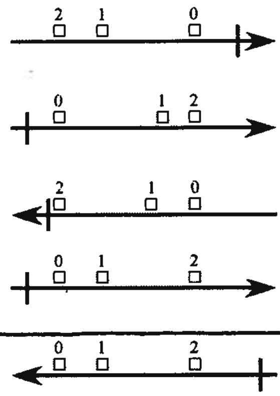
direction $= \tan^{-1}\left(\frac{|\Delta r_y|}{|\Delta r_x|}\right) = \boxed{59.6° \text{ S of W}}$

three-digit precision in final answer

Question value 8 points

(1) Each of the motion diagrams below displays three frames, numbered in sequential order 0→1→2. Each diagram also indicates a coordinate system, with a directional arrow denoting the positive direction and a crosshair indicating the location of the origin. Which of the diagrams displays a particle with positive position, negative velocity, and negative acceleration?

same sign implies: object must be slowing down



~~(a)~~ ⇒ all positions are negative (but object is slowing down)

~~(b)~~ ⇒ velocities are positive (but object is slowing down)

~~(c)~~ ⇒ positions are negative and velocities are positive (and object is speeding up)

~~(d)~~ ⇒ positions, velocities, and accelerations are all positive

(e)

→ left = positive; positions are left of origin so $\boxed{\text{positions are positive}}$

→ frames 0→1→2 are successively rightward so $\boxed{\text{velocities are negative}}$

→ object is speeding up, so $\vec{a}$ has same sign as $\vec{v}$

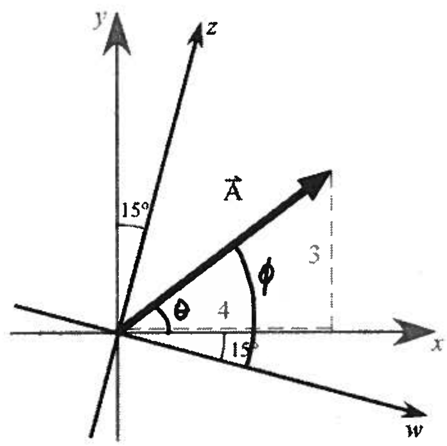
→ $\boxed{\text{acceleration is positive}}$

Question value 8 points

(2) In the diagram at right, vector $\vec{A}$ is shown decomposed into its x and y components:

$$\vec{A} = \langle x, y\rangle = \langle 4.00\,, 3.00\rangle$$

The diagram also shows a tilted "w–z coordinate system", that has been rotated 15.0° clockwise from the x–y coordinate system. What are the components of vector $\vec{A}$ in the w–z system?



(a) $\langle w, z\rangle = \langle 3.54\,, 3.54\rangle$

(b) $\langle w, z\rangle = \langle 1.86\,, 4.64\rangle$

(c) $\langle w, z\rangle = \langle 3.93\,, 3.09\rangle$

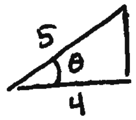
(d) $\langle w, z\rangle = \langle 4.64\,, 1.86\rangle$

(e) $\langle w, z\rangle = \langle 3.09\,, 3.93\rangle$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{16+9} = \sqrt{25}$$

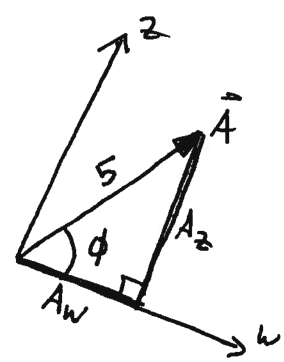
$$\boxed{|\vec{A}| = 5}$$

① 3-4-5 triangle in xy coords



$\sin\theta = \frac{3}{5} \rightarrow \theta = 36.9°$

② Angle of $\vec{A}$ relative to wz coords is $\phi = \theta + 15° = 51.9°$



③ Components of $\vec{A}$ in wz coords are

$$\boxed{\begin{aligned} A_w &= A\cos\phi = 3.085 \\ A_z &= A\sin\phi = 3.935 \end{aligned}}$$

Question value 8 points

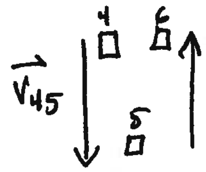
(3) A circus acrobat steps off a platform and drops straight down onto a trampoline, and then bounces straight back up to her original position. At the moment when she is at her lowest point of her "bounce" (i.e. with the maximum sag in the trampoline), the acrobat's acceleration is necessarily…

[Hint: draw a motion diagram for the acrobat!]

