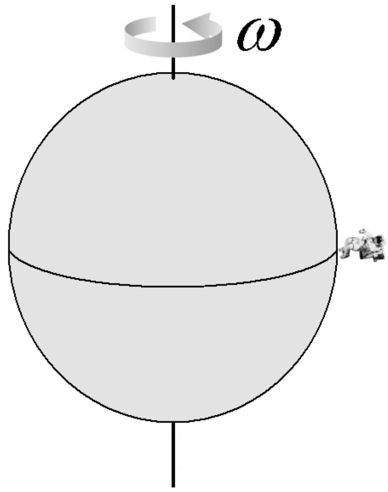
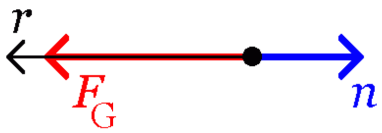
I. (16 points) Astronaut Arlene stands at the equator of the (fictional) planet Planteen and measures her apparent weight to be 220 N. Arlene knows her own mass to be 82 kg, while Planteen has a mass of 3.0×10^{23} kg and a radius of 2.5×10^{6} m. With what angular speed does Planteen rotate on its axis?





This is a Newton's Second Law problem with Uniform Circular Motion. Sketch a Free Body Diagram of Arlene. Choose the positive direction to be in the direction of the centripetal acceleration, toward the center of the planet. The only forces acting on her are the force of gravity, and a normal force. The normal force is her apparent weight. The force of gravity on Arlene will be calculated using the Law of Universal Gravitation—it can't be calculated from $w = mg$ because the acceleration of gravity on Planteen isn't known. Letting m_{P} be the mass of Planteen, and m_{A} be the mass of Arlene,

$$\sum F_r = F_{\text{G}} - n = m_{\text{A}} a_r \qquad \Rightarrow \qquad G\frac{m_{\text{P}} m_{\text{A}}}{r^2} - n = m_{\text{A}}\frac{v^2}{r} = m_{\text{A}} r \omega^2$$

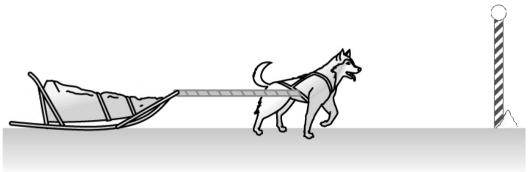
Arlene moves in a circle whose radius is that of Planteen. Solving for the angular speed ω,

$$\begin{aligned}\omega &= \sqrt{\frac{G m_{\text{P}}}{r^3} - \frac{n}{m_{\text{A}} r}} \\ &= \sqrt{\frac{(6.673 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)\,(3.0 \times 10^{23}\,\text{kg})}{(2.5 \times 10^{6}\,\text{m})^3} - \frac{220\,\text{N}}{(82\,\text{kg})\,(2.5 \times 10^{6}\,\text{m})}} \\ &= 4.6 \times 10^{-4}\,\text{rad/s}\end{aligned}$$

1. (6 points) It is the end of the dog sled race, and a dog approaches the finish line over level snow pulling a 55 kg sled. At 5.0 m from the finish line, the sled is travelling at 6.0 m/s. The coefficient of kinetic friction between the sled and the snow is 0.18 in these last 5.0 m. The tiring dog exerts a force, $\vec{F}$, on the sled whose magnitude decreases as the finish line is approached, according to

$$\left|\vec{F}(x)\right| = C\sqrt{x}$$

where C is a constant equal to $32\,\mathrm{N/m^{1/2}}$ and x is the distance from the sled to the finish line. Describe the speed of the sled as it approaches the finish line. *(On Earth.)*



The force of kinetic friction on the sled is always

$$f_\mathrm{k} = \mu_\mathrm{k} n = \mu_\mathrm{k} mg = 0.18\,(55\,\mathrm{kg})\,\left(9.8\,\mathrm{m/s^2}\right) = 97\,\mathrm{N}$$

At the finish line, the dog exerts zero force to the right, so the net force is to the left, opposite the direction of the velocity. The speed is decreasing at the finish line.

