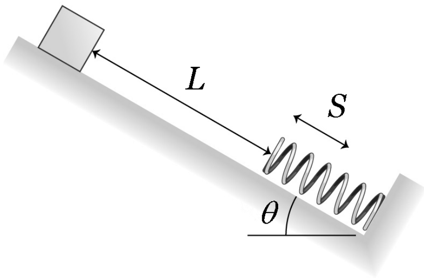

I. (16 points) A block of mass m slides down a plane that makes an angle θ with the horizontal. The block was initially at rest, and there is friction between the block and the plane. It slides a distance L where it contacts a spring with spring constant k. It continues sliding, compressing the spring to a distance S where it momentarily comes to rest. What is the coefficient of kinetic friction between the block and the plane? Express your answer in terms of parameters defined in the problem and physical or mathematical constants. (*On Earth.*)



. . . .

Use the Energy Principle.

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}}$$

Choosing the system to be the Earth, block, spring, and surface, no external forces do work on the system. Friction changes the thermal energy of the system. There are both gravitational and elastic potential energy changes in that system. The block begins and ends at rest, so there is no kinetic energy change between the initial and final states.

$$0 = 0 + \left(mgh_{\text{f}} - mgh_{\text{i}}\right) + \left(\tfrac{1}{2}k\left(\Delta s_{\text{f}}\right)^2 - \tfrac{1}{2}k\left(\Delta s_{\text{i}}\right)^2\right) + fd$$

where f is the force magnitude of friction on the block, d is the total displacement magnitude of the block, h_{i} and h_{f} and the initial and final heights of the block, and Δs_{i} and Δs_{f} and the initial and final compressions of the spring.

The frictional force on the block is proportional to the normal force, $f = \mu_{\text{k}} n$, where μ_{k} is the coefficient of kinetic friction and n is the normal force. The initial compression of the spring is zero, and the final compression is S. The final height of the block may be chosen to be zero. With that choice, $d = L + S$, and $h_{\text{i}} = (L+S)\sin\theta$.

$$0 = (0 - mg\,(L+S)\sin\theta) + \left(\tfrac{1}{2}kS)^2 - 0\right) + \mu_{\text{k}} n\,(L+S)$$

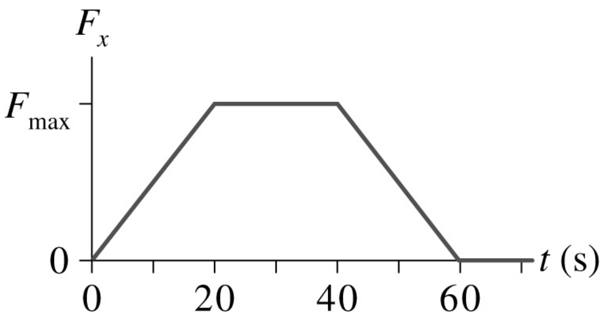
Using Newton's Second Law and sketching a Free-Body Diagram will show that $n = mg\cos\theta$, so

$$0 = -mg\,(L+S)\sin\theta + \tfrac{1}{2}kS^2 + \mu_{\text{k}} mg\,(L+S)\cos\theta$$

Solving for the coefficient of kinetic friction

$$\mu_{\text{k}} = \frac{mg\,(L+S)\sin\theta - \frac{1}{2}kS^2}{mg\,(L+S)\cos\theta} = \tan\theta - \frac{kS^2}{2mg\,(L+S)\cos\theta}$$

1. (6 points) Far out in space, where gravity is negligible, a 550 kg rocket traveling at 120 m/s in the $+x$ direction fires its engines. The figure shows the thrust force as a function of time, with a maximum thrust magnitude of $F_{\text{max}} = 960\,\text{N}$. At what time does the rocket reach its maximum speed?



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As long as there is force in the positive direction, the spaceship will accelerate in the positive direction. Since the velocity is in the positive direction the spaceship will continue to speed up until there is no force in the positive direction. The spaceship meets its maximum speed at ...

$t = 60\,\text{s}$

II. (16 points) In the problem above, what is the maximum speed of the rocket during these 60 s? (The mass lost by the rocket is negligible.)

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The impulse on the rocket is the change in its momentum.

$$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p} = m\,\Delta\vec{v}$$

Graphically, $\int F_x\,dt$ is represented by the area under a force *vs.* time graph, such as that in the illustration. That area (and thus the impulse) is 38400 N·s.

Letting the initial and final x-components of the rocket's velocity be v_{xf} and v_{xi}

$$\Delta v_x = \frac{J_x}{m} = v_{xf} - v_{xi}$$

and solving for the final velocity of the rocket

$$v_{xf} = v_{xi} + \frac{J_x}{m} = 120\,\text{m/s} + \left(\frac{38400\,\text{N}\cdot\text{s}}{550\,\text{kg}}\right) = 190\,\text{m/s}$$

2. (6 points) A block of mass $2m$ has a massless spring with spring constant k attached to its front, parallel to the ground. This block slides across a frictionless horizontal surface at speed v_i toward a stationary block of mass m. How does the speed of the block with mass $2m$ on the left, v_L, compare to that of the block with mass m on the right, v_R, at the instant of maximum compression?

