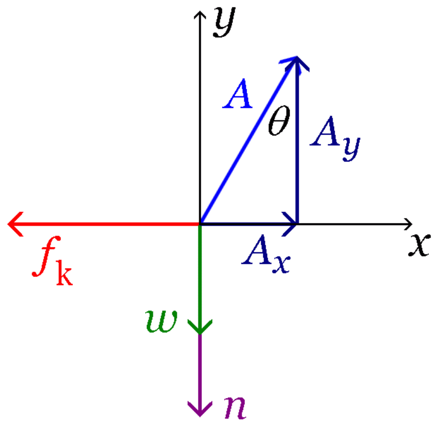
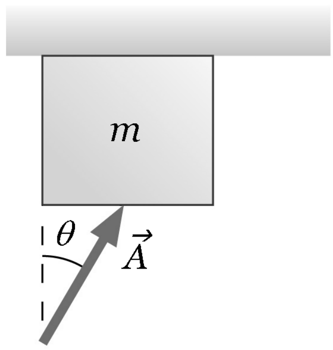

I. (16 points) A block of mass m is being slid to the right along a horizontal ceiling by an applied force of magnitude A that makes an angle θ with the vertical, as illustrated. The coefficient of static friction between the block and the ceiling is μ_{s}, while the coefficient of kinetic friction is μ_{k}. What is the acceleration magnitude the block, in terms of parameters defined in the problem, and physical or mathematical constants? (*On Earth.*)





Use Newton's Second Law. Sketch a Free Body Diagram, and choose a coordinate system. Write Newton's Second Law for each dimension. I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_x = A_x - f_{\mathrm{k}} = ma_x \qquad \Rightarrow \qquad A\sin\theta - \mu_{\mathrm{k}} n = ma_x$$

and

$$\sum F_y = A_y - n - w = ma_y = 0 \qquad \Rightarrow \qquad n = A\cos\theta - mg$$

Substituting this expression for the normal force, n, into the x-equation above,

$$A\sin\theta - \mu_{\mathrm{k}}\left(A\cos\theta - mg\right) = ma_x \qquad \Rightarrow \qquad a_x = \frac{A\sin\theta - \mu_{\mathrm{k}}\left(A\cos\theta - mg\right)}{m}$$

II. (16 points) In an electricity experiment, a plastic ball of mass $m = 65\,\mathrm{g}$ is attached to a string of length $L = 81\,\mathrm{cm}$ and given an electric charge. The string is tied to a charged plane, inclined at an angle $\phi 28°$ to the horizontal. The plane exerts an electrical force on the ball, perpendicular to the plane, causing the ball to swing up until the string is horizontal and remain there. What is the magnitude of the tension in the string? (*On Earth.*)

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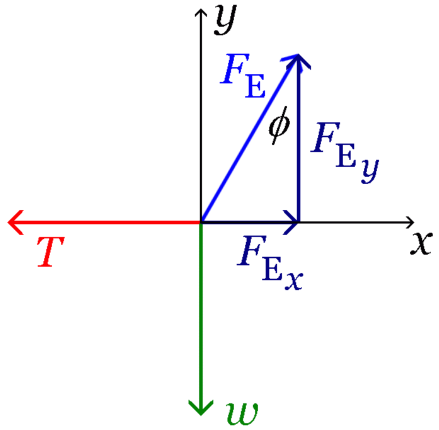

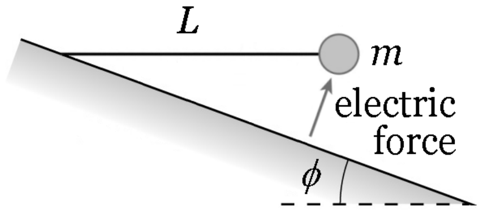

Use Newton's Second Law. Sketch a Free Body Diagram, and choose a coordinate system. Write Newton's Second Law for each dimension. I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_x = F_{\mathrm{E}_x} - T = ma_x = 0 \qquad \Rightarrow \qquad T = F_\mathrm{E} \sin\phi$$

and

$$\sum F_y = F_{\mathrm{E}_y} - w = ma_y = 0 \qquad \Rightarrow \qquad mg = F_\mathrm{E} \cos\phi$$

