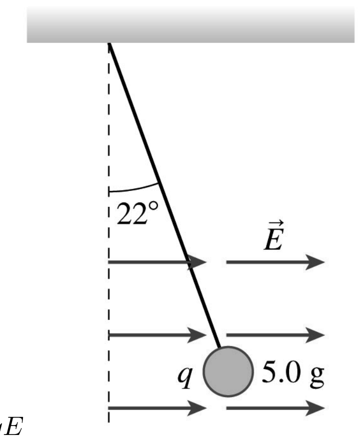
I. (16 points) The small sphere has a mass of 5.0 g, and a charge $q = +13\,\text{nC}$. It hangs by an ideal cord at an angle of 22° from the vertical, as shown. What is the magnitude of the horizontal electric field? (*On Earth, do NOT neglect gravity.*)



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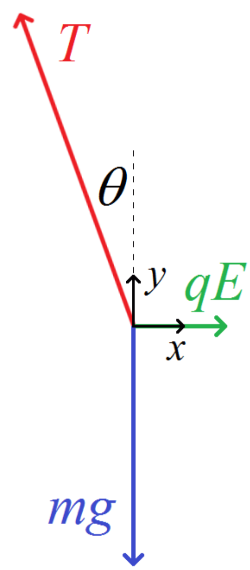
Use Newton's Second Law. Sketch a Free Body Diagram. There will be a tension force T parallel to the cord, an electric force qE to the right, and a gravitational force mg downward. Choose a coordinate system. I'll let x be positive to the right and y be positive upward. Resolve T into components. Write out Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes.

In the x direction:

$$\sum F_x = qE - T_x = ma_x = 0 \qquad \Rightarrow \qquad T\sin\theta = qE \qquad \Rightarrow \qquad T = \frac{qE}{\sin\theta}$$



In the y direction:

$$\sum F_y = T_y - mg = ma_y = 0 \qquad \Rightarrow \qquad T\cos\theta = mg \qquad \Rightarrow \qquad T = \frac{mg}{\cos\theta}$$

Eliminate the unknown tension by setting the two expressions equal to each other. Solve for E.

$$\frac{qE}{\sin\theta} = \frac{mg}{\cos\theta} \qquad \Rightarrow \qquad E = \left(\frac{mg}{q}\right)\frac{\sin\theta}{\cos\theta} = \left(\frac{mg}{q}\right)\tan\theta$$

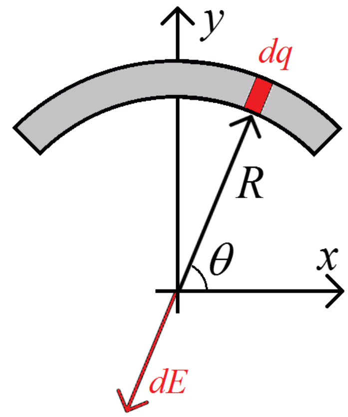
$$= \left(\frac{\left(5.0\times10^{-3}\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)}{13\times10^{-9}\,\text{C}}\right)\tan 22^\circ$$

$$= 1.5\times10^{6}\,\text{N/C}$$

1. (6 points) A non-uniform thin rod of charge is bent into an arc of radius R. It extends from $\theta = \pi/4$ to $\theta = 3\pi/4$, as shown. The linear charge density λ of the rod depends on θ according to

$$\lambda = \frac{\lambda_0}{\sin\theta}$$

where λ_0 is a positive constant. In what direction is the electric field at the origin?



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An element of charge dq produces an element of field $d\vec{E}$ at the origin. As the charge on the rod is positive, these elements of field point away from the rod, as shown. The charge distribution is symmetric about the y axis, so the x components of the elements of field will cancel (sum to zero). The net field at the origin, then, must be

In the $-y$ direction.

II. (16 points) In the problem above, what is the magnitude of the electric field at the origin? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

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An element of charge dq makes an element of field $d\vec{E}$. Since the element of charge is point-like, the magnitude of the element of field is

$$dE = K\frac{dq}{r^2}$$

As the rod is symmetrical about the y axis, the x components of the field cancel.

$$E = \int dE_y = \int dE\sin\theta = \int K\frac{dq}{r^2}\sin\theta$$

Since the arc is circular, $r = R$, a constant. The element of charge can be related to θ through an element of arc length ds.

