Solutions

Printed Name

Nine-digit GT ID

signature

Summer 2019 **PHYS 2211 M** **Test 03**

Test Form:

3A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on your test, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

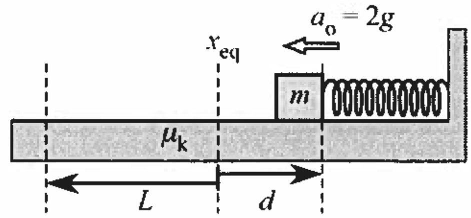
4 ⓐ ⓑ ⓒ ⓓ ⓔ

5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. *Show all supporting work for this problem.*

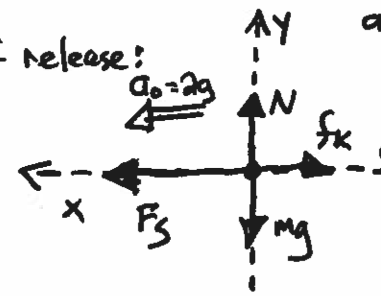
[1] *(20 points)* A horizontal spring with an unknown spring constant is compressed a distance d, and then used to launch a block of mass m across the floor. The coefficient of kinetic friction between the block and the floor is $\mu = 0.25$. The initial acceleration of the block, at the moment it is released, is $a_o = 2g$. How far beyond the spring's equlibrium position (L in the figure at right) will the block slide before coming to a stop? Express your answer as a numerical multiple of d.



Hint: Start by finding an expression for the sprink constant k, in terms of m, g and d

total distance travelled along surface is: $d+L$

① Analyze forces (2nd Law) at time of release:



choose "left = positive"

$Y: \langle +N \rangle + \langle -mg \rangle = 0$

$N = mg$, so $f_K = \mu_K N = \frac{1}{4} mg$

$x: \langle +F_S \rangle + \langle -f_K \rangle = m \langle +a_o \rangle$

$+kd - \frac{1}{4}mg = +2mg \longrightarrow kd = \frac{8}{4}mg + \frac{1}{4}mg = \frac{9}{4}mg \longrightarrow \boxed{k = \frac{9mg}{4d}}$

② Apply energy principle to the resulting motion

$\Delta E_{system} = W_{ext} \longrightarrow$ either "$\Delta K + \Delta U_s + \Delta E_{Th} = 0$" or "$\Delta K + \Delta U_s = W_{dissipative}$"

here: $\Delta K = 0$ (start and end at rest)

$\Delta U_s = U_{sf} - U_{si} = 0 - \frac{1}{2}kd^2 = -\frac{1}{2}kd^2$

$\Delta E_{Th} = \underbrace{+f_K(d+L)}_{\text{appears on left side}}$ or $W_{diss} = \underbrace{-f_K(d+L)}_{\text{appears on right side}}$

③ Substitute: $0 - \frac{1}{2}kd^2 + \left(\frac{1}{4}mg(d+L)\right) = 0$ or $0 - \frac{1}{2}kd^2 = -\left(\frac{1}{4}mg(d+L)\right)$

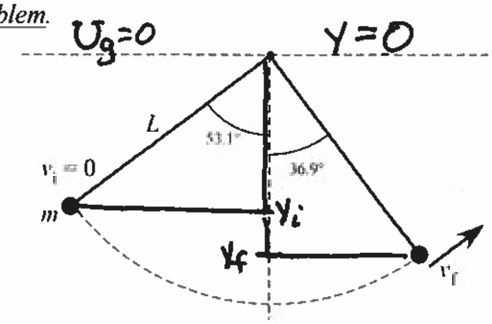
using $k = \frac{9mg}{4d}$:

$-\frac{1}{2}\frac{9mg}{4d}d^2 = -\frac{1}{4}mg(d+L)$

$\frac{9}{2}d = d + L \longrightarrow \boxed{L = \frac{7}{2}d}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

[II] *(20 points)* Jane (mass m) is swinging through the jungle from vine to vine, pursuing villains in an attempt to rescue Tarzan. Standing at rest on a high tree branch, she grabs a long vine that is inclined at an angle $\theta_0 = 53.1°$ from the vertical and swings downward. She swings through the lowest point in her arc, and then upward again until the vine makes an angle $\theta_1 = 36.9°$ from the vertical. At that moment she lets go of the vine, to grab another and continue her pursuit.



Determine the tension in the first vine, just before Jane lets go of it. Express your answer as a multiple of her true weight, mg.