Dividing the x-equation by the y-equation,

$$\frac{T}{mg} = \frac{F_\mathrm{E}\sin\phi}{F_\mathrm{E}\cos\phi} \qquad \Rightarrow \qquad T = mg\tan\phi = (0.065\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)\tan 28° = 0.34\,\mathrm{N}$$

1. (6 points) If it can be determined in the problem above, how is the electric force, $\vec{F}_\mathrm{E}$, related to the tension in the string, $\vec{T}$, and the weight of the ball, $\vec{w}$? (*Hint:* Remember the notation, $F_\mathrm{E} = \left|\vec{F}_\mathrm{E}\right|$, etc.)

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The weight, the electric force, and the tension are the only forces acting on the ball. Since the ball is in equilibrium, the net force (i.e., the vector sum of the forces) must be zero, $\vec{F}_\mathrm{E} + \vec{T} + \vec{w} = 0$, or

$$\vec{F}_\mathrm{E} = -\vec{T} - \vec{w}$$

III. A child is riding a merry-go-round (a horizontal circular ride found in an amusement park). The merry-go-round is rotating with a constant period T_0. The operator then applies the brakes, giving the merry-go-round a constant angular acceleration so it comes to a stop in time Δt. Through how many revolutions does it turn from the time the operator applies the brakes to the time it comes to a complete stop? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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Use constant angular acceleration kinematics.

$$\theta = \theta_0 + \omega_0\,\Delta t + \tfrac{1}{2}\alpha\,(\Delta t)^2 \qquad \text{and} \qquad \omega = \omega_0 + \alpha\,\Delta t$$

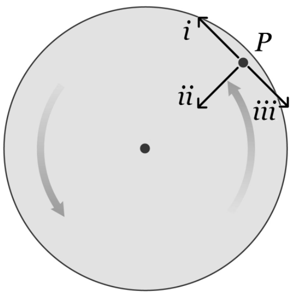
The merry-go-round comes to a stop, so $\omega = 0$. Therefore, $\alpha = -\omega_0/\Delta t$.

$$\theta - \theta_0 = \omega_0\,\Delta t + \tfrac{1}{2}\left(\frac{-\omega_0}{\Delta t}\right)(\Delta t)^2 = \tfrac{1}{2}\omega_0\,\Delta t$$

The initial period, T_0, like all periods, is in units of time per cycle (or revolution). Therefore, in units of revolutions per time, $\omega_0 = 1/T_0$.

$$\Delta\theta = \frac{\Delta t}{2T_0}$$

2. (6 points) The figure illustrates a top-view of the merry-go-round, as it rotates counterclockwise with a child sitting at point P. What is the direction of the child's acceleration as the merry-go-round slows down?



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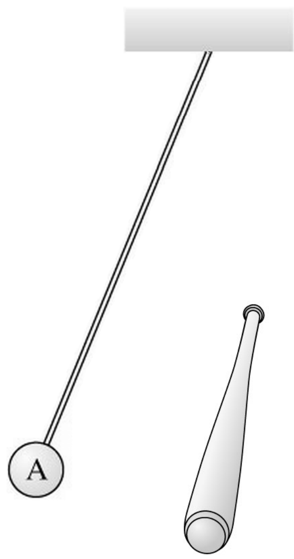
Since the merry-go-round is slowing down as it turns counterclockwise, the child's tangential acceleration is in direction *iii*. The child is still, however, moving in a circle, so there is a centripetal acceleration in direction *ii*. The child's total acceleration is the vector sum of these,

Somewhere between directions *ii* and *iii*.

