Physics 2212G/H
Spring 2017
Test 2

Test form 726

Name Solutions

Recitation Section (see back of test):________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

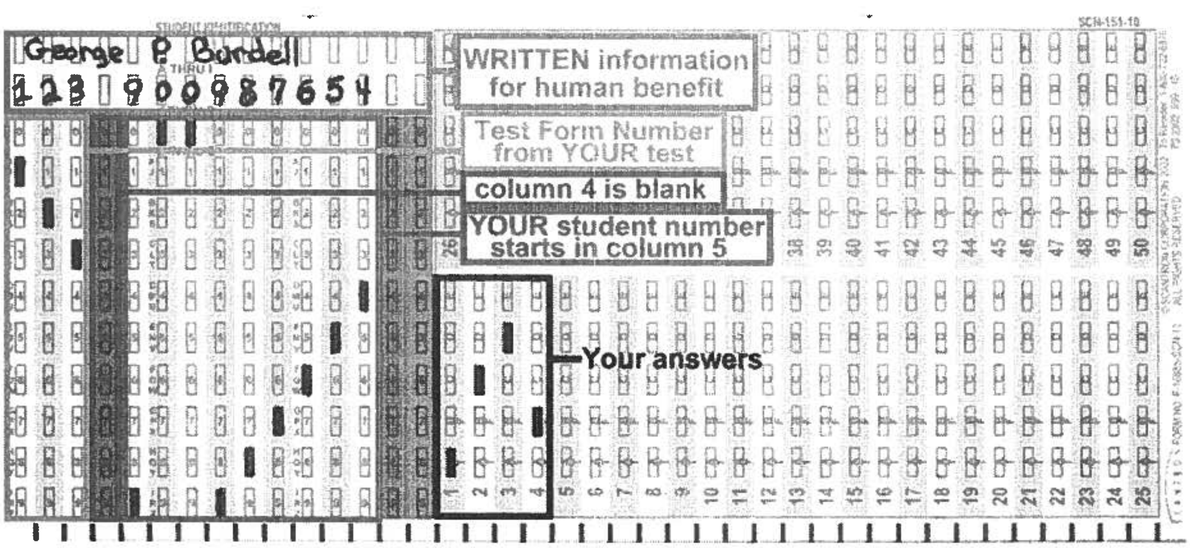
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$	$e = 1.60\times10^{-19}\ \text{C}$	$m_e = 9.11\times10^{-31}\ \text{kg}$
$\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$	$g = 9.81\ \text{m/s}^2$	$m_p = 1.67\times10^{-27}\ \text{kg}$

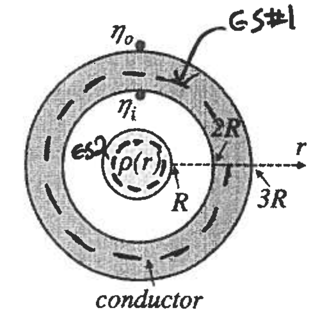
Your test form is: 726



Our next test will be on Tuesday, March 14

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[I] *(20 points)* An insulating sphere of radius R carries a non-uniform volume charge density $\rho(r) = C/r^2$, where C is a positively-valued constant The sphere is placed in the exact center uncharged hollow conducting sphere of inner radius $2R$ and outer radius $3R$.



(*i*) Calculate the surface charge density on the inner and outer walls of the hollow sphere.

① Find total charge on insulating sphere, by adding charge layer by layer

charge on one layer $= dQ = \rho\, dV$

$$= \left[\frac{C}{r^2}\right]\cdot\left[4\pi r^2 \cdot dr\right] \rightarrow \text{volume of thin layer = surface area × thickness}$$

$$\text{so } Q_{sph} = \int_{r=0}^{r=R} \frac{C}{r^2} 4\pi r^2 dr = 4\pi C \int_0^R dr = \boxed{4\pi C R}$$

② For Gaussian Surface #1 in figure, within conducting shell, where $E \equiv 0$

→ zero flux through GS, so $Q_{inside} = 0$

But $Q_{inside} = Q_{sphere} + Q_{inner\ surface} \rightarrow Q_{inner} = -Q_{sph} = -4\pi C R$

hence $\eta_{inner} = \frac{Q_{inner}}{4\pi(2R)^2}$ (using area of inner wall) $= \frac{-4\pi C R}{16\pi R^2} \Rightarrow \boxed{\eta_{in} = \frac{-C}{4R}}$

③ Since conductor is uncharged, $Q_{outer} = -Q_{inner} = -(-4\pi C R) = +4\pi C R$

$$\text{so } \eta_{outer} = \frac{+4\pi C R}{4\pi(3R)^2} = \boxed{\frac{+C}{9R}}$$

(4 points EXTRA CREDIT)

(*ii*) Apply Gauss's Law to find an expression for the electric field *within* the insulating sphere, at a distance d from the center of the sphere (*i.e.*, for $d < R$). Express your answer symbolically in terms of ε_0, C, d and/or R.

