Physics 2212G/H
Spring 2017
Test 1

Test form 711

Name Solutions

Recitation Section (see back of test):__________

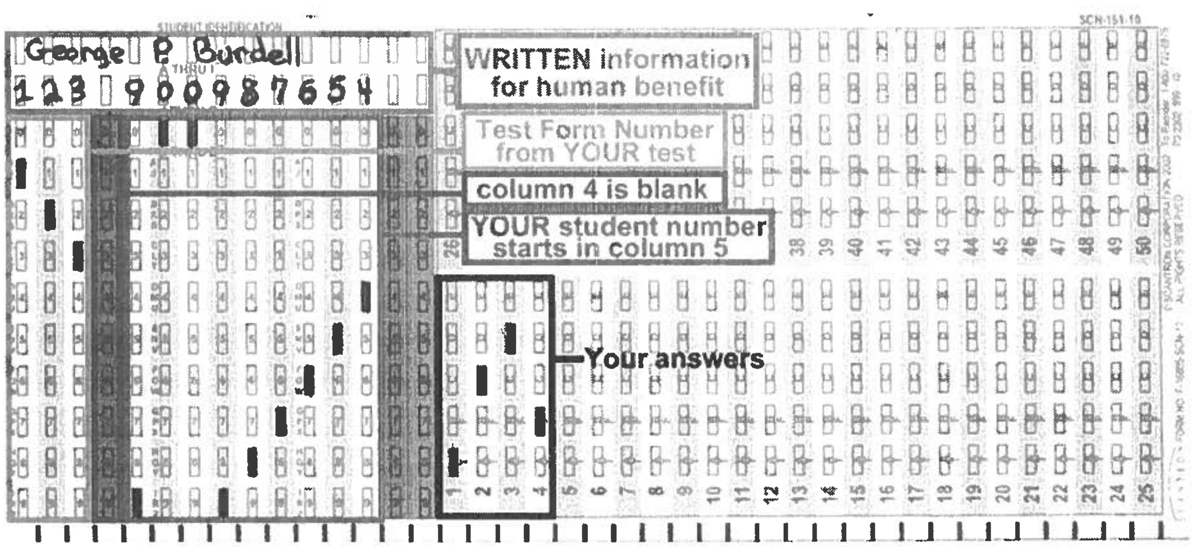
A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".
2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.
4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}$	$e = 1.60\times10^{-19}\ \mathrm{C}$	$m_e = 9.11\times10^{-31}\ \mathrm{kg}$
$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$	$g = 9.81\ \mathrm{m/s^2}$	$m_p = 1.67\times10^{-27}\ \mathrm{kg}$

Your test form is: 711



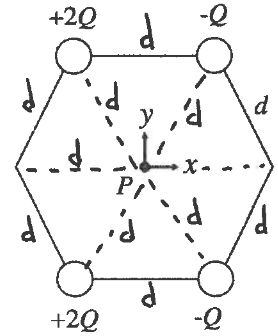
Our next test will be on Tuesday, February 21

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* Four point charges are placed at the vertices of a regular hexagon having sides of length $d = 4.00$ cm, as shown at right. A point charge $+3Q$ is then placed at point P, at the center of the hexagon. For all five charges, the parameter Q has the value $Q = 12.5$ nC.

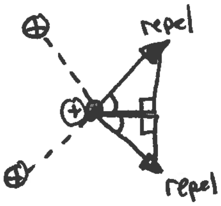
What is the magnitude and direction of the electrostatic force on charge $+3Q$? (*Hint*: The principle of superposition for the electric field is your friend!)

Express your answer by first solving algebraically in terms of the parameters Q and d, and then performing all numerical calculations at the very end. Your grader is likely to deduct points if you fill the page with numerical calculations!



- Hexagon can be broken into six equilateral triangles
  → all vertices are a distance d from center

- Charges +2Q create mirror image forces on 3Q:



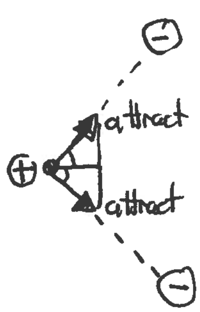
– directed at angle $\theta = 60°$ above/below x-axis

