Physics 2212G/H
Spring 2017
Test 4

Test form 742

Name Solutions

Recitation Section (see back of test):________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

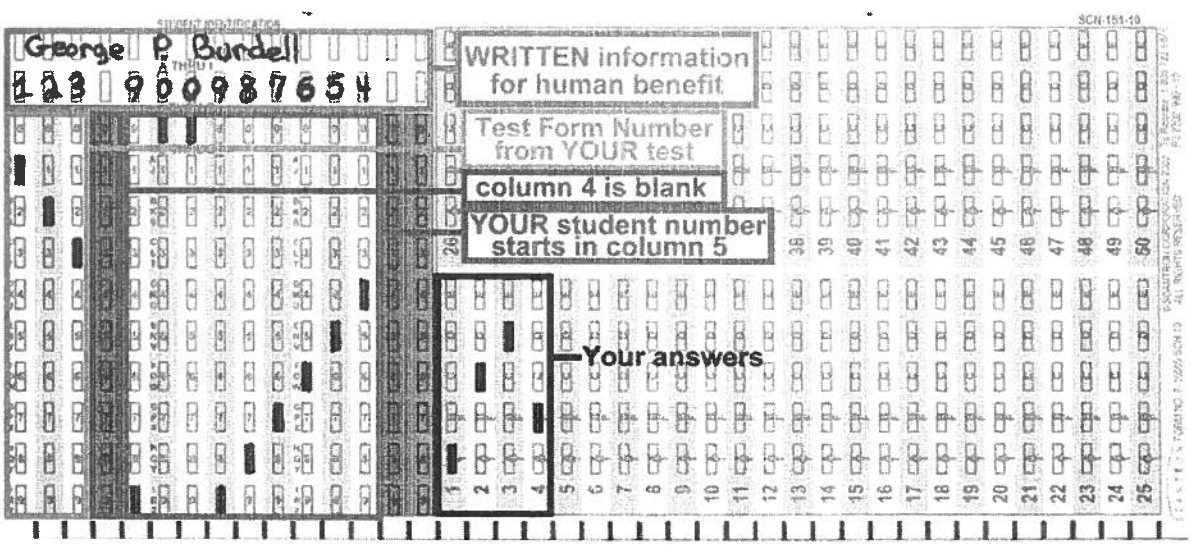
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to T-Square after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99 \times 10^{9}\ \mathrm{N{\cdot}m^2/C^2}$	$e = 1.60 \times 10^{-19}\ \mathrm{C}$	$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$
$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T{\cdot}m/A}$	$m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$

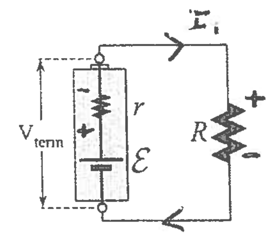
Your test form is: 742



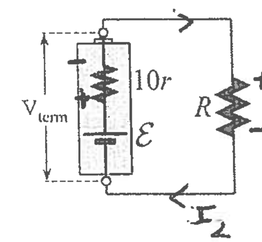
Our final exam will be on Monday, May 1
from 6:00 pm to 8:50 pm

The following problem will be hand-graded. Show all your work for this problem. *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* All *real* batteries have an internal resistance in addition to their emf. Consider a fresh battery with emf $\mathcal{E}$ and (small) internal resistance r. When the fresh battery is hooked up to a load resistance R, it is observed to have a *reduced* "terminal potential", $\Delta V_{term} = \frac{9}{10}\mathcal{E}$.



Suppose that a used battery—with the same emf but internal resistance $10r$—is hooked up to the same load resistance R. What will be the terminal potential across the used battery in that situation? Express your answer as a fraction of $\mathcal{E}$.



Hint: start by finding the internal resistance r of the fresh battery, as a fraction of R.