5.0 m from the finish line, the dog exerts a rightward force magnitude

$$F = C\sqrt{x} = \left(32\,\mathrm{N/m^{1/2}}\right)\sqrt{5.0\,\mathrm{m}} = 72\,\mathrm{N}$$

which is less than the magnitude of the leftward friction force, so the net force is to the left, opposite the direction of the velocity. The speed is decreasing 5.0 m the finish line.

The dog's force never increases in this last 5.0 m, so the net force is always to the left, opposite the direction of the velocity.

The speed of the sled always decreases.

II. (16 points) In the problem above, what is the speed of the sled as it crosses the finish line?

Let the sled and snow be the system, and use the Energy Principle:

$$W_\mathrm{ext} = \Delta E_\mathrm{sys} = \Delta K + \Delta E_\mathrm{th}$$

The gravitational force and the normal force each do no work on the system, as they are always perpendicular to the displacement. The tension, which is the same as the force exerted by the dog, is an external force parallel to the displacement, and the force of friction causes a change in thermal energy. So

$$\int_0^d F(x)\cos 0^\circ\,dx = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 + f_\mathrm{k} d$$

where d is the distance travelled, 5.0 m. Note that the normal force is equal in magnitude to the gravitational force in this situation, so the frictional force $f = \mu_\mathrm{k} n = \mu_\mathrm{k} mg$.

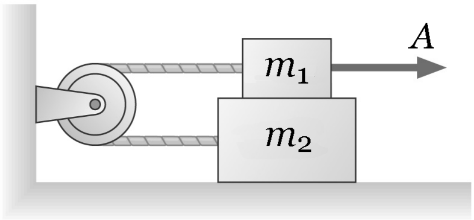
$$\int_0^d C\sqrt{x}\,dx = C\left.\frac{x^{3/2}}{3/2}\right|_0^d = \frac{2C}{3}\,d^{3/2} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 + \mu_\mathrm{k} mgd$$

Solving for the final speed

$$v_f = \sqrt{v_i^2 + \frac{4C}{3m}\,d^{3/2} - 2\mu_\mathrm{k} gd}$$

$$= \sqrt{(6.0\,\mathrm{m/s})^2 + \frac{4\cdot 32\,\mathrm{N/m^{1/2}}}{3\,(55\,\mathrm{kg})}\,(5.0\,\mathrm{m})^{3/2} - 2\,(0.18)\,(9.8\,\mathrm{m/s^2})\,(5.0\,\mathrm{m})} = 5.2\,\mathrm{m/s}$$

2. The upper block in the illustration, with mass m_1, is pulled horizontally by a rope with a tension of magnitude A. The lower block has mass m_2, and rests on a horizontal frictionless surface. The coefficient of kinetic friction between the two blocks is μ_k. How does the frictional force on the bottom block, $\vec{f}_2$, compare to that on the top block, $\vec{f}_1$? *(On Earth.)*



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These two forces constitute a force pair. From Newton's Third Law

$\vec{f}_2$ is in the opposite direction to $\vec{f}_1$, and $f_2 = f_1$.

III. In the problem above, what is the acceleration magnitude of the lower block, in terms of any or all of m_1, m_2, A, μ_k, and physical or mathematical constants?

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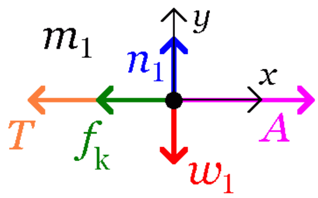
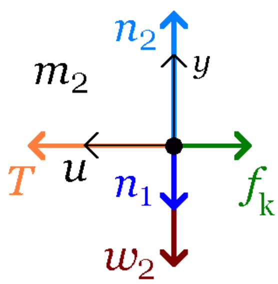
This is a Newton's Second Law problem. Make a Free Body Diagram of each block. Choose a coordinate system for each block in which one axis points in the direction of that block's acceleration.