3. (8 points) The pendulum bob, A, was struck by the bat, and is now swinging upward. Which items on this list are forces acting on the bob as it swings upward? Just as you do with "Choose all that apply" questions in class, express your answer as number, with your choices in numeric order. For example, if you believe that $m\vec{a}$ and the gravitational force are the forces on the bob, your answer would be "12". (*On Earth.*)

1 $m\vec{a}$
2 gravitational force
3 centripetal force
4 normal force
5 tension force
6 strike force
7 no forces act on the bob as it swings



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The bob isn't against a surface, so no normal force acts on it. Neither $m\vec{a}$ nor "centripetal force" are forces—instead they are what forces add to. The strike force no longer acts on the bob after it leaves contact with the bat. Only the gravitational force and tension force are acting on the bob as it swings up. In numerical order, those are choices

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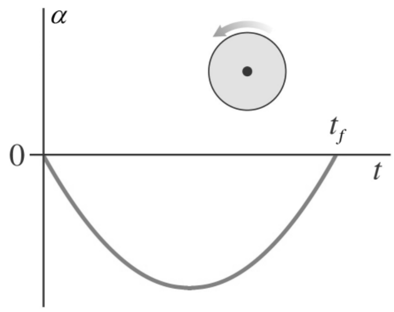
4. (8 points) A wheel is rotating counterclockwise with angular velocity ω_0. Let this direction be positive. It is then given a non-uniform angular acceleration, α, shown in the graph, from time $t = 0$ to time $t = t_f$. How does the *magnitude* of the angular velocity, ω_f, at time t_f, compare to the *magnitude* of ω_0?



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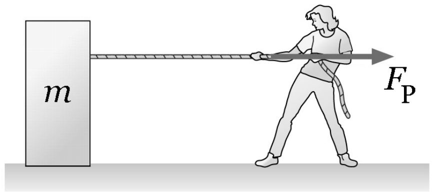
At time $t = 0$ the angular velocity of the wheel is positive. The graph shows that the wheel has negative angular acceleration from from time $t = 0$ to time $t = t_f$ (that is the only relevant information on the graph). Therefore, the wheel's angular velocity will be less positive at time $t = t_f$ than it was at time $t = 0$. There are many ways that condition can be satisfied, including having a lower positive angular velocity, an angular velocity of zero, or any negative angular velocity.

This cannot be determined from the information provided.

5. (8 points) Sue is pulling on the crate with a force magnitude F_P. The crate has mass m, and coefficient of static friction μ_s with the level ground. Because of this static friction, the crate does not move. Sue gets tired, and reduces her force to $F_\mathrm{P}/2$. What is the magnitude of the static friction force f_s on the crate, now that Sue has reduced her force? (*On Earth.*)

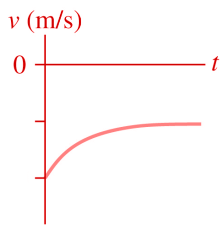


The block is in static equilibrium, so the net force on it is zero. The horizontal forces of static friction and Sue's pull must be opposite in direction and equal in magnitude.

$$f_\mathrm{s} = F_\mathrm{P}/2$$

6. (8 points) A ping-pong ball of mass m is thrown straight downward with a speed that is twice its terminal speed. If positive is chosen upward, which graph best represents the velocity of the ping-pong ball as a function of time? (*On Earth, do* **NOT** *neglect drag!*)

Since the drag force upward is initially greater than the weight, the graph of velocity as a function of time must have a positive initial slope. This slope must approach zero as the speed approaches the terminal speed (which is not zero).



7. (8 points) The truck can accelerate to the right over level ground without causing the crate on the bed to slide. If the acceleration is too great, however, the crate will slide and fall off the back of the truck. If the crate does slide and fall off the back, what horizontal force, if any, is acting on the crate while it slides, and in what direction? (*On Earth.*)

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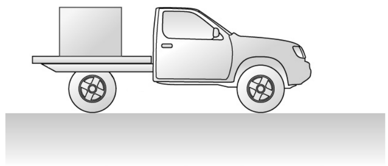

Since it is possible for the truck to accelerate without causing the crate to slide, there must be a frictional force between the truck and the crate. When the crate does slide, that frictional force must be kinetic. When the crate slides toward the rear of the truck, kinetic friction opposes this and must be toward the right. Or, when the truck accelerates toward the right, the crate also accelerates toward the right even if it slides, so once again

A force of kinetic friction acts toward the right.