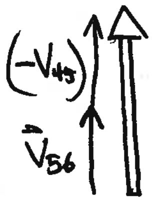
(a) zero.
~~(b)~~ positive.
~~(c)~~ negative.
$\boxed{\text{(d) upward.}}$
(e) downward.

no coord system was given, so we can't give signs to any vector!

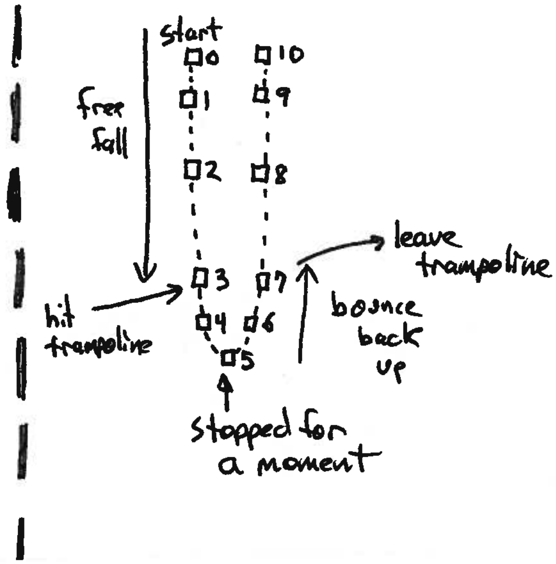
To find accel at frame 5, consider velocities before and after ($\vec{V}_{45}$ + $\vec{V}_{56}$)



$$\Delta\vec{V} = \vec{V}_{56} - \vec{V}_{45} = \vec{V}_{56} + (-\vec{V}_{45})$$



$\Delta\vec{V}$ is upward so $\boxed{\vec{a} = \frac{\Delta\vec{V}}{\Delta t} \text{ is upward}}$



Question value 8 points

(4) A stone is dropped from the roof of a very tall building, and then, two seconds later, a 2nd stone is also dropped. If we neglect air resistance, which of the following statements is most nearly correct.? *Hint: Drawside-by-side motion diagrams for both stones and label the frames of each diagram; 0-1-2-3… for the first stone and 2-3-4-5… for the second!*

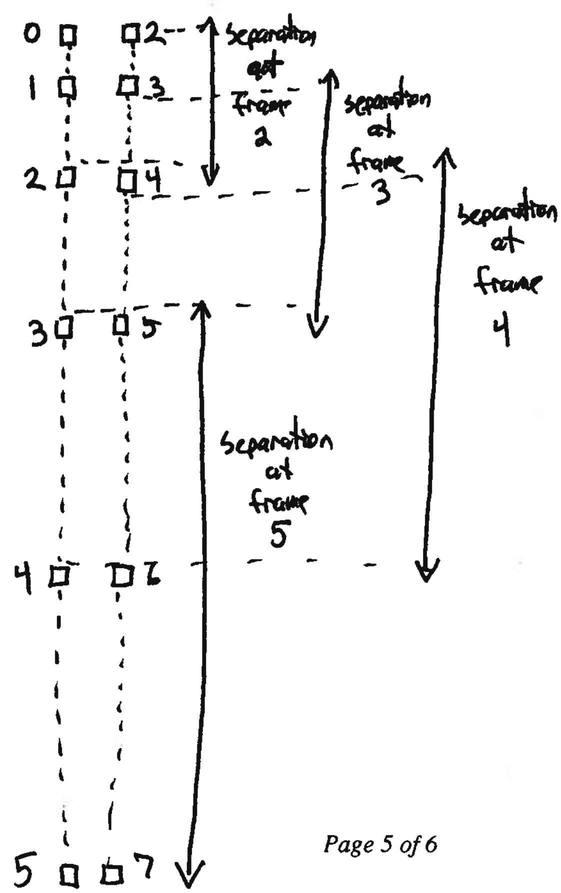
~~(a)~~ The stone that was dropped first will have a ~~greater acceleration~~ magnitude at all points in time. both have same accel