For the top block, the normal force and the weight act in the vertical direction.

$$\sum F_y = n_1 - w_1 = m_1 a_y = 0 \quad \text{so} \quad n_1 = m_1 g$$

The applied force, the tension, and the force of kinetic friction act in the horizontal direction.

$$\sum F_x = A - T - f_\mathrm{k} = m_1 a_x \qquad \Rightarrow \qquad A - T - \mu_\mathrm{k} n_1 = m_1 a_x \qquad \Rightarrow \qquad A - T - \mu_\mathrm{k} m_1 g = m_1 a_x$$

The acceleration is the answer to the question, but the tension is unknown.

For the bottom block, the tension and the force of kinetic friction act in the horizontal direction. Note that the tension has the same magnitude as the tension on the bottom block (ideal cord and pulley), and the force of kinetic friction is equal and opposite to the force of kinetic friction on the top block (Newton's Third Law).

$$\sum F_u = T - f_\mathrm{k} = m_2 a_u \qquad \Rightarrow \qquad T = m_2 a_u + \mu_\mathrm{k} m_1 g$$

Note that, by wise choice of coordinate systems, $a_x = a_u$. Let them both be a. Substituting this expression for T into the horizontal equation for the top block,

$$A - (m_2 a + \mu_\mathrm{k} m_1 g) - \mu_\mathrm{k} m_1 g = m_1 a$$

and solving for the acceleration

$$A - \mu_\mathrm{k} m_1 g - \mu_\mathrm{k} m_1 g = m_1 a + m_2 a \qquad \Rightarrow \qquad A - 2\mu_\mathrm{k} m_1 g = (m_1 + m_2)\, a \qquad \Rightarrow \qquad a = \frac{A - 2\mu_\mathrm{k} m_1 g}{m_1 + m_2}$$

3. (8 points) A 2.0 kg particle moving on the x axis is subject to the force shown on the graph. If the particle's velocity is $+5.0\,\mathrm{m/s}$ as it passes through the origin, what is its kinetic energy when it reaches $+10\,\mathrm{m}$?

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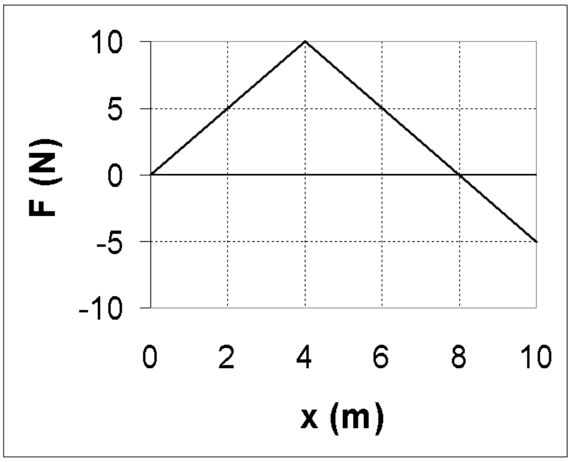


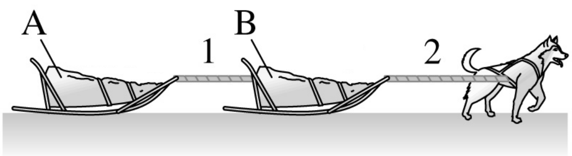
Since $W = \int \vec{F} \cdot d\vec{s}$, the work done is the area under the Force *vs.* Position graph, or $+35\,\mathrm{J}$. Work is also the change in kinetic energy. Initially, the kinetic energy is $\frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\,\mathrm{kg})(+5.0\,\mathrm{m/s})^2 = +25\,\mathrm{J}$. So, with an initial kinetic energy of $+25\,\mathrm{J}$ and a change of $+35\,\mathrm{J}$, the resulting kinetic energy must be

