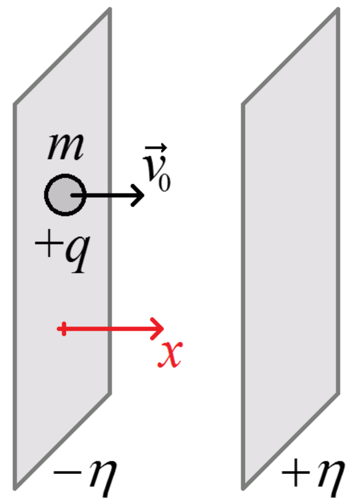

I. (16 points) A particle with mass $m = 36\,\mathrm{g}$ and charge $q = 2.0\,\mathrm{nC}$ is at rest against the negative plate of an ideal capacitor whose vertical plates have area charge density $\eta = \pm 6.0\times10^{-5}\,\mathrm{C/m^2}$. The particle is launched at $\vec{v}_0 = 0.40\,\mathrm{m/s}$ horizontally toward the positive plate. Assuming the distance between the plates is great enough that the particle does not strike the positive plate, how much time after launch is required for particle to return to the negative plate? (*On Earth, do* **NOT** *neglect gravity.*)



The electric field in an ideal capacitor is uniform, $E = \eta/\epsilon_0$, so the force on a charged particle is constant, and constant-acceleration kinematics may be used.

Choose a coordinate system. I'll set the origin at the negative plate, with positive x horizontally to the right. As the x component of the motion is independent of the motion along any other axis (in particular, the downward acceleration due to gravity), no other axis need be considered.

These are the constant-acceleration formulas related by calculus, and so worth remembering:

$$x = x_0 + v_{0x}, \Delta t + \tfrac{1}{2} a_x \left(\Delta t\right)^2 \qquad \text{and} \qquad v_x = v_{0x} + a_x\,\Delta t$$

The x component of the motion is symmetric, so $v_x = -v_{0x}$, and

$$-v_{0x} = v_{0x} + a_x\,\Delta t \qquad \Rightarrow \qquad \Delta t = \frac{-2v_{0x}}{a_x}$$

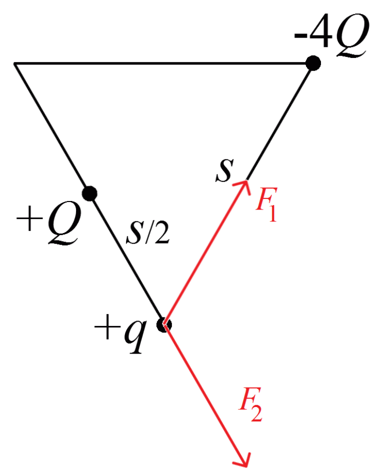
where

$$a_x = \frac{F_x}{m} = \frac{qE_x}{m} = \frac{-q\eta}{m\epsilon_0}$$

so (and not forgetting to convert the mass to kilograms),

$$\Delta t = \frac{-2v_{0x}}{-q\eta/m\epsilon_0} = \frac{2v_0 m\epsilon_0}{-q\eta} = \frac{2\,(0.40\,\mathrm{m/s})\left(36\times10^{-3}\,\mathrm{kg}\right)\left(8.854\times10^{-12}\,\mathrm{C^2/N{\cdot}m^2}\right)}{\left(2.0\times10^{-9}\,\mathrm{C}\right)\left(6.0\times10^{-5}\,\mathrm{C/m^2}\right)} = 2.1\,\mathrm{s}$$

1. (6 points) Three charged particles are placed on an equilateral triangle with sides of length s. A positive charge $+q$ is placed at one vertex, and a negative charge $-4Q$ is placed at another. Finally, a positive charge $+Q$ is placed at the mid-point of one side, as shown. What is the direction of the electric force on the particle with charge $+q$?



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There will be a repelling force $\vec{F}_2$ down and to the right on $+q$ from $+Q$. There will be an attractive force $\vec{F}_1$ on $+q$ up and to the right from $-4Q$. Note that $-4Q$ is four times the charge magnitude of $+Q$, but is twice as far from $+q$. The force magnitudes, therefore, are equal. The components up and down the page cancel, resulting in a net force

directly rightward

II. (16 points) In the problem above, what is the magnitude of the force on the particle with charge $+q$? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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Use Coulomb's Law. The magnitude of the force F_2 on $+q$ from $+Q$ is

$$F_2 = K\frac{q_1 q_2}{r^2} = K\frac{Qq}{(s/2)^2} = 4K\frac{Qq}{s^2}$$

The magnitude of the force F_1 on $+q$ from $-4Q$ is the same, as explained above, but for completeness,