Fresh battery: loop rule

$$+\mathcal{E} - I_1 r - I_1 R = 0 \rightarrow \text{unknowns } r, I_1$$

but also: terminal potential = total ΔV across battery:

$$V_{Term} = \Delta V_{batt} = +\mathcal{E} - I_1 r = \frac{9}{10}\mathcal{E} \Rightarrow \underline{I_1 r = \frac{1}{10}\mathcal{E}}$$

back to loop rule equation $\rightarrow$ $\mathcal{E} - \left(\frac{1}{10}\mathcal{E}\right) - I_1 R = 0 \Rightarrow \frac{9}{10}\mathcal{E} = I_1 R$

$$\boxed{I_1 = \frac{9\mathcal{E}}{10R}}$$

Now that we know I_1, find r

V_{Term} (fresh) $= \mathcal{E} - I_1 r = \frac{9}{10}\mathcal{E}$

$$\mathcal{E} - \frac{9\mathcal{E}}{10R} r = \frac{9}{10}\mathcal{E} \rightarrow 1 - \frac{9r}{10R} = \frac{9}{10} \rightarrow \frac{9}{\cancel{10}}\frac{r}{R} = \frac{1}{\cancel{10}}$$

$$\boxed{r = \frac{R}{9}}$$

Now look at circuit with used battery

$$+\mathcal{E} - I_2(10r) - I_2 R = 0$$

$$\mathcal{E} - I_2\left(\frac{10}{9}R + R\right) = 0 \rightarrow \boxed{I_2 = \frac{9\mathcal{E}}{19R}}$$

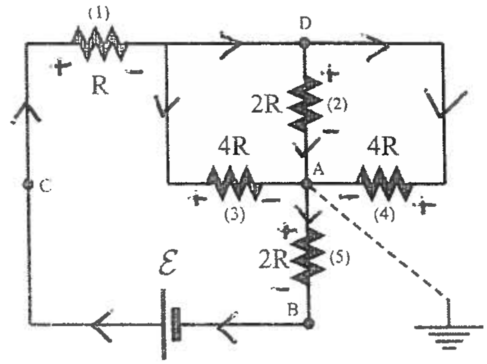
knowing current, we can now compute V_{Term} for used battery

$$V_{Term} = \Delta V_{batt} = \mathcal{E} - I_2(10r) = \mathcal{E} - \left(\frac{9\mathcal{E}}{19R}\right)\left(\frac{10}{9}R\right) = \mathcal{E} - \frac{10}{19}\mathcal{E}$$

$$\boxed{\Delta V_{batt\,(used)} = \frac{9}{19}\mathcal{E}} \approx 0.47\,\mathcal{E}$$

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] *(20 points)* In the circuit diagram at right, R represents an unspecified resistance, and each of the five labeled resistors, (1)—(5), has a resistance that is an integer multiple of R. The network is hooked up to an ideal battery having emf $\mathcal{E}$.

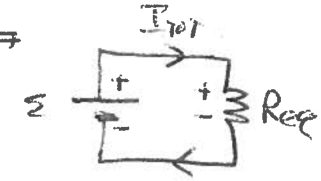


(*i*) Determine the currents—magnitude AND direction through each of the resistors. Express each answer in terms of $\mathcal{E}$ and R.

① Note that 234 are in parallel

$$R_{234} = \left(\frac{1}{2R} + \frac{1}{4R} + \frac{1}{4R}\right)^{-1} = \left(\frac{1}{R}\right)^{-1} = R$$

② Note that 1, (234), and 5 are in series $R_{TOT} = R + R + 2R = \underline{4R}$



$\Rightarrow$ Total current through emf is found from simple loop

$$+\mathcal{E} - I_{TOT} R_{eq} = 0 \qquad \boxed{I_{TOT} = \frac{\mathcal{E}}{4R}}$$

$\rightarrow$ Current through (1) is $\boxed{\frac{\mathcal{E}}{4R}, \text{ rightward}}$

$\rightarrow$ current through (5) is $\boxed{\frac{\mathcal{E}}{4R}, \text{ downward}}$