$+60\,\mathrm{J}$

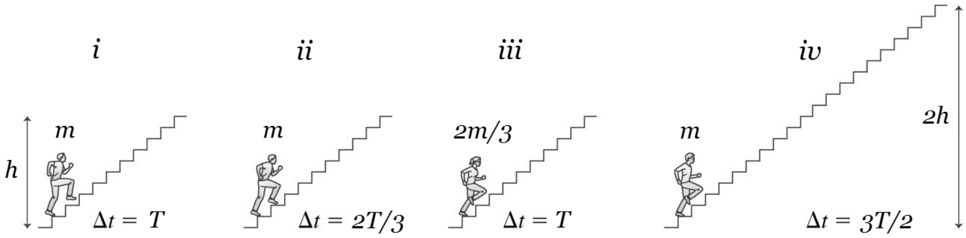
4. (8 points) It is the beginning of the dog sled race, and the illustrated dog accelerates toward the right as it leaves the starting line over level snow, pulling two sleds of equal mass. The runners of one sled have been waxed to reduce the coefficient of kinetic friction between that sled and the snow. How is the tension in rope 2 related to the tension in rope 1? (*On Earth.*)

Rope 2 must pull the mass of both sleds, while rope 1 must only pull the rear sled. The waxing of the runners is irrelevant.

Tension in rope 2 is greater than tension in rope 1, regardless of which sled has waxed runners.



5. (8 points) Four students with the indicated masses run up their staircases in the indicated times. Rank, from greatest to least, their power outputs.



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Power is the time rate of energy transformation, $P = dE/dt$ or $\Delta E/\Delta t$. For each of the students, the energy transformation is an increase in the gravitational potential energy of the student-Earth system.

$$P_i = \frac{mgh}{T} \qquad P_{ii} = \frac{mgh}{2T/3} = \frac{3}{2}\left(\frac{mgh}{T}\right) \qquad P_{ii} = \frac{(2m/3)\,mgh}{T} = \frac{2}{3}\left(\frac{mgh}{T}\right) \qquad P_{iv} = \frac{mg\,(2h)}{3T/2} = \frac{4}{3}\left(\frac{mgh}{T}\right)$$

so

$$ii > iv > i > iii$$

6. (8 points) A block of mass m is given an initial push so that it slides up the left side of a ramp with kinetic friction coefficient μ_{up}. When it reaches the top it slides smoothly over the peak and then down the right side. The kinetic friction coefficient on the right side is μ_{down}. Compare the time for the block to slide up the left side with that for it to slide down the right side. (Hint: consider the block's kinetic energy.) (*On Earth.*)

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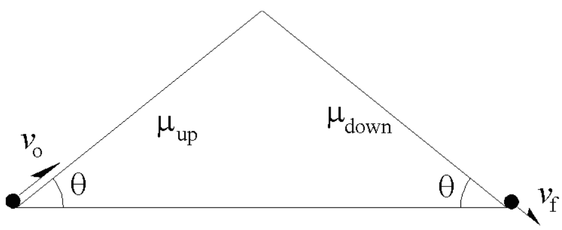
Consider two points at the same height on the left and right side of the ramp. The gravitational potential energy of the Earth-block system is the same at these two points. The non-conservative force of friction has removed mechanical energy from the system as the block moved from the point on the left side to the point on the right side. Therefore, the kinetic energy of the block at that point going up the left is greater than



at the point going down the right. The two points at the same height on the left and right side were chosen arbitrarily, so the block must have greater speed at every point going up the left side than it does coming down through the corresponding point on the right. Since the angles are the same, the lengths of the two ramps are the same, and

The block takes more time to slide down the right side than up the left side.

7. (8 points) A merry-go-round is spinning at a constant angular velocity. A man walks from the center of the merry-go-round out to the edge along a radius. If the man does not slip, what must be true? (*On Earth.*)

As the man approaches the edge . . .

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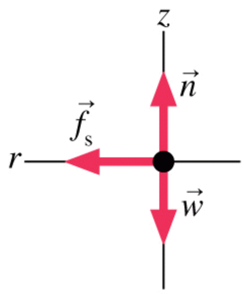
From the Free-Body Diagram,



$$\sum F_r = f_s = ma_r = mr\omega^2$$

The mass, m, and the angular velocity, ω, are constant. As the man approaches the edge, increasing r,

the force of static friction must increase.