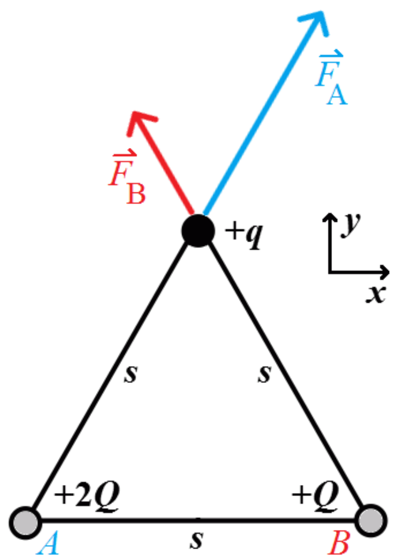

I. (16 points) Three point particles with charges $+q = +1.6\,\mu\text{C}$, $+Q = +2.2\,\mu\text{C}$, and $+2Q = +4.4\,\mu\text{C}$ lie on the vertexes of an equilateral triangle with sides of length $s = 2.5\,\text{cm}$, as shown. What is the electric force on the particle with charge $+q$?



Use Coulomb's Law. Let the force from the particle with charge $+Q$ be $\vec{F}_B$ and the force from the particle with charge $+2Q$ be $\vec{F}_A$. In terms of magnitudes,

$$F = K\frac{|q_1|\,|q_2|}{r^2} \qquad \Rightarrow \qquad F_B = K\frac{|Q|\,|q|}{s^2}$$

so

$$F_B = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2.2\,\mu\text{C})\,(1.6\,\mu\text{C})}{(0.025\,\text{m})^2} = 50.6\,\text{N}$$

The components of $\vec{F}_B$ are

$$F_{Bx} = F_B\cos 120^\circ = (50.6\,\text{N})\left(\frac{-1}{2}\right) = -25.3\,\text{N} \qquad \text{and} \qquad F_{By} = F_B\sin 120^\circ = (50.6\,\text{N})\left(\frac{\sqrt{3}}{2}\right) = 43.8\,\text{N}$$

Since $F_A = 2F_B$, and since the sketch shows that both components of $\vec{F}_A$ are positive,

$$F_x = F_{Ax} + F_{Bx} = -2F_{Bx} + F_{Bx} = -F_{Bx} = -(-25.3\,\text{N}) = 25.3\,\text{N}$$

$$F_y = F_{Ay} + F_{By} = 2F_{Bx} + F_{Bx} = 3F_{Bx} = 3\,(43.8\,\text{N}) = 132\,\text{N}$$

So

$$\vec{F} = (25\hat{\imath} + 132\hat{\jmath})\ \text{N}$$

Or, if you prefer magnitude and direction

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(25.3\,\text{N})^2 + (132\,\text{N})^2} = 134\,\text{N} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{132\,\text{N}}{25.3\,\text{N}}\right) = 79^\circ$$

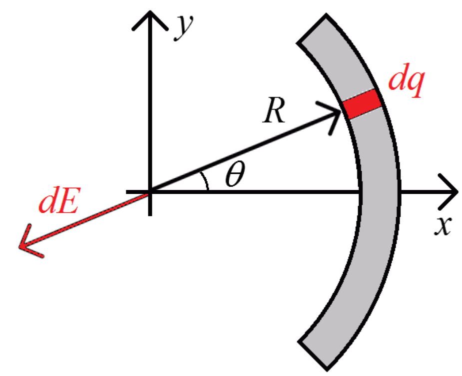
So

$$\vec{F} = 134\,\text{N} \;@\; 79^\circ$$

1. (6 points) A non-uniform thin rod of charge is bent into an arc of radius R. It extends from $\theta = -\pi/4$ to $\theta = +\pi/4$, as shown. The linear charge density λ of the rod depends on θ according to

$$\lambda = \frac{\lambda_0 \theta^2}{\cos\theta}$$

where λ_0 is a positive constant. In what direction is the electric field at the origin?



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An element of charge dq produces an element of field $d\vec{E}$ at the origin. As the charge on the rod is positive, these elements of field point away from the rod, as shown. The charge distribution is symmetric about the x axis, so the y components of the elements of field will cancel (sum to zero). The net field at the origin, then, must be

In the $-x$ direction.