$\sin(53.1°) = \frac{4}{5}$ $\quad \cos(53.1°) = \frac{3}{5}$

$\sin(36.9°) = \frac{3}{5}$ $\quad \cos(36.9°) = \frac{4}{5}$

① Mechanical energy is conserved

$$K_i + U_i = K_f + U_f$$

setting $y=0$ at top of vine, we have:

$$\vec{y}_i = -L\cos 53° = -\tfrac{3}{5}L \qquad \vec{y}_f = -L\cos 36.9° = -\tfrac{4}{5}L$$

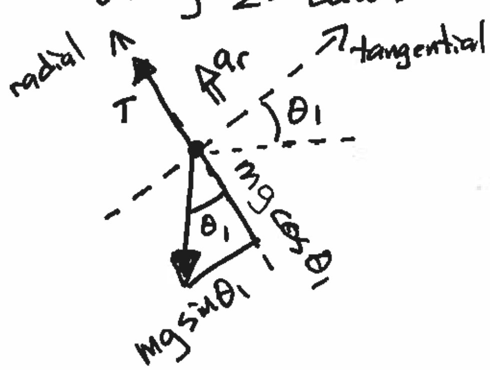
so $U_{gi} = -\tfrac{3}{5}mgL \quad U_{gf} = -\tfrac{4}{5}mgL$

and $K_i = 0 \quad K_f = \tfrac{1}{2}mv_f^2$

so: $$0 - \tfrac{3}{5}mgL = \tfrac{1}{2}mv_f^2 - \tfrac{4}{5}mgL \rightarrow \tfrac{1}{2}mv_f^2 = \tfrac{1}{5}mgL$$

$$\boxed{v_f^2 = \tfrac{2}{5}gL}$$

② At release point, analyze circular motion (before release) using 2nd Law:



$$\Sigma \vec{F}_r = m\vec{a}_r$$

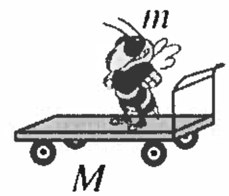
$$\langle +T \rangle + \langle -mg\cos\theta_1 \rangle = m\langle + v_f^2/L \rangle$$

$$T - \tfrac{4}{5}mg = m\left(\tfrac{2}{5}g\right)$$

$$\boxed{T = \tfrac{6}{5}mg}$$

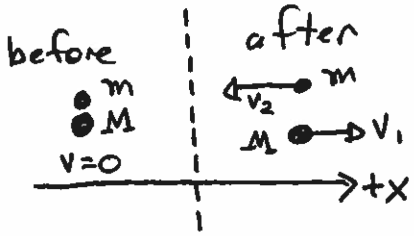
The following problem will be hand-graded. *Show all supporting work for this problem.*

[III] *(20 points)* Buzz (mass m) stands on a large, flat pushcart (mass M) that is initially stationary. He leaps off horizontally with a speed v_0 *measured relative to the cart* (not relative to the ground). Determine (1) the velocity of the cart relative to the ground, as it recoils away from Buzz's leap; and (2) the velocity of Buzz relative to the ground while he is airborne. Express each answer as a 1D-vector, in terms of the parameters m, M, and v_0.



① Let "system" = Buzz + cart

→ system is isolated, so total momentum is conserved



$\vec{P}_i = \vec{P}_f$

$0 = \langle +MV_1 \rangle + \langle -mV_2 \rangle$

$\boxed{MV_1 = mV_2}$ one equation in two unknowns (speeds V_1 and V_2)

② We are given a speed for Buzz relative to cart, V_0

→ set up a relative velocity problem

$\vec{V}_{\text{buzz to ground}} = \vec{V}_{\text{buzz to cart}} + \vec{V}_{\text{cart to ground}}$

$\langle -V_2 \rangle = \langle -V_0 \rangle + \langle +V_1 \rangle$

or $\boxed{V_2 = V_0 - V_1}$ second equation in unknowns V_1 V_2
→ problem can now be solved

③ substitute

$MV_1 = m(V_0 - V_1) \rightarrow (M+m)V_1 = mV_0$

$V_1 = \frac{m}{M+m} V_0$ (speed)

so velocity of cart is $\boxed{\vec{V}_{cg}} \; \boxed{\left\langle + \frac{m}{M+m} V_0 \right\rangle}$

④ substitute again:

$V_2 = V_0 - V_1 = \left(\frac{M+m}{M+m}\right) V_0 - \left(\frac{m}{M+m} V_0\right)$

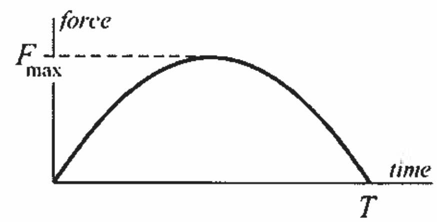
$V_2 = \frac{M}{M+m} V_0$ (speed)

so velocity of Buzz is

$\boxed{V_{Bg} = \left\langle -\frac{M}{M+m} V_0 \right\rangle}$

The next two questions involve the following situation:
A rubber ball of mass m is traveling horizontally at speed v when it strikes a wall. It rebounds from the wall traveling in the opposite direction, at the same speed v. While in contact with the wall, the ball experiences a time-dependent force with a magnitude given by the expression:

$$F(t) = 4F_{max}\left(\frac{t}{T} - \frac{t^2}{T^2}\right) \quad \text{for} \quad 0 \le t \le T$$



Question value 4 points

(1) What is the magnitude of the peak force F_{max} experienced by the ball while in contact with the wall?