Look at insulating sphere closely, let Gaussian Surface #2 have radius $d < R$

① Flux through GS#2:

$$\oint \vec{E}\cdot d\vec{A} \longrightarrow \oint E(d)\,dA \longrightarrow E(d)\oint dA \longrightarrow E(d)\cdot \underbrace{4\pi d^2}_{\text{Total area of GS\#2}}$$

($\vec{E}$ and $d\vec{A}$ are radial) (E is constant on GS#2)

② Find charge inside GS#2

= all charge at radii $r < d$

$$Q_{in} = \int_{r=0}^{r=d} \rho(r)\,dV = \int_0^d \frac{C}{r^2}\cdot 4\pi r^2 dr = 4\pi C \int_0^d dr = 4\pi C d$$

③ Gauss's Law:

$$\oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{in} \rightarrow E(d)\cdot 4\pi d^2 = \frac{1}{\varepsilon_0} 4\pi C d$$

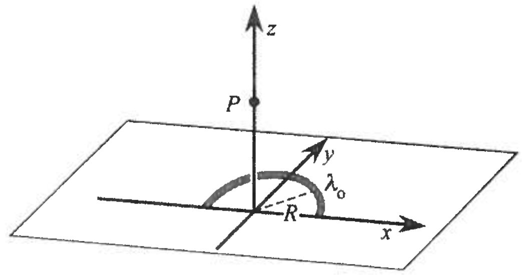
$$E(d) = \frac{C}{\varepsilon_0 d} \quad \text{or} \quad \boxed{\vec{E} = +\frac{C}{\varepsilon_0 d}\hat{r}}$$

ie radially outward

question asks for field, so answer should be a vector!

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

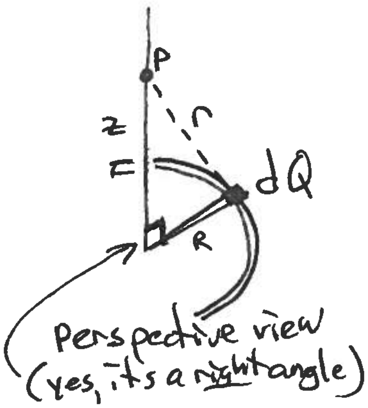
[II] *(20 points)* A thin insulating rod is bent into a semicircle of radius R. It is placed in the xy-plane as shown at right, with its center of curvature at the origin. Positive charge is distributed uniformly along the rod, with a linear density $+\lambda_o$.



(*i*) Use integration methods (i.e. *not* a memorized formula) to derive an expression for the electric potential at point P on the z-axis, a distance $z = 2R$ from the origin. Express your answer in terms of λ_o, R, and ε_o.

(*ii*) Suppose you tried to <u>approximate</u> the potential at $z = 2R$, by treating the entire rod as if it were a point particle (with the same total charge as the rod) located at the origin. What percent error would this approximation generate?

[Recall that %error = 100% × (approx – exact) / exact]



distance from a dQ <u>on</u> arc to fixed point z is:

$$r = \sqrt{R^2+z^2}$$

so potential <u>at</u> z, due to dQ is

$$dV = \frac{dQ}{4\pi\varepsilon_o r} = \frac{dQ}{4\pi\varepsilon_o\sqrt{R^2+z^2}}$$

$$\text{Then } V_{TOT} = \int_{\text{around arc}} \frac{dQ}{4\pi\varepsilon_o\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_o\sqrt{R^2+z^2}}\int_{\text{around arc}} dQ$$

[We have exploited the fact that <u>all</u> points on arc are <u>same</u> distance from P!]

→ at this point, you don't need to formally integrate, but you <u>can</u> write

$dQ = \lambda_o ds$ where ds = an arc length

$$\text{so } V = \frac{\lambda_o}{4\pi\varepsilon_o\sqrt{R^2+z^2}}\int ds \quad \text{but } \int ds = \underline{\text{half}} \text{ a circle} = \pi R$$

$$V = \left.\frac{\lambda_o \cancel{\pi} R}{4\cancel{\pi}\varepsilon_o\sqrt{R^2+z^2}}\right|_{\text{at } z=2R} = \frac{\lambda_o R}{4\varepsilon_o\sqrt{R^2+4R^2}} = \boxed{\frac{\lambda_o}{4\sqrt{5}\,\varepsilon_o}}$$