③ After resistor (1), current splits

$$\boxed{I_1 = I_2 + I_3 + I_4}$$

but 234 in parallel means $\Delta V_2 = \Delta V_3 = \Delta V_4$

$$\Rightarrow -I_4(4R) = -I_3(4R) = -I_2(2R)$$

so $\boxed{I_3 = I_4 = \frac{1}{2} I_2}$

these two current equations imply:

$$I_2 = \frac{1}{2} I_1 = \boxed{\frac{\mathcal{E}}{8R}, \text{ downward}}$$

$$I_3 = \frac{1}{2} I_2 = \frac{1}{4} I_1 = \boxed{\frac{\mathcal{E}}{16R}, \text{ rightward}} \quad \text{and} \quad \boxed{I_4 = \frac{\mathcal{E}}{16R}, \text{ leftward}}$$

Extra Credit: 4 points

(*ii*) If the circuit is grounded at position *A*, determine the voltages at positions *B*, *C*, and *D*. Express each answer as a multiple or fraction of $\mathcal{E}$. *Be sure to include the proper sign!*

If $\boxed{V_A \equiv 0}$ then $V_B = V_A + \Delta V(A \rightarrow B) = 0 + \Delta V_{(5)} = 0 + (-I_5 R_5)$

$$\Rightarrow V_B = 0 - \left(\frac{\mathcal{E}}{4R}\right)(2R) \rightarrow \boxed{V_B = -\frac{\mathcal{E}}{2}}$$

If $V_B = -\frac{\mathcal{E}}{2}$, then $V_C = V_B + \Delta V(B \rightarrow C) = -\frac{\mathcal{E}}{2} + \mathcal{E} \rightarrow \boxed{V_C = +\frac{\mathcal{E}}{2}}$

If $V_C = +\frac{\mathcal{E}}{2}$ then $V_D = V_C + \Delta V(C \rightarrow D) = +\mathcal{E}/2 - I_1 R_1$

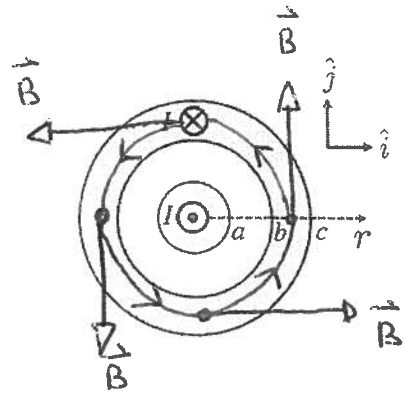
$$= +\frac{\mathcal{E}}{2} - \left(\frac{\mathcal{E}}{4R}\right)R \rightarrow \boxed{V_D = +\frac{\mathcal{E}}{4}}$$

(one could also work backward, from A to D to C to B, climbing "upstream" through resistors)

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* A coaxial cable consists of a conducting wire of radius $a = R$ and an outer conducting shell, having inner radius $b = 2R$ and outer radius $c = 3R$. The space between the wire and the shell ($a < r < b$) is empty. The central wire carries a total current I in the $+\hat{k}$ direction (i.e. out of the page). The outer shell carries the same current I in the opposite direction, $-\hat{k}$ (i.e. into the page).

Use Ampere's Law to determine the magnetic field at a distance $r = 5R/2$ from the central axis of the cable. Be sure to specify the direction of the field, as well as the magnitude.



Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{through}$

↳ Choose a circular path at distance $r = \frac{5R}{2}$

⇒ Since path will enclose all outward current, but only some inward current, we expect $\vec{B}$ will form $\boxed{\text{ccw loops}}$

so, choose our path to be ccw circle

$\rightarrow \oint \vec{B}\cdot d\vec{s} = \oint B\,ds = B\oint ds = B\cdot 2\pi r \overset{\text{using } r=\frac{5}{2}R}{=} \boxed{B\cdot 5\pi R}$

To find $I_{through}$:

① All current on inner wire counts, as positive current: $\boxed{I_{in} = +I}$

② Only some of current on outer wire counts, as negative current

$I_{out} = -(\text{fraction of } I)$

Current can be assumed to be uniformly distributed, so

$I_{out} = -(J\cdot A_{out}) = -\left(\frac{I_{TOT}}{A_{TOT}}\right)A_{out} = -I_{TOT}\,\frac{A_{out}}{A_{TOT}}$