(a) $F_{max} = \frac{mv}{T}$

(b) $F_{max} = \frac{2mv}{T}$

(c) $F_{max} = \frac{3mv}{T}$

(d) $F_{max} = \frac{mv}{2T}$

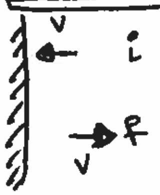
(e) $\boxed{F_{max} = \frac{mv}{3T}}$

Impulse - momentum theorem: $\vec{J} = \Delta\vec{p}$

where $\vec{J} = \int \vec{F}(t)\,dt$ and $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$\vec{J} = \int_0^T 4F_{max}\left(\frac{t}{T} - \frac{t^2}{T^2}\right)dt$

$= 4F_{max}\left[\frac{t^2}{2T} - \frac{t^3}{3T}\right]_0^T = 4F_{max}\left[\frac{T}{6}\right] = \frac{2}{3}F_{max}T$



$\Delta\vec{p} = \langle +mv\rangle - \langle -mv\rangle$
$= \langle +2mv\rangle$

$\vec{J} = \Delta\vec{p}$ gives $\frac{2}{3}F_{max}T = 2mV$

$$\boxed{F_{max} = \frac{mV}{3T}}$$

Question value 4 points

(2) What is the magnitude of the average force F_{avg} experienced by the ball while in contact with the wall?

(a) $\boxed{F_{avg} = \frac{2}{3}F_{max}}$

(b) $F_{avg} = \frac{4}{3}F_{max}$

(c) $F_{avg} = 0$

(d) $F_{avg} = \frac{3}{4}F_{max}$

(e) $F_{avg} = \frac{1}{2}F_{max}$

F_{avg} = force that creates same impulse in time T

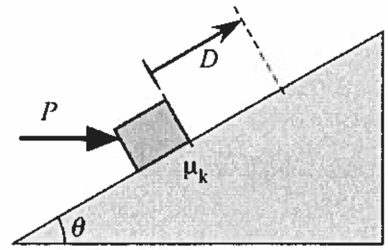
$F_{avg} \cdot T = \vec{J} = \int \vec{F}(t)\,dt$

$F_{avg} \cdot T = \frac{2}{3}F_{max}T$ (from above)

$$\boxed{F_{avg} = \frac{2}{3}F_{max}}$$

Question value 8 points

(3) A crate is pushed up a loading ramp that is inclined at an angle θ above the horizontal, by a pushing force P that is itself *horizontal*. The ramp is rough, with a coeffiecient of kinetic friction $\mu_k = 0.20$. How much work is done by the pushing force, as the crate moves a distance D up the ramp?



(a) $W_P = +0.8\, PD \sin\theta$

(b) $W_P = +0.8\, PD \cos\theta$

(c) $\boxed{W_P = +PD\cos\theta}$

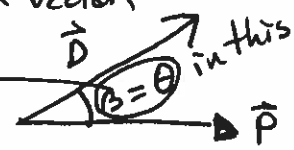
(d) $W_P = +PD$

(e) $W_P = +PD\sin\theta$

Work by $\vec{P}$:

$W_P = \vec{P} \cdot \vec{\Delta s} = |\vec{P}|\,|\vec{\Delta s}| \cos\beta$

β = angle between vectors



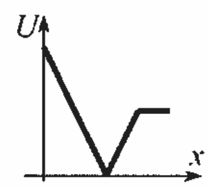
in this case!