Now, approximate rod as a point charge $Q = \lambda_o\pi R$, at origin

$$\tilde{V} = \frac{Q}{4\pi\varepsilon_o r} = \frac{\lambda_o\pi R}{4\pi\varepsilon_o(2R)} = \frac{\lambda_o}{8\varepsilon_o}$$

$$\text{Hence, error would be } \frac{\tilde{V}-V}{V} = \frac{\tilde{V}}{V} - 1 = \frac{\cancel{\lambda_o}/8\varepsilon_o}{\cancel{\lambda_o}/4\sqrt{5}\,\varepsilon_o} - 1 = \frac{\sqrt{5}}{2} - 1$$

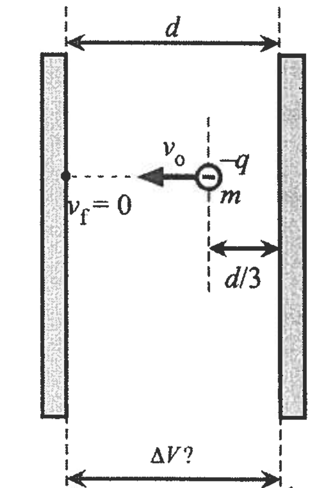
$$\text{so, } \%\text{ error} = 100\% \cdot \left[\frac{\sqrt{5}}{2} - 1\right] = 100\% \cdot [1.118 - 1]$$

$$= \boxed{11.8\%}$$

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* In the figure at right, a charged capacitor has plates separated by a distance d. A particle of mass m and charge $-q$ is initially held at a distance $d/3$ from the right plate. (Note that the symbol "q" is inherently positive, so the particle itself has negative charge!) The particle is launched directly toward the *left* plate with an initial speed v_0. It is observed to slow down, stopping *just* as it reaches the left plate.

Determine the electric potential difference ΔV between the two capacitor plates. Express your answer symbolically in terms of the parameters given in this problem. **Also, be sure to indicate which of the plates is at high potential and which is at low potential!**



lower V on left

higher V on right

Energy is conserved within capacitor

$$\Delta K + \Delta U = 0$$

so $\Delta U = -\Delta K = -(\cancel{K_f} - K_i)$ (zero)

$= -\left(-\tfrac{1}{2} m v_0^2\right)$

$\rightarrow \Delta U = +\frac{1}{2} m v_0^2$ [positive value: charge has gained PE)

then $\underbrace{\Delta V_{i \to f} = \frac{\Delta U}{-q} = \frac{-m v_0^2}{2q}}_{\Delta V_{\text{of charge}}}$, charge has moved to lower electric potential

$\boxed{\text{Right Plate is at High Potential}}$

Note that this change in potential occurs for charge moving through $\Delta x = -\frac{2}{3} d$

$\Rightarrow \Delta V_{cap}$ = full difference, over full distance $\Delta x = -d$

recall: in uniform field, $\Delta V = -\vec{E} \cdot \vec{\Delta x}$

$\Rightarrow$ linear relationship between ΔV and Δx

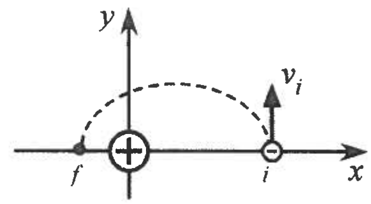
So, if $\Delta x = -\frac{2}{3} d$ results in $\Delta V_{charge} = -\frac{m v_0^2}{2q}$

then $\Delta V_{cap} = \frac{3}{2} \Delta V_{charge} = \frac{3}{2}\left(-\frac{m v_0^2}{2q}\right) = \boxed{-\frac{3 m v_0^2}{4q}}$

minus sign tells us: potential decreases as we move right-to-left

$\rightarrow$ confirms right = high, left = low

The next two questions both involve the following situation:
In the figure at right, a positive source charge is held fixed at the origin, and a negative test charge is initially at location *i*, moving vertically with speed v_i. The test charge follows the dotted trajectory, reaching position *f*—at which point it is moving vertically *downward.*



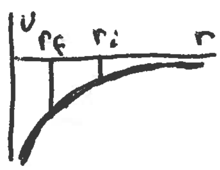
No external forces act, so mechanical energy will be conserved: $K + U = \text{constant}$

Question value 4 points

(1) What can you say about the energy changes of the test charge, as it moves from *i* to *f*?

(a) The charge has lost kinetic energy and lost potential energy.
(b) The charge has gained kinetic energy and lost potential energy.
(c) The kinetic and potential energies of the charge have remained unchanged.
(d) The charge has gained kinetic energy and gained potential energy.
(e) The charge has gained kinetic energy and lost potential energy.