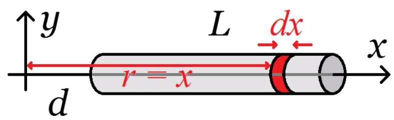
$$F_1 = K\frac{q_1 q_2}{r^2} = K\frac{4Qq}{s^2} = 4K\frac{Qq}{s^2}$$

The vertical components cancel, and the horizontal components add. Remember that an equilateral triangle has 60° angles.

$$F_{\text{total}} = F_1 \cos(60°) + F_2 \cos(60°) = 2F_1 \cos(60°) = 2F_1\left(\frac{1}{2}\right) = F_1 = 4K\frac{Qq}{s^2}$$

2. (6 points) A thin rod of length L lies on the x axis with one end at $x = d$, as shown. It has a non-uniform linear charge density λ the depends on position according to

$$\lambda = \lambda_0 \left(\frac{\pi x^2}{2L^2}\right) \cos\left[\left(\frac{\pi}{2}\right)\frac{x-d}{L}\right]$$



Where λ_0 is a constant. What is the direction of the electric field at the origin?

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The left end of the rod is at $x = d$, so

$$\cos\left[\left(\frac{\pi}{2}\right)\frac{x-d}{L}\right] = \cos\left[\left(\frac{\pi}{2}\right)\frac{d-d}{L}\right] = \cos[\,0\,] = 1$$

The right end of the rod is at $x = d + L$, so

$$\cos\left[\left(\frac{\pi}{2}\right)\frac{x-d}{L}\right] = \cos\left[\left(\frac{\pi}{2}\right)\frac{d+L-d}{L}\right] = \cos\left[\left(\frac{\pi}{2}\right)\frac{L}{L}\right] = \cos\left[\frac{\pi}{2}\right] = 0$$

Except at the very far right end where it's zero, then, the cosine factor is always positive. The factor $\left(\frac{\pi x^2}{2L^2}\right)$ is always positive. The sign of the rod's charge, therefore, will be the same as the sign of λ_0. If the rod were positive, there would be a repelling force on a positive test charge at the origin, but if the rod were negative, the force would be attractive.

Field is in the $-x$ direction if λ_0 is positive, and in the $+x$ direction if λ_0 is negative.

III. (16 points) If λ_0 is a *positive* constant in the problem above, what is the magnitude of the electric field at the origin?

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Divide the rod into point-like bits, as shown. Each bit of charge dq has width dx and is a distance $r = x$ from the origin. Add (integrate) the contribution to the field at the origin due to all the bits. The field due to each bit is in the same ($\pm x$) direction, so only the x component need be considered.

$$E = E_x = \int K\frac{dq}{r^2} = K\int \frac{\lambda\, dx}{x^2}$$

$$= K\int_d^{d+L} \lambda_0 \left(\frac{\pi x^2}{2L^2}\right)\cos\left[\left(\frac{\pi}{2}\right)\frac{x-d}{L}\right]\frac{dx}{x^2} = \frac{K\lambda_0}{L}\int_d^{d+L}\left(\frac{\pi}{2L}\right)\cos\left[\left(\frac{\pi}{2}\right)\frac{x-d}{L}\right]dx$$

where $\lambda = dq/dx \quad \Rightarrow \quad dq = \lambda\, dx$. At this point, u substitution with $u = \left(\frac{\pi}{2}\right)\frac{x-d}{L}$ reveals that the integral is just $\int \cos u\, du$. So

$$E = \frac{K\lambda_0}{L}\sin\left[\left(\frac{\pi}{2}\right)\frac{x-d}{L}\right]_d^{d+L} = \frac{K\lambda_0}{L}\left\{\sin\left[\left(\frac{\pi}{2}\right)\frac{d+L-d}{L}\right] - \sin\left[\left(\frac{\pi}{2}\right)\frac{d-d}{L}\right]\right\}$$

$$= \frac{K\lambda_0}{L}\left\{\sin\left[\frac{\pi}{2}\right] - \sin[\,0\,]\right\} = \frac{K\lambda_0}{L}\{1-0\} = \frac{K\lambda_0}{L}$$

3. (8 points) Particles with charges q_1 and q_2 are separated by a distance s. There is a position "X" a distance d beyond q_2, as shown, where the electric field is zero. Compare q_1 and q_2.

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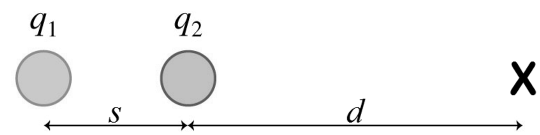

If the field at "X" is zero, the fields due to q_1 and q_2 must be equal in magnitude but opposite in direction.