II. (16 points) In the problem above, what is the magnitude of the electric field at the origin? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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An element of charge dq makes an element of field $d\vec{E}$. Since the element of charge is point-like, the magnitude of the element of field is

$$dE = K\frac{dq}{r^2}$$

As the rod is symmetrical about the x axis, the y components of the field cancel.

$$E = \int dE_x = \int dE\cos\theta = \int K\frac{dq}{r^2}\cos\theta$$

Since the arc is circular, $r = R$, a constant. The element of charge can be related to θ through an element of arc length ds.

$$\lambda = \frac{dq}{ds} \qquad \Rightarrow \qquad dq = \lambda\, ds = \frac{\lambda_0\theta^2}{\cos\theta} R\, d\theta$$

So

$$E = \int_{-\pi/4}^{+\pi/4} K\frac{\frac{\lambda_0\theta^2}{\cos\theta} R\, d\theta}{R^2}\cos\theta = \frac{K\lambda_0}{R}\int_{-\pi/4}^{+\pi/4}\theta^2\, d\theta = \frac{K\lambda_0}{R}\left[\frac{\theta^3}{3}\right]_{-\pi/4}^{+\pi/4}$$

$$= \frac{K\lambda_0}{3R}\left[\left(\frac{\pi}{4}\right)^3 - \left(\frac{-\pi}{4}\right)^3\right] = \frac{K\lambda_0}{3R}\left[\frac{\pi^3}{32}\right] = \frac{K\lambda_0\pi^3}{96R}$$

III. (16 points) An infinitely long hollow insulating cylinder has uniform volume charge density ρ. Its inner radius is R, and its outer radius is $2R$. What is the magnitude of the electric field at point P, a distance $3R$ from the cylinder axis? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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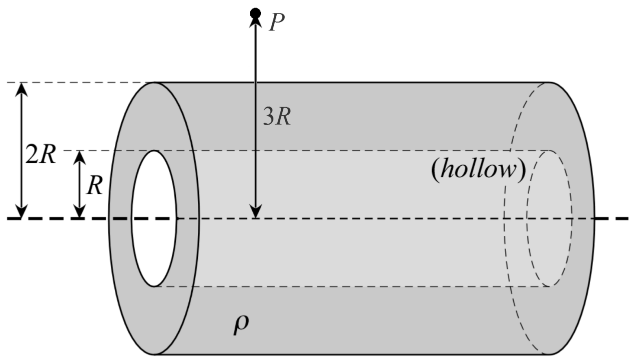

Use Gauss' Law, $\epsilon_0\Phi = q_{\text{in}}$. Choose a Gaussian Surface that passes through the point at which the field is to be found, and that has the symmetry of the charge distribution. In this case, the Gaussian Surface will be a cylinder co-axial with the charge, having a radius $3R$, and an arbitrary length L.

The flux through the ends of this cylindrical Gaussian Surface is zero, as the field and area vectors are perpendicular. The flux through the curved side of the cylindrical Gaussian Surface is EA_{side}, as the field has uniform magnitude on the curved side, and the field and area vectors are parallel at every point.

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \int_{\text{ends}} \vec{E}\cdot d\vec{A} + \int_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = E\,2\pi rL = E\,2\pi\,(3R)\,L = E\,6\pi RL$$

The charge within the Gaussian Surface is

$$q_{\text{in}} = \rho V = \rho\left[\pi\,(2R)^2\,L - \pi R^2 L\right] = \rho\,3\pi R^2 L$$

Putting these together and solving for the field

$$\epsilon_0\Phi = q_{\text{in}} \qquad \Rightarrow \qquad \epsilon_0 E\,6\pi RL = \rho\,3\pi R^2 L \qquad \Rightarrow \qquad E = \frac{\rho R}{2\epsilon_0}$$

2. (6 points) In the problem above, let the magnitude of the electric field at point P be E_0. What is the magnitude of the electric field at a distance $R/3$ from the cylinder axis?

. . . .

A cylindrical Gaussian Surface co-axial with the charge, having a radius $R/3$ and an arbitrary length L, contains no charge. There is no flux through this surface, so the electric field magnitude is

Zero

3. (8 points) An electric dipole is near a positively-charged particle, as shown. If the particle is fixed in place but the dipole is free to move, what will the motion of the dipole be?