$\boxed{\text{(b) Once both stones are falling, the separation distance between them will constantly increase.}}$

~~(c)~~ Once both stones are falling, the separation distance between them will constantly de~~crease~~. implies #2 could catch up to #1

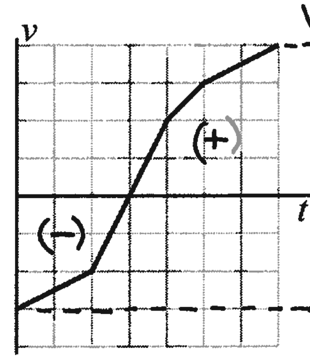
~~(d)~~ The stone that was dropped second will eventually catch up to the first stone…before either one hits the ground.

(e) Once both stones are falling, the separation distance between them will re~~main constant~~.



The next two questions involve the following situation:
The graph at right depicts the velocity of a toy car as it moves in a straight line. Each gridline along the horizontal axis corresponds to one second of elapsed time, and each gridline along the vertical axis corresponds to 4 m/s of speed. The car is observed to be at position $\vec{x}_i = \langle -4.0 \text{ m} \rangle$ at time $t = 0$.

$\vec{V}_f = \langle +16 \text{ m/s} \rangle$



$\vec{V}_i = \langle -12 \text{ m/s} \rangle$

Question value 4 points

(5) Where will the car be located at the very end of the time interval displayed in the graph?

(a) $\vec{x}_f = \langle +6.0 \text{ m} \rangle$

(b) $\vec{x}_f = \langle +18.0 \text{ m} \rangle$

(c) $\vec{x}_f = \langle +10.0 \text{ m} \rangle$

$\boxed{\text{(d) } \vec{x}_f = \langle +14.0 \text{ m} \rangle}$

(e) $\vec{x}_f = \langle +28.0 \text{ m} \rangle$

displacement = area under curve
[areas below v=0 are negative]

$\vec{\Delta X} = (-6 \text{ squares}) + (+10.5 \text{ squares})$
$= (+4.5 \text{ squares})$

→ each square has area = height × width
$= (4 \text{ m/s}) \times (1 \text{ s}) = 4 \text{ m}$

so: $\vec{\Delta X} = \langle +18 \text{ m} \rangle \rightarrow \vec{X}_f = \vec{X}_i + \vec{\Delta X} = \langle -4.0 \text{ m} \rangle + \langle +18 \text{ m} \rangle$

$\boxed{\vec{X}_f = \langle +14 \text{ m} \rangle}$

Question value 4 points

(6) What is the average acceleration of the car, during the full time interval displayed in the graph?

(a) $\vec{a} = \langle -3.0 \text{ m/s}^2 \rangle$

(b) $\vec{a} = \langle +5.0 \text{ m/s}^2 \rangle$

$\boxed{\text{(c) } \vec{a} = \langle +4.0 \text{ m/s}^2 \rangle}$

(d) $\vec{a} = \langle 0 \rangle$

(e) $\vec{a} = \langle +7.0 \text{ m/s}^2 \rangle$

$$\vec{a} = \frac{\vec{\Delta V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t}$$

→ from graph, $\vec{V}_f = \langle +16 \text{ m/s} \rangle$
$\vec{V}_i = \langle -12 \text{ m/s} \rangle$

→ $\vec{\Delta V} = \langle +16 \text{ m/s} \rangle - \langle -12 \text{ m/s} \rangle$
$= \langle +28 \text{ m/s} \rangle$

→ from graph, $\Delta t = 7$ sec

so $\vec{a} = \dfrac{\langle +28 \text{ m/s} \rangle}{7 \text{ sec}} = \boxed{\langle +4 \text{ m/s}^2 \rangle}$