→ ratio of cross-sectional areas is

$$\frac{A_{out}}{A_{TOT}} = \frac{\pi(\frac{5}{2}R)^2 - \pi(2R)^2}{\pi(3R)^2 - \pi(2R)^2}$$

(don't include cavity cross-section!)

$$= \frac{\frac{25}{4} - \frac{16}{4}}{9-4} = \frac{\frac{9}{4}}{5} = \frac{9}{20} \Rightarrow \boxed{I_{out} = -\frac{9}{20}I}$$

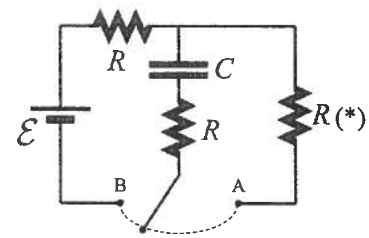
so: $I_{Through} = +I - \frac{9}{20}I$

➔ $I_{Through} = \frac{11}{20}I$

So Ampere's Law gives: $B\cdot 5\pi R = \mu_0\left[\frac{11}{20}I\right] \rightarrow \boxed{B = \frac{11\mu_0 I}{100\pi R}}$

Positive value confirms direction

$\boxed{\vec{B} \text{ forms ccw loop}}$



The next two questions involve the following situation:
A two-mode *RC* circuit is illustrated at right. A switch allows the circuit to be toggled back and forth bewteen the two modes of operation. All resistances are identical, and all wires are ideal.

Question value 4 points

(1) Assume the switch has been at position *A* for a very long time. It is then is suddenly flipped over to position *B*. Which of the expressions below describes the initial rate at which charge on the capacitor is changing, immediately after the switch is flipped? Assume that Q_o denotes the charge on the capacitor immediately *before* the switch is flipped.

(a) $\frac{dQ}{dt} = -\frac{Q_o}{2R}$

(b) $\frac{dQ}{dt} = +\frac{\varepsilon}{R}$

(c) $\frac{dQ}{dt} = 0$

(d) $\frac{dQ}{dt} = +\frac{Q_o}{2RC}$

(e) $\boxed{\frac{dQ}{dt} = +\frac{\varepsilon}{2R}}$

at A: capacitor discharges fully $Q \to 0$

switch to B: capacitor starts charging, from $Q_o = 0$

$\Rightarrow \Delta V_C = Q/C = 0$, initially

so, loop rule is $+\varepsilon - I_o R - \cancel{\frac{Q_o}{C}}^{0} - IR = 0$

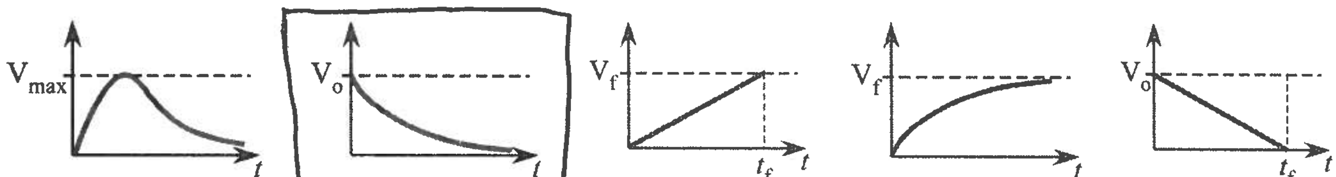
$\Rightarrow I = \varepsilon/2R$ = rate of flow of charge in wires

$\Rightarrow$ rate of accumulation of charge on capacitor: $\frac{dQ}{dt} = +I$

$\Rightarrow \boxed{\frac{dQ}{dt} = +\frac{\varepsilon}{2R}}$

Question value 4 points

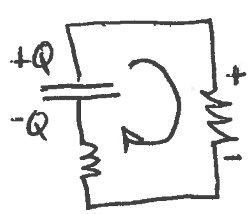
(2) Assume the switch has been at position *B* for a very long time. It is then is suddenly flipped over to position *A*. Which one of the graphs below best depicts the potential difference across the right-hand resistor (*) as a function of time?