For the fields due to q_1 and q_2 to be opposite in direction, the charges must have opposite signs.

For the fields due to q_1 and q_2 to be equal in magnitude, considering that the electric field due to point charges follows in inverse square law, the more distant charge must have greater charge magnitude.

q_1 and q_2 have opposite signs, with $|q_2| < |q_1|$.

4. (8 points) A disk of radius R has uniform charge Q. Describe the electric field magnitude E on its axis z, near and far from the disk.

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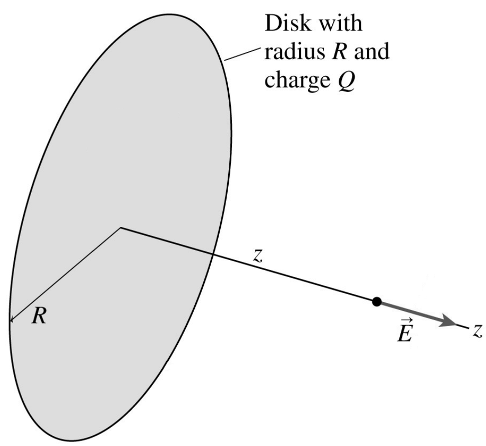
Near the disk, the field must approach that of an infinite sheet,

$$E_{\text{near}} \approx \frac{\eta}{2\epsilon_0} = \frac{Q/A}{2\epsilon_0} = \frac{Q}{2\pi R^2 \epsilon_0}$$

Far from the sheet, the field must approach that of a point charge, which follows an inverse square law.

The field approaches $Q/2\pi R^2\epsilon_0$ near the disk. Far from the disk, $E \propto 1/z^2$.

or, $E \propto 1/R^2$ near the disk. Far from the disk, $E \propto 1/z^2$.



5. (8 points) The leaves of an initially neutral electroscope will spread apart if it is touched by a charged rod, or if the rod is just brought nearby. What happens to the leaves in those two situations when the charged rod is removed?

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If the rod touches the electroscope, the electroscope will become charged, and will remain so after the rod is removed. But if the rod is just brought nearby, the electroscope will become polarized. Once the rod is removed, though, the electroscope will not remain polarized. After removal, then ...

If the rod touched, the leaves remain spread. If it was brought nearby, the leaves collapse.

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6. (8 points) A solid sphere of radius R has uniformly distributed charge $+Q$. A particle with charge $+q$ is embedded within it, a distance $R/3$ from the center. What is the magnitude of the electric force of the sphere on the particle?

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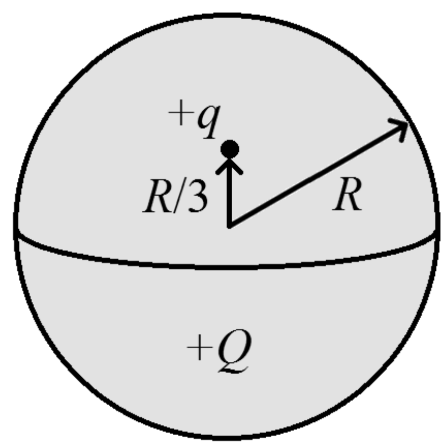

As the sphere is uniform, every "layer" of the sphere may be considered a uniform thin spherical shell. The particle is inside all the shells with radius greater than $R/3$, so those layers have no net force on it. The particle is outside all the shells with radius less than $R/3$, so each of those layers acts as if it were a point at it's own center.

The force on the particle with charge $+q$, therefore, is the same as the force from a particle $R/3$ away that has charge equal to the amount of charge less than $R/3$ from the center. That is,

$$F = K\frac{Q'q}{r'^2}$$

where $r' = R/3$, and Q' can be determined from the volume charge density, ρ. Because the sphere is uniform,

$$\rho = \frac{Q}{V} = \frac{Q'}{V'} \qquad \Rightarrow \qquad \frac{Q}{\frac{4}{3}\pi R^3} = \frac{Q'}{\frac{4}{3}\pi\left(R/3\right)^3} \qquad \Rightarrow \qquad Q' = Q/27$$

So

$$F = K\frac{\left(Q/27\right)q}{\left(R/3\right)^2} = K\frac{Qq}{3R^2}$$

7. (8 points) Assume the charge separation distance s in the dipole shown is small compared to the distance r from the negative point charge to the dipole. If the dipole is released from rest in the position shown, what will its subsequent motion, if any, be?

The positive end of the dipole will be attracted to the negative particle, and the negative end will be repelled. This will align the dipole moment with the particle's field. Then the positive end of the dipole will be in a stronger field than the negative end, so the attractive force will be greater than the repulsive force.

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