
I. (16 points) An electron is orbiting a charged dust speck, in a circular orbit of radius $880\,\mu$m. If the dust speck has a charge of $+2.2 \times 10^{-12}$ C, what is the speed of the electron?

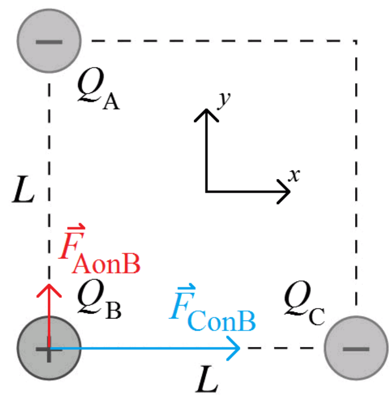
Apply Newton's Second Law to the electron. The only force on the electron is the electric force from the dust speck. Choose a coordinate system. I'll choose the "c" axis to point toward the center of the orbit, as that is the known direction of the electron's acceleration.

$$\sum F_c = F_E = ma_c \qquad \Rightarrow \qquad K\frac{q_1 q_2}{r^2} = m\frac{v^2}{r}$$

Solve for v.

$$v = \sqrt{K\frac{q_1 q_2}{mr}} = \sqrt{8.988 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}\frac{(2.2 \times 10^{-12}\,\mathrm{C})\,(1.602 \times 10^{-19}\,\mathrm{C})}{(9.109 \times 10^{-31}\,\mathrm{kg})\,(880 \times 10^{-6}\,\mathrm{m})}} = 2.0 \times 10^6\ \mathrm{m/s}$$

II. (16 points) Three particles are located on the vertexes of a square with sides having length $L = 12\,\text{cm}$, as shown. If $Q_\text{A} = -1.0\,\text{nC}$, $Q_\text{B} = +2.0\,\text{nC}$, and $Q_\text{C} = -3.0\,\text{nC}$, what is the magnitude of the force on particle B?



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As Q_A and Q_C are negative, while Q_B is positive, the forces on Q_B are attractive, as shown. The magnitude of the force of Q_A on Q_B is

$$F_\text{AonB} = K\frac{Q_\text{A}Q_\text{B}}{r^2} = 8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\,\frac{(1.0\,\text{nC})\left(2.0\times10^{-9}\,\text{C}\right)}{(0.12\,\text{m})^2}$$

$$= 1.25\times10^3\,\text{nN} = 1.25\mu\text{N}$$

As $Q_\text{C} = 3Q_\text{A}$, $F_\text{ConB} = 3F_\text{AonB}$.

$$F_\text{ConB} = 3\,(1.25\mu\text{N}) = 3.75\mu\text{N}$$

As F_AonB and F_ConB are perpendicular, the Pythagorean Theorem can be used to find the the total force.

$$F_\text{tot} = \sqrt{F^2_\text{AonB} + F^2_\text{ConB}} = \sqrt{(1.25\mu\text{N})^2 + (3.75\mu\text{N})^2} = 3.9\mu\text{N}$$

1. (6 points) What is the direction of the force on particle B?

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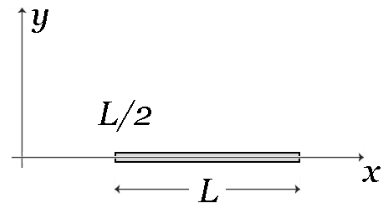
From the forces sketched on the diagram, it can be seen that the total force must lie in the first quadrant, and be less than 45°. The only choice in that range is

$$18°$$

III. (16 points) A thin rod of length L lies on the $+x$ axis, with one end at $+L/2$ and the other at $+3L/2$, as illustrated. The linear charge density, λ, of the rod depends on position, x, according to

$$\lambda = \lambda_0 \left(\frac{L}{x}\right)$$



where λ_0 is a positive constant. What is the magnitude of the electric field at the origin? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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Imagine the rod broken into point-like bits, each with length dx and charge dq. Each bit would produce an element of field at the origin, with magnitude $dE = K\,dq/r^2$. The distance, r, from each bit to the origin is x, and the charge of each bit can be related to its length by $dq = \lambda\,dx$. Adding up the contributions by each bit is an integral.

$$E = \int dE = \int \frac{K\,dq}{r^2} = \int_{L/2}^{3L/2} \frac{K\lambda\,dx}{x^2} = K \int_{L/2}^{3L/2} \frac{\lambda_0 L/x}{x^2}\,dx = K\lambda_0 L \int_{L/2}^{3L/2} \frac{dx}{x^3} = K\lambda_0 L \int_{L/2}^{3L/2} x^{-3}\,dx$$

$$= K\lambda_0 L \left[\frac{x^{-2}}{-2}\right]_{L/2}^{3L/2} = \frac{-K\lambda_0 L}{2}\left[\frac{1}{(3L/2)^2} - \frac{1}{(L/2)^2}\right] = \frac{-K\lambda_0 L}{2}\left[\frac{4}{9L^2} - \frac{4}{L^2}\right]$$

$$= \frac{-2K\lambda_0 L}{L^2}\left[\frac{1}{9} - 1\right] = \frac{-2K\lambda_0}{L}\left[-\frac{8}{9}\right] = \frac{16K\lambda_0}{9L}$$

Each factor is positive, so the magnitude is also

$$\frac{16K\lambda_0}{9L}$$

2. (6 points) In the problem above, what is the direction, if any, of the electric field at the origin?

