
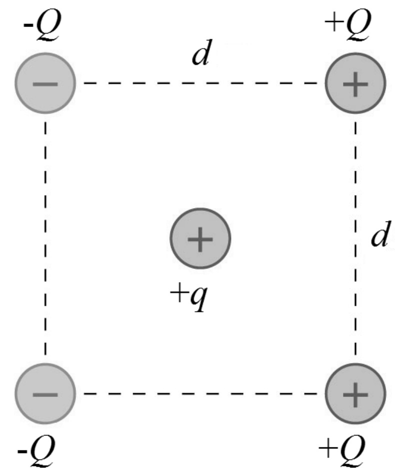

I. (16 points) Five point particles are arranged on the corners and center of a square, as shown. If the square has sides of length $d = 2.5\,\text{cm}$, and $q = +1.5\,\mu\text{C}$ and $Q = +2.2\,\mu\text{C}$, what is the magnitude of the electric force on the particle with charge $+q$ in the center?

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Use Coulomb's Law. By symmetry, we can see that the components of the force up or down the page cancel, while the components of the force to the left add. The leftward component of the force from one corner charge Q on q is

$$F_{\text{left,one}} = K\frac{Qq}{r^2}\cos\theta$$

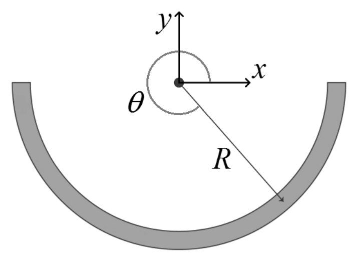
where $r = d\sqrt{2}/2$ and $\theta = 45°$ so $\cos\theta = \sqrt{2}/2$. So

$$F_{\text{left,one}} = K\frac{Qq}{\left(d\sqrt{2}/2\right)^2}\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}K\frac{Qq}{d^2}$$

Since each of the four corner charges Q exerts this same leftward component of force,

$$F_{\text{left,four}} = 4F_{\text{left,one}} = 4\sqrt{2}K\frac{Qq}{d^2} = 4\sqrt{2}\left(8.988\times10^9\,\text{N·m}^2/\text{C}^2\right)\frac{\left(2.2\times10^{-6}\,\text{C}\right)\left(1.5\times10^{-6}\,\text{C}\right)}{\left(2.5\times10^{-2}\,\text{m}\right)^2} = 270\,\text{N}$$

1. (6 points) A thin insulating rod is bent into a semi-circle of radius R about the origin from $\theta = \pi$ to $\theta = 2\pi$, as shown. It has a non-uniform linear charge density, λ, that depends on angular position according to



$$\lambda = \frac{\lambda_0}{\sin\theta}$$

where λ_0 is a positive constant. In what direction is the electric field at the origin?

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As $\sin\theta$ is negative and symmetric about the y-axis, the x components of the field due to each element of charge cancel, while the y components are all in the negative direction. The field is

In the $-y$ direction.

II. (16 points) In the problem above, what is the magnitude of the electric field at the origin? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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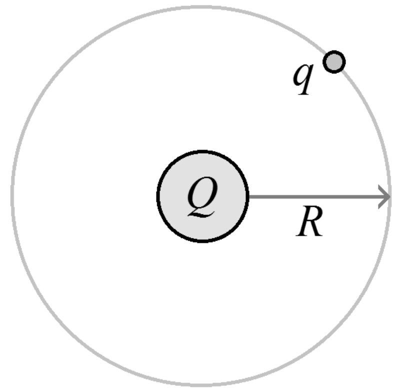
Choose point-like elements of charge $dq = \lambda\, ds$. As only the y components contribute to the field

$$E = \int dE_y = \int dE \sin\theta = \int \frac{K\,dq}{r^2}\sin\theta = \int \frac{K\lambda\,ds}{r^2}\sin\theta$$

The element of arc length $ds = r\,d\theta$, and every element of charge is the same distance $r = R$ from the point at which the field is to be determined.

$$E = \int_{\pi}^{2\pi} \frac{K\left(\frac{\lambda_0}{\sin\theta}\right) R\,d\theta}{R^2}\sin\theta = \int_{\pi}^{2\pi} \frac{K\lambda_0\,d\theta}{R} = \frac{K\lambda_0}{R}\int_{\pi}^{2\pi} d\theta = \frac{K\lambda_0}{R}\theta\Bigg|_{\pi}^{2\pi} = \frac{K\lambda_0}{R}(2\pi - \pi) = \frac{K\lambda_0\pi}{R}$$

III. (16 points) A dust speck with mass $m = 22\,\mu\text{g}$ and charge $q = +0.17\,\mu\text{C}$ is in a circular orbit around a uniform sphere with mass $M = 0.25\,\text{kg}$ and charge $Q = -3.2\,\mu\text{C}$. If the orbit has radius $R = 7.5\,\text{cm}$, what is the speed of the dust speck?



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Use Newton's Second Law. Choose an axis, c, that points in the direction of the known acceleration, toward the center of the orbit. The only force in that direction is the electric force. In terms of magnitudes,

$$\sum F_c = F_E = ma_c \qquad \Rightarrow \qquad K\frac{Qq}{r^2} = m\frac{v^2}{r}$$

Note that it is the dust speck, of mass $m = 22 \times 10^{-9}\,\text{kg}$, that is accelerating. Solve for v.

$$v = \sqrt{K\frac{Qq}{mr}} = \sqrt{(8.988 \times 10^9\,\text{N·m}^2/\text{C}^2)\,\frac{(3.2 \times 10^{-6}\,\text{C})\,(0.17 \times 10^{-6}\,\text{C})}{(22 \times 10^{-9}\,\text{kg})\,(7.5 \times 10^{-2}\,\text{m})}} = 1.7 \times 10^3\,\text{m/s}$$

2. (6 points) In the problem above, let the speed of the dust speck be v_0. If the charge of the dust speck were doubled, but the charge of the sphere were halved, what would the new speed v' of the dust speck need to be, for the same orbit to be maintained?