(a) (b) (c) (d) (e)

at B: capacitor is charging
for a long time: capacitor is fully charged
switch to A: capacitor discharges via clockwise current

loop rule: $+\frac{Q_o}{C} - I_o R - I_o R = 0 \quad I_o = \frac{Q_o}{2RC}$



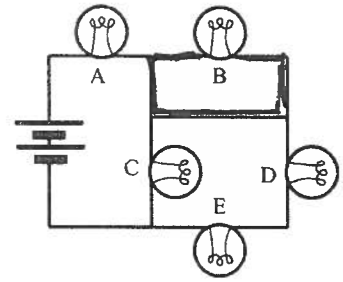
$\to$ as capacitor discharges: $Q \to 0$ so $\Delta V_C \to 0$

by loop rule, then, $\Delta V_R \to 0$ also $\boxed{\Delta V_R \text{ decays to zero}}$

[Final loop rule: $+0-0-0=0$]

Question value 8 points

(3) In the circuit diagrammed at right, all bulbs are identical and all wires are ideal. Rank, from greatest to least, the brightnesses of bulbs A through E



(a) A = C > B = D = E
(b) A = B > C = D > E
(c) A > C > D = E > B
(d) A > C = D > E > B
(e) A > B > D = E > C

① All current in circuit passes through A : no other bulb is brighter than A

② B has been "shorted out" by horizontal wire; $\Delta V_B = 0$ so $\boxed{\text{B is unlit}}$

③ Current from A splits: some through C, some through D+E

$\boxed{\text{A is brightest}}$

④ resistance of D+E is greater than resistance of C

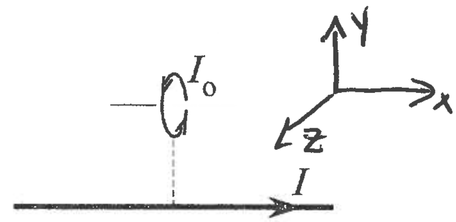
⇒ more current through C than through D+E $\boxed{\text{C brighter than D, E}}$

⑤ Same current through D and E : $\boxed{\text{D and E are equally bright}}$

so: $\boxed{A > C > D = E > B}$

Question value 8 points

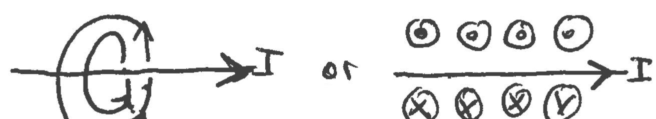
(4) A small circular wire loop carrying current I_0 is placed near a very long straight wire, in the orientation shown at right. The long wire carries a *rightward* current *I*. If the loop is released and allowed to move freely, how will it move? You may assume that the size of the loop is small in comparison to the distance from loop to wire. (Ignore gravity.)



(a) The loop will rotate around the negative y-axis.
(b) The loop will move in a straight line, away from the wire (i.e. the in the positive y-direction).
(c) The loop will rotate around the negative z-axis.
(d) The loop will not move.
(e) The loop will move in a straight line, toward the wire (i.e. in the negative y-direction).

① loop is a dipole :

② long wire generates looping $\vec{B}$:



⇒ at location of dipole, $\vec{B}$ = out of page

hence, dipole experiences a torque : $\vec{\tau} = \vec{\mu} \times \vec{B}$

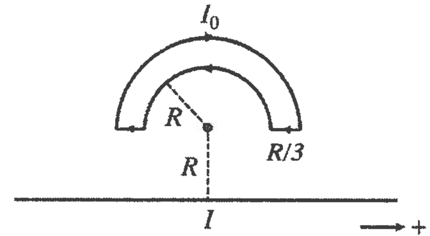
by RHR, $\vec{\tau}$ is down in plane of page $= \boxed{-\hat{j}}$

[This rotation will turn $\vec{\mu}$ to point out of page]

[Note : since size of loop is tiny, $\vec{B} \simeq$ uniform at location of loop
⇒ no net force on loop, in that situation]

Question value 8 points

(5) A wire is bent into a half-annulus, having inner radius R and outer radius $4R/3$. It carries a current I_0 in the indicated direction around its perimeter. A very long straight wire, carrying an unknown current I is placed at distance R from the center of curvature of the annulus. If the net magnetic field at the center of curvature is *zero*, what is the magnitude and direction of the current in the straight wire? (Note the sign convention for I, shown in the figure!).