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The negative end of the dipole will be attracted to the positively-charged particle, while the positive end will be repelled, causing the dipole to rotate counterclockwise. The negative end of the dipole, now closer to the particle, will be in a region of greater field strength than the positive end, so the attractive force will have a greater magnitude then the repulsive force.

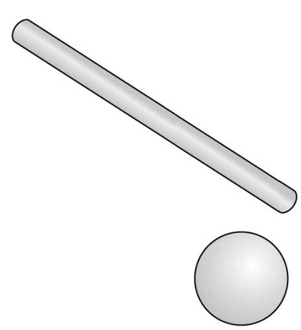
The dipole will rotate and move toward the particle.

4. (8 points) A rod is rubbed with fabric to give it a non-zero charge. When the rod is brought near a neutral object, the neutral object will ...

. . . .

The charged rod will polarize the neutral object. The non-uniform field due to the rod will produce a greater attractive force on the near edge of the sphere and a lesser repulsive force on the far side of the sphere. The net effect is that the neutral object will ...

be attracted to the rod.

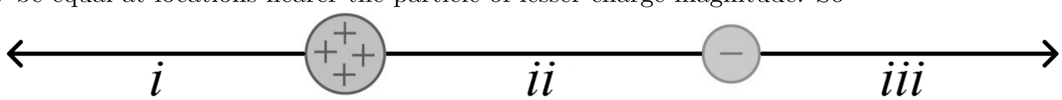


5. (8 points) Consider two charged particles, one with positive charge $+4Q$ and one with negative charge $-Q$, as shown. Is there any point along a line through the two particles, other than at infinite distance, at which the electric field due to the two particles is zero?

. . . .

If there is such a point, it cannot be between the particles. A probe charge placed between the particles will be attracted to one particle and repelled by the other. The force on that probe charge will not be zero at any point between the particles, so the electric field cannot be zero at any point between the particles.

On the line to the left or right of the particles, the forces of the particles on a probe charge will be in opposite directions. At a location with zero field, there will be no net force on a probe charge, so the magnitudes of the forces from the particles must be equal. Because Coulomb's Law is an inverse-square law, the magnitudes can only be equal at locations nearer the particle of lesser charge magnitude. So



Yes, somewhere to the right of the negatively charged particle (region *iii*).

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6. (8 points) An electron is given an initial velocity $\vec{v}_0$ inside an ideal parallel-plate capacitor, as shown. Describe the path of the electron's motion.

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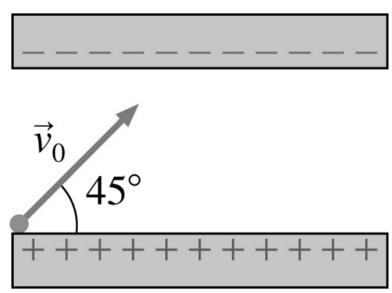
The ideal parallel-plate capacitor has a uniform electric field within it. The electron, then, is subject to a constant force and acceleration on one axis, and zero force and acceleration on the other axis. Like an object launched in a uniform gravitational field near the Earth's surface,

The electron's path will be a parabola.



7. (8 points) A hollow conducting sphere has a solid conducting sphere at its center, as shown in cross-section. The outer, hollow, sphere has a net positive charge $+Q_0$, while the inner, solid, sphere has a net positive charge $+q_0$. The solid sphere is now moved until it touches the inner surface of the hollow sphere, then is returned to the center. How, if at all, will charge have moved between the spheres?

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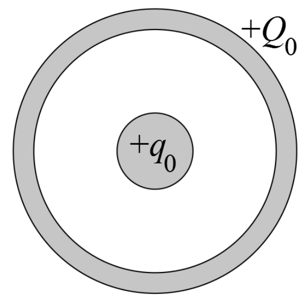
Once the two spheres come into contact, they act as a single conductor. There is no electric field in a conductor at equilibrium, so no electric flux through *any* Gaussian Surface inside the conductor, so no net charge within *any* Gaussian Surface inside the conductor.

All the charge on the inner, solid, sphere moves to the outer, hollow, sphere.