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The electric force depends on the product of the two charges. If one charge is doubled while the other is halved, the product remains the same. Therefore, the force remains the same, the acceleration remains the same, and the speed remains the same.

$$v' = v_0$$

3. (8 points) Identical metal spheres A and B are initially neutral and touching. A positively charged rod is brought near sphere A, but not touching. Sphere B is briefly grounded, then the charged rod is removed. What is the resulting charge on each sphere?

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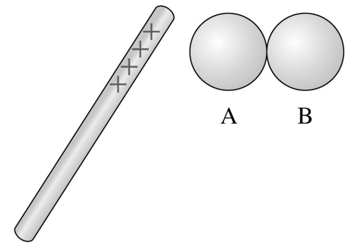

As the spheres are touching, they can be treated like one object. When the positively charged rod is brought near sphere A, the two spheres together become polarized, with sphere A becoming negatively charged and sphere B becoming positively charged. When sphere B is grounded, it will become neutral. When the rod is removed, the negative charge on sphere A will spread over both spheres.

Sphere A has negative charge. Sphere B has negative charge.

4. (8 points) A thin spherical shell has uniformly distributed positive charge $+Q$. Centered within it, a uniform solid sphere has positive charge $+2Q$. In the gap between them lies a particle with positive charge $+q$, a distance R from the center of the spheres. What is the electric force, if any, on the particle?

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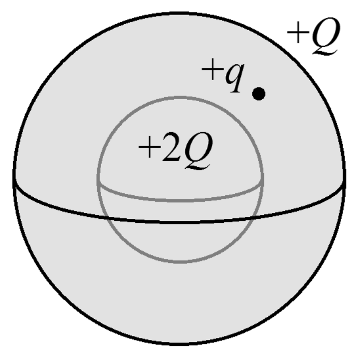

Apply the Shell Theorems. The thin spherical shell has no net force on the point charge within it. The uniform solid sphere exerts a force on the charge outside it, as if the sphere were a point in its own center. Both the solid sphere and the particle are positively charged, so the force on the particle is

$2KQq/R^2$ away from the center

5. (8 points) The particle on the left has negative charge $-q$. The particle on the right has positive charge $+4q$. At which of the indicated locations is the electric field magnitude zero?

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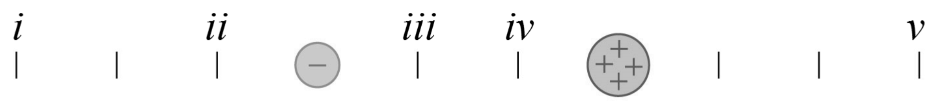

The location must be to the left of the negative charge. At locations between the charges, the field due to each charge points in the same direction (leftward), and so the net field cannot be zero. At locations to the right of the positive charge, the rightward field due to the positive charge will always be greater in magnitude than the leftward field due to the negative charge, as the charge of greater magnitude will always be closer.

As Coulomb's Law is an inverse square law, the location to the left of the negative charge, at which the fields due to each charge cancel, must be twice as far from the charge with four times the magnitude. This is

At location i.

6. (8 points) Three identical electric dipoles lie in a uniform electric field. The dipoles are far apart, and do not interact with each other. Rank the torque magnitudes about the center of each dipole, from greatest to least.

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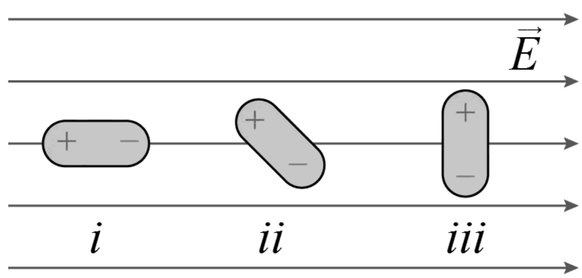


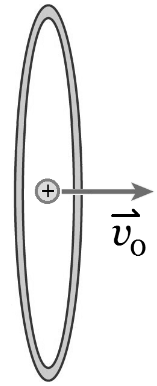
The torque on a dipole depends the dipole moment, the electric field, and the angle between them.

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \Rightarrow \qquad \tau = pE \sin\phi$$

where the direction of the dipole moment is from the negative toward the positive end of the dipole. For dipole i the angle between $\vec{p}$ and $\vec{E}$ is 180°, so the magnitude torque is zero. For dipole ii the angle between $\vec{p}$ and $\vec{E}$ is 135°, so the torque magnitude is $pE\sqrt{2}/2$. For dipole iii the angle between $\vec{p}$ and $\vec{E}$ is 90°, so the torque magnitude is pE.

$$iii > ii > i$$

7. (8 points) The ring in the illustration carries a uniformly distributed positive charge. A small charged particle moves along the axis of the ring, having speed v_0 at the instant it passes through its center. Describe the acceleration of the particle as it moves away from the ring.



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When the particle passes through the center of the ring, the electric field is zero (and thus the force on and acceleration of the particle is also zero). As the distance of the particle from the ring gets large, the electric field tends toward zero (and once again, the force on and acceleration of the particle tends toward zero). Somewhere between the center of the ring and infinity, the field, force, and acceleration must reach a maximum.

The acceleration of the particle first increases, then decreases.