(a) $I = +\frac{\pi}{4} I_0$

(b) $I = 0$

(c) $I = -\frac{\pi}{8} I_0$

(d) $I = +\frac{\pi}{8} I_0$

(e) $I = -\frac{\pi}{4} I_0$

Circular loop: $B_{center} = \frac{\mu_0 I_0}{2R}$

$\rightarrow$ half-loop: $B = \frac{\mu_0 I}{2R} \times \frac{1}{2} = \frac{\mu_0 I_0}{4R}$

So: arc at R: $B_R = \frac{\mu_0 I_0}{4R}$, out of page

" " $\frac{4}{3}R$: $B_{\frac{4}{3}R} = \frac{\mu_0 I_0}{4(4/3R)} = \frac{3\mu_0 I_0}{16R}$ into page

] directions via RH curl

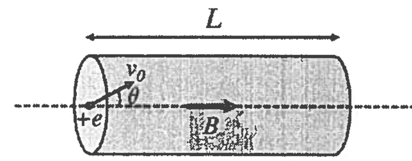
so $\vec{B}_{loop} = +\frac{\mu_0 I_0}{4R} - \frac{3\mu_0 I_0}{16R} = +\frac{\mu_0 I_0}{16R}$ out of page

Thus, require $\vec{B}_{long\ wire}$ = { into page: I flows to left / I is negative; $\frac{\mu_0 |I|}{2\pi R} = \frac{\mu_0 I_0}{16R}$: magnitudes cancel: $|I| = \frac{\pi}{8} I_0$ }

so $\boxed{I = -\frac{\pi}{8} I_0}$

The next two questions involve the following situation:

A solenoid has radius R and length L along its axis, with a uniform magnetic field B within its interior. A proton (mass m, charge $+e$) is fired into the solenoid with an initial speed v_0, traveling at an angle θ relative to the solenoid axis. (Assume that θ is small enough that the proton never strikes the solenoid walls.)



Question value 4 points

(6) What type of path does the proton follow, as it passes through the solenoid?

(a) A sinusoidal up-and-down trajectory as it moves from left to right.

(b) A symmetric parabola that enters and exits exactly on the solenoid axis.

(c) A clockwise helix, when viewed from behind (i.e. from the left).

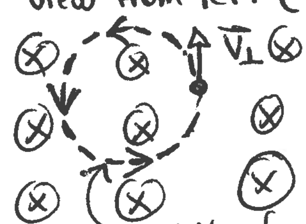
(d) A counter-clockwise helix, when viewed from behind (i.e. from the left).

(e) A circular orbit that never allows it to leave the solenoid.

general path of charged particle in a uniform B is a helix

- drift along field direction
- circle around field direction

View from left (behind charge):



$\vec{V}$ = up

$\vec{B}$ = away

$\vec{F} = q\vec{v}\times\vec{B}$ = to left

orbit of proton = $\boxed{\text{CCW}}$

Question value 4 points

(7) How much time does the proton spend within the solenoid?

(a) $\Delta t = 2\pi R/v_0$

(b) $\Delta t \rightarrow \infty$; the proton never escapes from the solenoid.

(c) $\Delta t = L/v_0 \sin\theta$

(d) $\Delta t = L/v_0$

(e) $\Delta t = L/v_0 \cos\theta$

Along axis $\vec{V}_\parallel = V_0 \cos\theta$ is constant [no component of force along axis, so $\vec{a}_\parallel = a_x = 0$]

time to travel $\Delta X = +L$ is:

$\Delta X = V_x \Delta t \Rightarrow \Delta t = \frac{\Delta X}{V_x} = \boxed{\frac{L}{V_0 \cos\theta}}$