Quiz and Exam Formulæ & Constants

$$K = \frac{1}{4\pi\epsilon_0}$$
$$\Delta V = -\int \vec{E}\cdot d\vec{s}$$
$$V = K\frac{q}{r}$$
$$\Delta U = q\,\Delta V$$
$$I = dq/dt$$
$$P = I\,\Delta V$$
$$R = \frac{\Delta V}{I}$$

Series :

$$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$$
$$R_{\text{eq}} = \sum R_i$$

Parallel :

$$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_i}$$
$$C_{\text{eq}} = \sum C_i$$

$$\vec{E} = K\frac{q}{r^2}\hat{r}$$
$$\vec{F} = K\frac{q_1 q_2}{r^2}\hat{r}$$
$$\vec{F} = q\vec{E}$$
$$\vec{p} = q\vec{d}$$
$$\vec{\tau} = \vec{p}\times\vec{E}$$
$$U = -\vec{p}\cdot\vec{E}$$
$$|\vec{E}| \propto \frac{|\vec{p}|}{r^3}$$
$$\Phi_{\text{E}} = \int \vec{E}\cdot d\vec{A}$$
$$\epsilon_0 \oint \vec{E}\cdot d\vec{A} = q_{\text{enclosed}}$$
$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_{\text{B}}}{dt}$$
$$C = \frac{Q}{\Delta V}$$
$$C = \epsilon_0\frac{A}{d}$$
$$U = \tfrac{1}{2}C\,[\Delta V]^2$$
$$R = \rho\frac{\ell}{A}$$
$$\tau_{\text{C}} = RC$$
$$u_{\text{E}} = \tfrac{1}{2}\epsilon_0 E^2$$

$$\vec{B} = \frac{\mu_0 q}{4\pi}\,\frac{\vec{v}\times\hat{r}}{r^2}$$
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{\ell}\times\hat{r}}{r^2}$$
$$\vec{F} = q\vec{v}\times\vec{B}$$
$$\vec{F} = I\vec{\ell}\times\vec{B}$$
$$\vec{\mu} = NI\vec{A}$$
$$\vec{\tau} = \vec{\mu}\times\vec{B}$$
$$U = -\vec{\mu}\cdot\vec{B}$$
$$\Phi_{\text{B}} = \int \vec{B}\cdot d\vec{A}$$
$$\oint \vec{B}\cdot d\vec{A} = 0$$
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_{\text{c}} + I_{\text{d}})$$
$$L = \frac{\Phi_{\text{B}}}{I}$$
$$L = \mu_0 N^2\frac{A}{\ell}$$
$$U = \tfrac{1}{2}LI^2$$
$$B = \mu_0 n I$$
$$\tau_{\text{L}} = L/R$$
$$u_{\text{B}} = \frac{1}{2\mu_0}B^2$$

$$q = q_{\max}\left(1 - e^{-t/\tau_{\text{C}}}\right)$$
$$q = q_0 e^{-t/\tau_{\text{C}}}$$
$$I = I_{\max}\left(1 - e^{-t/\tau_{\text{L}}}\right)$$
$$I = I_0 e^{-t/\tau_{\text{L}}}$$
$$I = \int \vec{J}\cdot d\vec{A}$$
$$\vec{J} = \sigma\vec{E}$$
$$\mathcal{E} = -N\frac{d\Phi_{\text{B}}}{dt}$$
$$I_d = \epsilon_0\,\frac{d\Phi_{\text{E}}}{dt}$$
$$\mathcal{E} = -L\,\frac{dI}{dt}$$
$$c = f\lambda = \frac{|\vec{E}|}{|\vec{B}|}$$
$$\vec{S} = \frac{1}{\mu_0}\,\vec{E}\times\vec{B}$$

Fundamental Charge $e = 1.602\times10^{-19}\,\text{C}$
Earth's gravitational field $g = 9.81\,\text{N/kg}$
Coulomb constant $K = 8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2$
Speed of Light $c = 2.998\times10^{8}\,\text{m/s}$

Mass of an Electron $m_{\text{e}} = 9.109\times10^{-31}\,\text{kg}$
Mass of a Proton $m_{\text{p}} = 1.673\times10^{-27}\,\text{kg}$
Vacuum Permittivity $\epsilon_0 = 8.854\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$
Vacuum Permeability $\mu_0 = 4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}$

Unless otherwise directed, friction, drag, and gravity should be neglected, and all batteries and wires are ideal.
All derivatives and integrals in free-response problems must be evaluated.