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As the linear charge density, $\lambda = \lambda_0 L/x$ always positive in the region $x = L/2$ to $x = 3L/2$, all the charge on the rod is positive. A positive test charge at the origin would be repelled by each bit of this positive charge, so the electric field there is

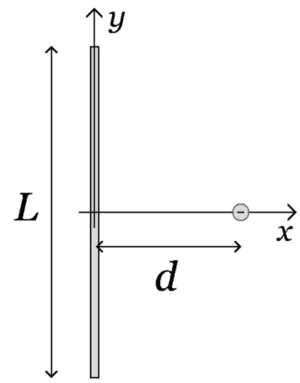
In the $-x$ direction.

3. (8 points) A non-uniform line segment of charge with length L lies centered on the y axis. Its linear charge density, λ, depends on position, y, according to

$$\lambda = \lambda_0 y^5$$

where λ_0 is a positive constant.

An electron is placed at a position $+d$ on the x axis. What is the direction of the electric field due to the line segment of charge at the location of the electron?



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Announced during the quiz: The line segment is centered on the y axis.

The linear charge density is an odd function of y. That is, for each point at a position $+y$ on the positive y axis, there is a point at a position $-y$ on the negative y axis that has a charge density of the same magnitude, but opposite sign.

The electric field at a location is in the same direction as the electric force on an imagined positive probe charge. If there were a positive probe charge at the location of the electron (the electron itself is irrelevant), it would be repelled by the positively charged portion of the rod on the $+y$ axis, and attracted to the negatively charged portion of the rod on the $-y$ axis. From the symmetry of the problem, the x components of these forces add to zero, and only the y components, which all point in the $-y$ direction, remain.

The electric field at the electron is in the $-y$ or $-\hat{\jmath}$ direction.

4. (8 points) A dipole is released near a negatively-charged particle, as shown. What is the subsequent motion of the dipole?

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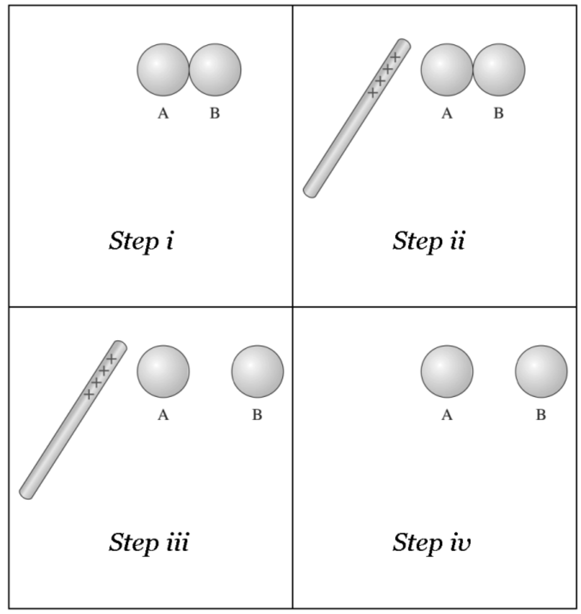
The negative end of the dipole will be repelled from the particle, while the positive end will be attracted. The positive end of the dipole, now closer to the particle, will be in a region of greater field strength that the negative end, so the attractive force will have a greater magnitude then the repulsive force.

It rotates clockwise and moves closer to the negative charge.

5. (8 points) Two neutral conducting spheres, A and B, are in contact (*Step i*). A positively charged rod is brought near, but not touching, sphere A (*Step ii*). The spheres are separated (*Step iii*), and then the rod is removed (*Step iv*). Describe the charge of each sphere after *Step iv*.



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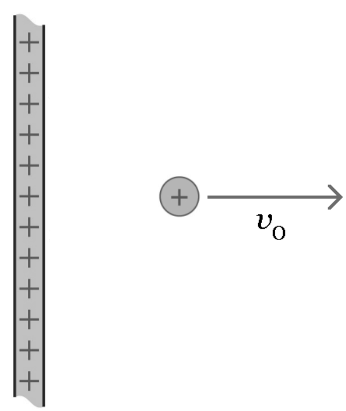
The spheres are in contact in *Step i*, and so will act as a single conductor. In *Step ii* that conductor is polarized, with sphere A being the negative end (electrons are attracted to the positively charged rod), and sphere B being the positive end (as it is now deficient in electrons). Separating the spheres in *Step iii* traps those charges on the spheres, and they remain that way in *Step iv* when the rod is removed.

A is negative, B is positive.

6. (8 points) A particle with positive charge q and mass m has initial velocity $\vec{v}_0$, moving away from the positively and uniformly charged infinite **rod** shown in the figure. Which statement best describes the subsequent motion of the particle?

There is always a net force on the particle, directed away from the rod.

It will keep moving away from the rod, with increasing speed.



7. (8 points) How does the electric field magnitude due to an infinite uniformly charged sheet depend on the distance from the sheet?

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An infinite uniformly charged sheet creates uniform field.