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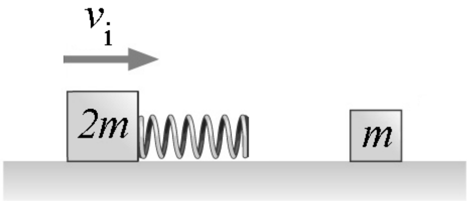
If the velocities were different, the separation would already be increasing or decreasing, which would indicate that compression was not maximum.

$$v_\mathrm{L} = v_\mathrm{R}$$



III. (16 points) In the problem above, what is the maximum compression of the spring during the collision? Express your result in terms of any or all of m, k, v_i, and physical or mathematical constants. (*On Earth.*)

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Let the two blocks and the spring be the system. There are no net external forces on the system, so linear momentum is conserved. Letting "L" represent the block on the left, and "R" represent the block on the right, and dropping the vector symbolism as the collision takes place in one dimension,

$$p_i = p_f \qquad \Rightarrow \qquad m_\mathrm{L} v_{\mathrm{L}i} + m_\mathrm{R} v_{\mathrm{R}i} = m_\mathrm{L} v_{\mathrm{L}f} + m_\mathrm{R} v_{\mathrm{R}f}$$

$v_{\mathrm{L}i}$ is given to be v_i. Since the block on the right is initially stationary, $v_{\mathrm{R}i} = 0$. At the moment of maximum compression the velocities of the two blocks must be the same, $v_{\mathrm{L}f} = v_{\mathrm{R}f} = v_f$. Substituting $m_\mathrm{L} = 2m$, $m_\mathrm{R} = m$,

$$m_\mathrm{L} v_{\mathrm{L}i} = (m_\mathrm{L} + m_\mathrm{R})\, v_f \qquad \Rightarrow \qquad 2mv_i = (2m + m)\, v_f \qquad \Rightarrow \qquad v_f = \frac{2mv_i}{2m+m} = \tfrac{2}{3} v_i$$

Next, use the Energy Principle. There are no thermal energy changes, and the external forces of gravity and the normal force do no work as they are perpendicular to the displacements. Both blocks have kinetic energy, and the only potential energy change is due to the spring, $U_\mathrm{s} = \frac{1}{2}k\,(\Delta s)^2$.

$$W_\mathrm{ext} = \Delta K + \Delta U + \Delta E_\mathrm{th} \qquad \Rightarrow \qquad 0 = \left(\tfrac{1}{2} m_\mathrm{L} v_{\mathrm{L}f}^2 - \tfrac{1}{2} m_\mathrm{L} v_{\mathrm{L}i}^2\right)$$
$$+ \left(\tfrac{1}{2} m_\mathrm{R} v_{\mathrm{R}f}^2 - \tfrac{1}{2} m_\mathrm{R} v_{\mathrm{R}i}^2\right) + \left(\tfrac{1}{2}k\,(\Delta s_f)^2 - \tfrac{1}{2}k\,(\Delta s_i)^2\right) + 0$$

The spring is initially relaxed, so $\Delta s_i = 0$. The the block on the right is initially stationary, so $v_{\mathrm{R}i} = 0$. Again, at the moment of maximum compression the velocities of the two blocks must be the same, $v_{\mathrm{L}f} = v_{\mathrm{R}f} = \frac{2}{3} v_i$. Substituting $m_\mathrm{L} = 2m$, $m_\mathrm{R} = m$,

$$0 = \left(\tfrac{1}{2} 2m \left(\tfrac{2}{3} v_i\right)^2 - \tfrac{1}{2} 2m v_i^2\right) + \left(\tfrac{1}{2} m \left(\tfrac{2}{3} v_i\right)^2\right) + \left(\tfrac{1}{2} k\,(\Delta s_f)^2\right)$$

Solving for Δs_f,

$$0 = \tfrac{4}{9} m v_i^2 - m v_i^2 + \tfrac{2}{9} m v_i^2 + \tfrac{1}{2} k\,(\Delta s_f)^2 = \left(\tfrac{6}{9} - 1\right) m v_i^2 + \tfrac{1}{2} k\,(\Delta s_f)^2 = -\tfrac{3}{9} m v_i^2 + \tfrac{1}{2} k\,(\Delta s_f)^2$$

So

$$\tfrac{1}{3} m v_i^2 = \tfrac{1}{2} k\,(\Delta s_f)^2 \qquad \Rightarrow \qquad (\Delta s_f)^2 = \tfrac{2m}{3k} v_i^2 \qquad \Rightarrow \qquad \Delta s_f = v_i \sqrt{\frac{2m}{3k}}$$

3. (8 points) A particle in a system moves along the x axis under the influence of a conservative force within the system. The graph shows the potential energy of the system as a function of the position of the particle. At what values of x does the force have the greatest positive value?