Electrical PE: $U = \dfrac{k(+Q)(-q)}{r}$



→ $(-q)$ ends up closer to $(+Q)$
→ $r_f < r_i$
→ $U_f < U_i$

$\boxed{\text{charge loses PE and gains KE}}$

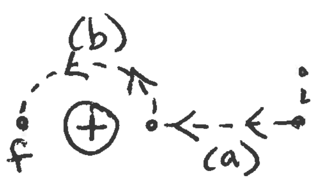
Question value 4 points

(2) What can you say about the work done by the electric field of the (positive) source charge, and the electric potential difference moved through by the (negative) test charge, as the test charge moves from *i* to *f*?

(a) The field has done positive work and the charge has moved to higher electric potential.
(b) The field has done zero work and the charge has moved to lower electric potential.
(c) The field has done negative work and the charge has moved to higher electric potential.
(d) The field has done negative work and the charge has moved to lower electric potential.
(e) The field has done positive work and the charge has moved to lower electric potential.

Since work is path-independent, we can imagine any alternate path that makes $W_{i \to f}$ easier to deduce

• choose straight line + circular arc



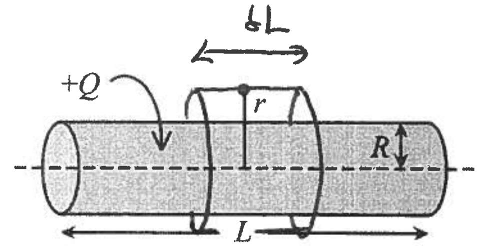
on (a): Force is attractive, displacement is inward] $\int_{(a)} \vec{F} \cdot d\vec{r}$ will be positive

on (b): Force is radial, displacement is ⊥ to $\vec{F}$] $\int_{(b)} \vec{F} \cdot d\vec{r}$ will be zero

Net $\boxed{\text{work by field is POSITIVE}}$

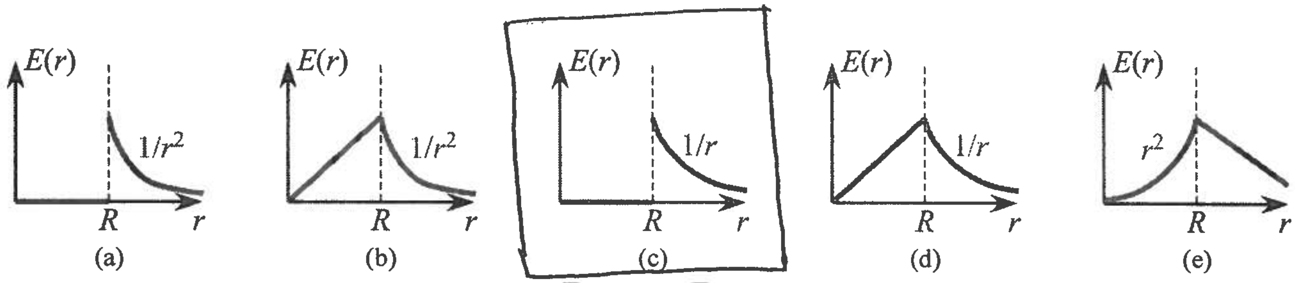
Then $\Delta V = \dfrac{\Delta U}{(-q)} = \dfrac{(-W_{field})}{(-q)} = \dfrac{W_{field}}{q} = \dfrac{\text{positive}}{\text{positive}}$

$\boxed{\text{charge moved to higher potential}}$



Question value 8 points

(3) A cylindrical conductor of length L and radius R carries a total charge $+Q$. Which graph below best represents the magnitude of the electric field $E(r)$ as a function of the distance r from the cylinder's axis. *Hint:* You may assume that $L >> r$, and that the conductor has been allowed to reach electrostatic equilibrium.



① cylinder is a conductor so $\boxed{\vec{E}_{inside} \equiv 0}$

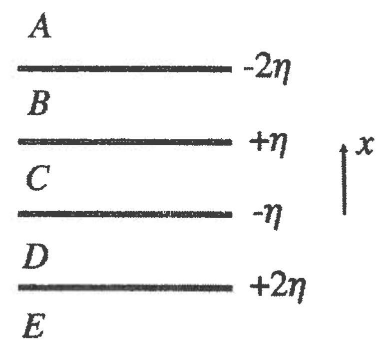
② Outside, use a cylindrical Gaussian surface of some length ΔL

$\oint \vec{E} \cdot d\vec{A} \rightarrow E(r) \cdot 2\pi r \delta L = \frac{1}{\epsilon_0} \frac{Q}{L} \delta L \rightarrow E(r) = \frac{\text{stuff}}{r}$ so $\boxed{E \sim \frac{1}{r}}$

$Q_{in} = \frac{Q}{L} \cdot \delta L$ (fraction of total charge)

Question value 8 points

(4) Four very large, uniformly-charged sheets are arranged as shown at right. Rank the magnitude of the electric field in regions A to E, from the greatest to the least.