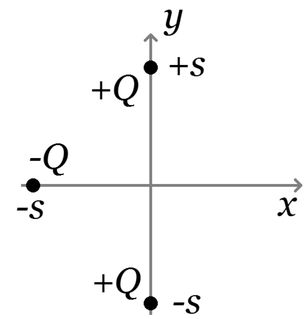
$$\lambda = \frac{dq}{ds} \qquad \Rightarrow \qquad dq = \lambda\, ds = \frac{\lambda_0}{\sin\theta}R\,d\theta$$

So

$$E = \int_{\pi/4}^{3\pi/4} K\frac{\frac{\lambda_0}{\sin\theta}R\,d\theta}{R^2}\sin\theta = \frac{K\lambda_0}{R}\int_{\pi/4}^{3\pi/4} d\theta = \frac{K\lambda_0}{R}\theta\Big|_{\pi/4}^{3\pi/4}$$

$$= \frac{K\lambda_0}{R}\left[\frac{3\pi}{4} - \frac{\pi}{4}\right] = \frac{K\lambda_0}{R}\left[\frac{\pi}{2}\right] = \frac{K\lambda_0\pi}{2R}$$

III. (16 points) Two small insulating spheres are centered at $y = \pm s$, as shown. These spheres have a uniformly distributed positive charge $+Q$. A test charge is centered at $x = -s$ with negative charge $-Q$. What is the electric **field** at the center of the test charge, in terms of parameters defined in the problem and physical or mathematical constants?



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The electric field magnitude due to a point charge is

$$E = \frac{Kq}{r^2}$$

The charge $+Q$ located at $y = +s$ is a distance $r = s\sqrt{2}$ from the test charge, so it contributes field with magnitude

$$E_{+s} = \frac{KQ}{\left(s\sqrt{2}\right)^2} = \frac{KQ}{2s^2}$$

This contribution points directly away from the charge at $y = +s$, so its components are

$$E_{+s,x} = \frac{KQ}{2s^2}\cos\left(225°\right) = \frac{-KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right)$$

$$E_{+s,y} = \frac{KQ}{2s^2}\sin\left(225°\right) = \frac{-KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right)$$

Similarly, the charge at $y = -s$ contributes field that points directly away from it, so its components are

$$E_{+s,x} = \frac{KQ}{2s^2}\cos\left(135°\right) = \frac{-KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right)$$

$$E_{+s,y} = \frac{KQ}{2s^2}\sin\left(135°\right) = \frac{+KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right)$$

The total field at the location of the test charge, then, is

$$E_x = E_{+s,x} + E_{-s,x} = \frac{-KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right) + \frac{-KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right) = -2\frac{KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right) = \frac{-KQ}{\sqrt{2}s^2}$$

$$E_y = E_{+s,y} + E_{-s,y} = \frac{-KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right) + \frac{KQ}{2s^2}\left(\frac{1}{\sqrt{2}}\right) = 0$$

So

$$\vec{E} = \frac{-KQ}{\sqrt{2}s^2}\hat{\imath}$$

2. (6 points) In the problem above, what happens to the electric field at the point $x = -s$ if the sphere of charge $-Q$ is replaced with a positive test charge $+Q$?

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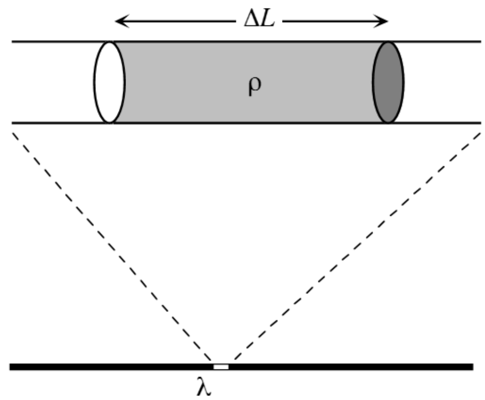
As the electric field at a point in space does not depend on what, is anything, is located there,

It remains the same in magnitude and direction.

3. (8 points) An insulating rod of radius R has some charge *uniformly* distributed throughout its volume, as a density ρ. At large distances from the rod, the cross-sectional area will not be noticeable, and the charge will seem to be a linear density λ. How is this effective λ related to the actual ρ for the rod?