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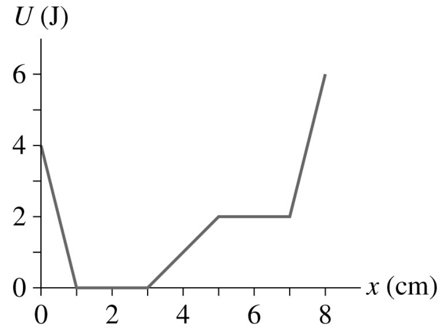
Between $x = 0\,\text{cm}$ and $x = 1\,\text{cm}$. Since $F = -dU/ds$, the greatest positive force occurs where there is the greatest magnitude negative slope of the graph.



4. (8 points) A system with total energy E has a potential energy PE that depends on the position, x, of a particle within it. At a particular instant, the particle is at $x = 0\,\text{m}$ and moving to the right. What are the particle's turning points in the range shown by the graph?

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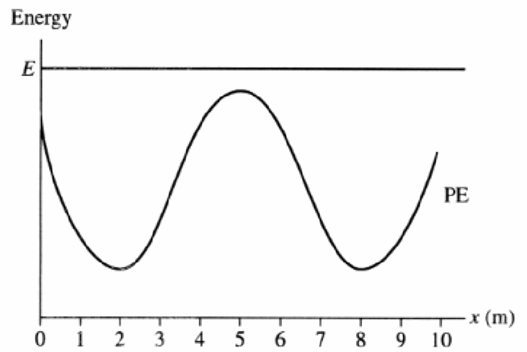
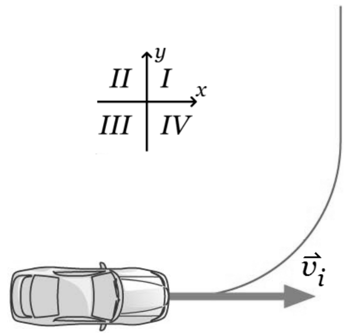
The particle has no turning points in the range $0 \leq x \leq 10\,\text{m}$. Turning points occur where the kinetic energy is zero, that is, when the potential energy equals the total energy. There is no such point on the graph.



5. (8 points) The automobile in the top-down illustration is traveling in the $+x$ direction with velocity $\vec{v}_i$. It then turns to travel in the $+y$ direction *at constant speed*. What direction, if any, is the impulse on the car for this process?



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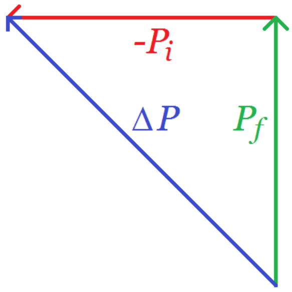
Remember that the impulse on an object is the change in momentum of the object

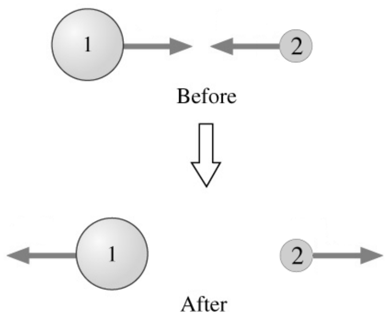
$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

This relationship is illustrated to the right. Note that Δp is directed



Somewhere in quadrant II.

6. (8 points) Object 1, with mass m_1, is travelling to the right with 70 J of kinetic energy. Object 2, with mass m_2, is travelling to the left with 30 J of kinetic energy. They undergo a perfectly elastic collision, after which object 1 is travelling to the left with 16 J of kinetic energy and object 2 is travelling to the right. If $m_1 > m_2$, what is the kinetic energy of object 2 after the collision?



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Since the collision is elastic, kinetic energy is conserved. Before the collision, there is a total of $70\,\mathrm{J} + 30\,\mathrm{J} = 100\,\mathrm{J}$ in the two-object system. Afterward, object 1 has 16 J, so object 2 must have 84 J. Kinetic energy is a non-negative scalar, so the directions are irrelevant.

$+84\,\mathrm{J}$

7. (8 points) Taking the zero point of gravitational potential energy to be at infinite separation, under what conditions will a space probe eventually escape the Earth's gravity?

   It will escape the Earth's gravity if . . .

   . . . .

   To escape from the Earth's gravity, the probe must have at least enough kinetic energy achieve infinite separation. At that point, the gravitational potential energy of the Earth/probe system is zero, and the probe's kinetic energy must be at least zero. So, it will escape the Earth's gravity if . . .

   the mechanical energy of the Earth/probe system is greater than or equal to zero.