(a) $E_C > E_D = E_B > E_A = E_E$

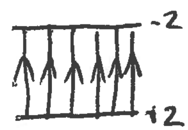
(b) $E_B = E_D > E_C > E_A = E_E$

(c) $E_A = E_E > E_D = E_B = E_C$

(d) $E_C > E_A = E_E > E_B = E_D$

(e) $E_B = E_D = E_C = E_A = E_E$

$+2\eta / -2\eta$ form a capacitor



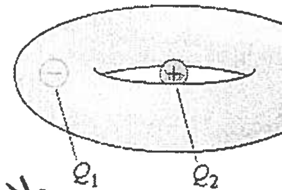
$+\eta / -\eta$ also form a capacitor

- Note that outside both capacitors $E = 0$, so $\boxed{E_A = E_E = 0 = \text{least}}$
- In regions B/D, only capacitor $\pm 2\eta$ contributes $\boxed{E_B = E_D}$
- In region A, capacitor $\pm\eta$ partially cancels capacitor $\pm 2\eta$ $\boxed{E_C < E_B \text{ or } E_D}$

So, overall: $\boxed{E_B = E_D > E_C > E_A = E_E}$

Question value 8 points

(5) What is the next flux through the torus of the figure at right, if $Q_1 = -2.2$ nC and $Q_2 = +4.4$ nC?



inside the 'donut'

not inside the 'donut'

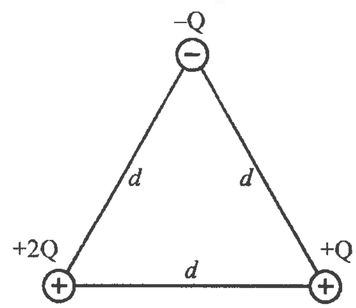
(a) $+750\ \text{N}\cdot\text{m}^2/\text{C}$
(b) $-250\ \text{N}\cdot\text{m}^2/\text{C}$
(c) $+250\ \text{N}\cdot\text{m}^2/\text{C}$
(d) $-750\ \text{N}\cdot\text{m}^2/\text{C}$
(e) $+500\ \text{N}\cdot\text{m}^2/\text{C}$

Gauss's law flux $= \frac{1}{\varepsilon_0} Q_{inside}$, where only $Q_{in} = Q_1 = -2.2e^{-9}$ C

so $\Phi = \frac{Q_1}{\varepsilon_0} = \boxed{-249\ \text{Nm}^2/\text{C}}$ rounds to $-250\ \text{Nm}^2/\text{C}$

Question value 8 points

(6) Three charges are initially placed at the corners of a equilateral triangle having sides of length d, as shown at right. How much work must be done by an external agent in order to remove charge $-Q$ to a distant location? Assume all charges begin and end at rest.



(a) $-\frac{2kQ^2}{d}$
(b) $+\frac{kQ^2}{d}$
(c) $+\frac{3kQ^2}{d}$
(d) $+\frac{2kQ^2}{d}$
(e) $-\frac{kQ^2}{d}$

Work by an external agent will change the mechanical energy of system

$W_{ext} = \Delta E_{mech} = \Delta K + \Delta U$

but $K_i = K_f = 0$, so $\Delta K = 0$

$W_{ext} = \Delta U = U_f - U_i = \left(\sum \frac{kQ_iQ_j}{r_{ij}}\right)_f - \left(\sum \frac{kQ_iQ_j}{r_{ij}}\right)_i$

Final state: $-Q$ is infinitely far; only $+2Q$, $+Q$ contribute to PE

$$U_f = k\frac{(2Q)(Q)}{d} = \frac{2kQ^2}{d}$$

Initial state all three contribute to PE

$$U_i = \frac{k(2Q)(Q)}{d} + \frac{k(2Q)(-Q)}{d} + \frac{k(Q)(-Q)}{d} = \frac{-kQ^2}{d}$$

$$\text{so } W_{ext} = U_f - U_i = \frac{2kQ^2}{d} - \left(\frac{-kQ^2}{d}\right) = \boxed{\frac{+3kQ^2}{d}}$$

Positive work had to be done, to pull $-Q$ away from the pos charges that attract it