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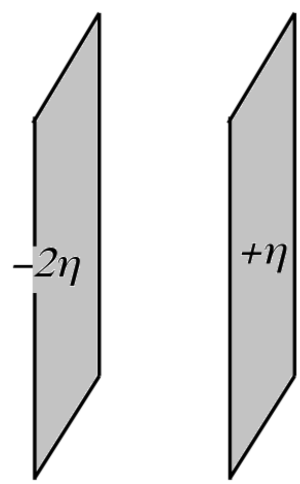
Consider a segment of length ΔL, containing a charge Q. That charge Q must be the same regardless of whether it is calculated as a linear charge density times a length, or a volume charge density times a volume.

$$Q = \lambda \, \Delta L = \rho V$$

Substitute the volume of a cylinder $V = \pi R^2 \, \Delta L$, and solve for λ.

$$\lambda \, \Delta L = \rho \, \pi R^2 \, \Delta L \qquad \Rightarrow \qquad \lambda = \rho \, \pi R^2$$

4. (8 points) A pair of charged infinite sheets aligned parallel to one another, as shown in the diagram below. What will be the magnitude and direction of the electric field at a point to the left of both sheets?



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In general, the electric field due to a uniform infinite sheet of charge of area charge density η is

$$E = \frac{\eta}{2\epsilon_0}$$

The left sheet has negative charge, so the field it creates points toward it. The right sheet has positive charge, so the field it creates points away from it. Therefore, the field due to each sheet individually is

$$E_{\text{left}} = 2\frac{\eta}{2\epsilon_0} \quad \text{rightward, and} \qquad E_{\text{right}} = \frac{\eta}{2\epsilon_0} \quad \text{leftward}$$

Adding these together, the net field is

$$\frac{\eta}{2\epsilon_0} \quad \text{to the right}$$

5. (8 points) An electron moves through a uniform electric field which points directly to the right. Which of the following diagrams could **NOT** represent a possible trajectory for the electron at any time? Assume each straight-line path is perfectly straight, and that each curved path is parabolic in shape. Do **NOT** assume the electron is simply released from rest in every case.

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An electron, having negative charge, will experience an leftward force in a rightward electric field. It will, therefore, have a leftward acceleration. The electron could move directly to the left at increasing speed, or directly to the right at decreasing speed. Its path could curve to the left in a parabolic arc. Its path could *not*, however, curve to the right in a parabolic arc like the path shown.

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6. (8 points) Charged particle q_1 exerts a force magnitude $F_{1\text{on}2}$ on charged particle q_2. If the charge of each particle is tripled (increased by a factor of 3), and the distance between the particles is doubled (increased by a factor of 2), what is the new force magnitude $F'_{1\text{on}2}$?

. . . .

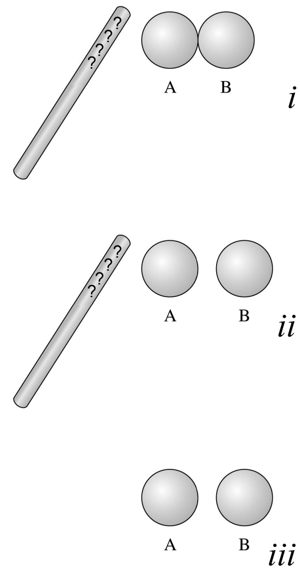
Use Coulomb's Law. In terms of magnitudes,

$$F_{1\text{on}2} = K\frac{q_1 q_2}{r_{1\text{to}2}^2}$$

The new force is

$$F'_{1\text{on}2} = K\frac{q'_1 q'_2}{r'^2_{1\text{to}2}} = K\frac{3q_1\, 3q_2}{(2r_{1\text{to}2})\, 2} = \frac{9}{4}K\frac{q_1 q_2}{r_{1\text{to}2}^2} = \frac{9}{4}F_{1\text{on}2}$$

7. (8 points) Two identical neutral conducting spheres are in contact when a rod with non-zero charge is brought nearby (*i*). The spheres are then separated (*ii*), and the rod is taken away (*iii*). Compare the resulting charge of each sphere to that of the rod.



When the charged rod is brought nearby, the two spheres will become polarized as if they were one object. Charge like the rod will be repelled to sphere B, and charge unlike the rod will be attracted to sphere A. When the spheres are separated, these charges becomes trapped on each sphere, so the spheres will retain those charges when the rod is removed.

Sphere A has charge opposite the sign of the rod, while
Sphere B has charge of the same sign as the rod.