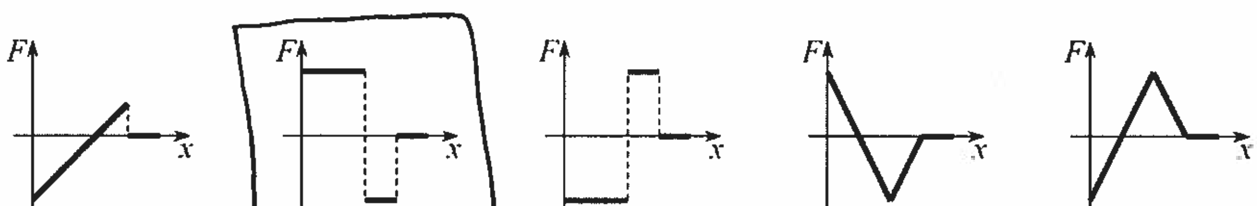
$\boxed{W_P = {+}PD\cos\theta}$

(Note that the possible presence of friction does not impact the value of work done by push)

Question value 8 points

(4) A particle moves along the x-axis subject to a conservative force $F(x)$. The potential energy function for this force is shown in the figure. Which of the graphs below best characterizes the force F as a function of position?





(a) (b) (c) (d) (e)

$$\vec{F}_x = \left\langle -\frac{dU}{dx} \right\rangle = \text{negative of slope of } U$$

- initially: U has constant (large) negative slope
  $\rightarrow$ constant (large) positive force
- then: U has constant ($\simeq$ same) positive slope
  $\rightarrow$ constant ($\simeq$ same) negative force
- finally: U has zero slope
  $\rightarrow$ zero force

Question value 8 points

(5) What is the difference between a *stable equilibrium point* and an *unstable equilibrium point*?

(a) A stable equlibrium point is a place where the PE graph has a *local maximum;* an unstable equilibrium point is a place where the graph has a *local minimum.*

(b) A stable equlibrium point is a place where the slope of the PE graph is *zero*; an unstable equilibrium point is a place where the slope is *non-zero.*

(c) A stable equlibrium point is a place where the slope of the PE graph is *positive*; an unstable equilibrium point is a place where the slope is *negative.*

(d) A stable equlibrium point is a place where the PE graph has a *local minimum;* an unstable equilibrium point is a place where the graph has a *local maximum.*

(e) A stable equlibrium point is a place where the slope of the PE graph is *negative*; an unstable equilibrium point is a place where the slope is *positive.*

$\vec{F} = \langle -\frac{dU}{dx} \rangle \rightarrow$ equilibrium, $\Sigma\vec{F} = 0$ means $\frac{dU}{dx} = 0$, meaning $\boxed{U = \text{min or max at equilibrium}}$

max = unstable: Force near max pushes away from max

min = stable: force near min pushes toward min

Question value 8 points

(6) A car having mass m is on the interstate traveling due east with speed $4v$. It slows down while entering a broad curve, and exits the curve traveling due south with speed $3v$. What is the magnitude of the impulse delivered to the car during the turn?

(a) $3\,mv$

(b) $2\,mv$

(c) $1\,mv$

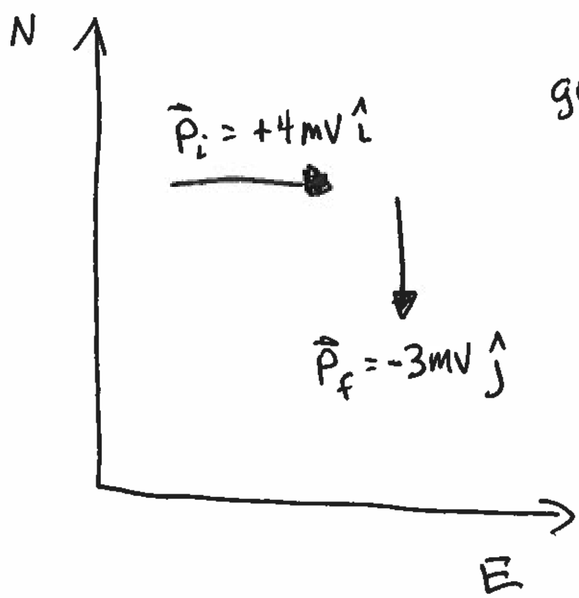
(d) $4\,mv$

(e) $5\,mv$

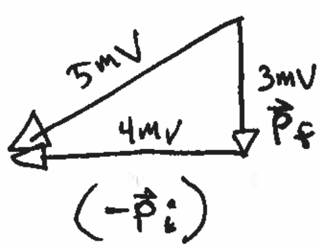
$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$

so $|\vec{J}| = |\Delta\vec{p}| = |\vec{p}_f - \vec{p}_i|$

(NOT the same thing as $|\vec{p}_f| - |\vec{p}_i|$!!)



graphical construction: $\Delta\vec{p} = \vec{p}_f + (-\vec{p}_i)$



We have a 3-4-5 triangle, here!

$|\Delta\vec{p}| = \sqrt{(p_i)^2 + (p_f)^2}$

$= \sqrt{16m^2v^2 + 9m^2v^2}$

$= mv\sqrt{25}$

$|\vec{J}| = |\Delta\vec{p}| = \boxed{5mv}$