$\vec{F}_y$'s cancel, $\vec{F}_x$'s combine:

$$\vec{F}_{+2Qs} = 2|\vec{F}_{2Q}|\cos 60°\,\hat{\imath} = \cancel{2}|\vec{F}_{2Q}|\cdot\frac{1}{\cancel{2}}\,\hat{\imath} = \frac{k(2Q)(3Q)}{d^2}\,\hat{\imath}$$

$$= \frac{6kQ^2}{d^2}\,\hat{\imath}$$

- Charges $-Q$ also create mirror image forces, also directed at 60° above/below positive x-axis



$$\vec{F}_{-Qs} = 2|\vec{F}_{-Q}|\cos 60°\,\hat{\imath} = \cancel{2}|\vec{F}_{-Q}|\cdot\frac{1}{\cancel{2}}\,\hat{\imath}$$

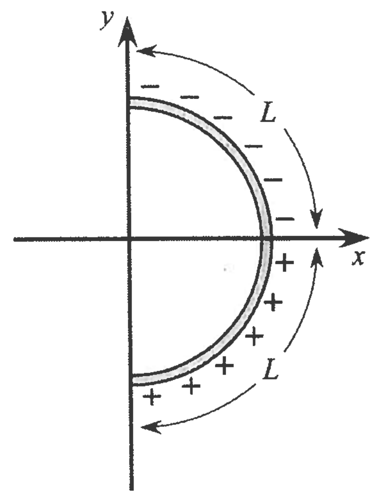
$$\Rightarrow \vec{F}_{-Qs} = \frac{k|-Q||3Q|}{d^2}\,\hat{\imath} = \frac{3kQ^2}{d^2}\,\hat{\imath}$$

- Total force is thus $\vec{F}_{net} = \vec{F}_{+2Qs} + \vec{F}_{-Qs} = \left(\frac{6kQ^2}{d^2} + \frac{3kQ^2}{d^2}\right)\hat{\imath}$

$$\vec{F}_{net} = \boxed{\frac{9kQ^2}{d^2}\,\hat{\imath} = \frac{9Q^2}{4\pi\varepsilon_0 d^2}\,\hat{\imath} = 7.90\times10^{-3}\,\text{N}}$$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

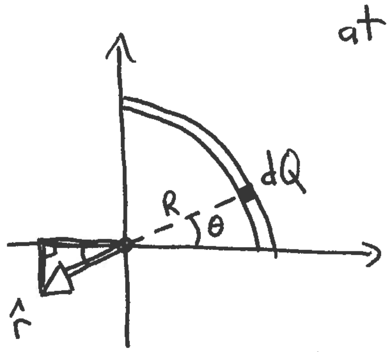
[II] *(20 points)* An insulating rod of length L is uniformly charged with a total charge $-Q$, and then bent into a quarter-circle arc. An identical rod is uniformly charged with a total charge $+Q$, and also bent into a quarter-circle arc. The two rods are placed as shown at right.



(*i*) Use integration techniques to determine the electric field vector at the origin **due to the negative rod only.** Your final answer should be an algrebraic vector expression (i.e. evaluate any integrals in your answer).

(*ii*) Use the principle of superposition to determine the net electric field at the origin, **due to both rods** (positive and negative).

Express both answers in terms of the parameters k, Q, and L.



at position θ, consider a small bit of rod $ds = R\,d\theta$ (arc length)

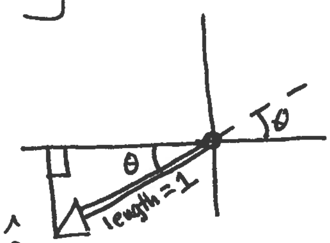
$\Rightarrow$ charge on bit is

$$dQ = \lambda\,ds = \lambda R\,d\theta$$

where $\lambda = \dfrac{\text{total charge}}{\text{total length}} = \dfrac{-Q}{L} \Rightarrow \lambda = \dfrac{-2Q}{\pi R}$

But: L is related to arc radius:

$$L = \tfrac{1}{4}\text{ circle} = \tfrac{1}{4}(2\pi R) = \tfrac{\pi}{2}R$$



We thus have $dQ = \lambda R\,d\theta = -\dfrac{2Q}{\pi}d\theta$

Then, distance from charge bit to origin is $R = \dfrac{2L}{\pi}$

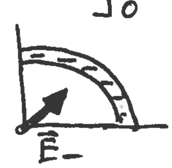
And: unit vector at origin, away from charge bit is $\hat{r} = (-\cos\theta)\hat{\imath} + (-\sin\theta)\hat{\jmath}$

so, field at origin by this bit is:

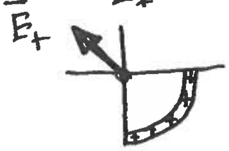
$$d\vec{E} = \frac{k\,dQ}{R^2}\hat{r} = \frac{k\left(-\frac{2Q}{\pi}d\theta\right)}{(2L/\pi)^2}\left[-\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}\right] = \frac{k\pi Q}{2L^2}\left[\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}\right]d\theta$$

$$\Rightarrow \vec{E}_{\text{minus rod}} = \int_0^{\pi/2} d\vec{E} = \frac{k\pi Q}{2L^2}\int_0^{\pi/2}\left(\cos\theta\,d\theta\,\hat{\imath} + \sin\theta\,d\theta\,\hat{\jmath}\right) = \frac{k\pi Q}{2L^2}\left[\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}\right]_0^{\pi/2}$$

$$= \frac{k\pi Q}{2L^2}\left[(1-0)\hat{\imath} - (0-1)\hat{\jmath}\right] \quad \boxed{\vec{E}_- = \frac{k\pi Q}{2L^2}\left[+\hat{\imath} + \hat{\jmath}\right]}$$



$\vec{E}_+$ will be identical in magnitude, but away from + rod (ie Quadrant II)



$$\vec{E}_{\text{both}} = \frac{k\pi Q}{2L^2}\Big[\underbrace{(+\hat{\imath}+\hat{\jmath})}_{\text{minus}} + \underbrace{(-\hat{\imath}+\hat{\jmath})}_{\text{plus}}\Big] \quad \boxed{\vec{E}_{\text{both}} = +\frac{k\pi Q}{L^2}\hat{\jmath}}$$

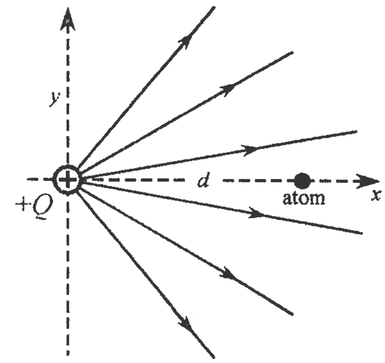
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* A neutral atom placed in an electric field can become polarized. This strength of this effect is quantified by the ***polarizability*** of the atom. For such an atom, the induced electric dipole moment is given by:

$$\vec{p} = \alpha\vec{E}$$ [note that α is always positive]

where α is the polarizability of the atom (i.e. a fixed constant for that atom), and $\vec{E}$ is the electric field vector at the location of the atom. Note that this is a vector expression— the dipole is aligned parallel to the field that induces it.

Consider an atom placed at a distance d from a point charge $+Q$, as shown in the figure at right.



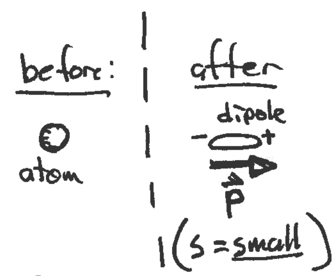
(*i*) Determine the induced electric dipole moment $\vec{p}$ of the atom, **due to the field of the point charge.** Express you answer as a vector, using the parameters Q, d, α, and the electrostatic constant k. (Coulomb's Law is your friend, here).

(*ii*) Determine the electric field **due to the induced dipole**, back at the location of the original charge Q. You may assume that the distance d is much greater that the size of the induced dipole, so you can use the large-distance dipole approximation. Again, your answer should be a vector expression involving Q, d, α, and the electrostatic constant k.

(*iii*) Determine the force **on the original point charge**, by the induced dipole field. In particular, how does the strength of the force vary with the separation distance d? (Recall that for two point charges, their mutual force varies as "one over distance squared"...but this is not a "two point charge" situation!)

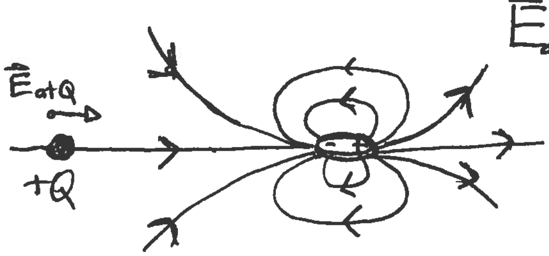
(i) at atom field by charge is simply $\vec{E}_Q = \dfrac{+kQ}{d^2}\hat{i}$

so, induced dipole moment is $\vec{p} = \alpha\vec{E} = \dfrac{+\alpha kQ}{d^2}\hat{i}$



(ii) dipole field on dipole axis, at large distance from the dipole, is:

$$\vec{E}_{\text{dipole axis}} = \frac{2k\vec{p}}{d^3}$$ [ie same direction as $\vec{p}$]



so $\vec{E}_{\text{dipole, at }Q} = \dfrac{2k}{d^3}\left[+\dfrac{\alpha kQ}{d^2}\hat{i}\right] = \dfrac{2\alpha k^2 Q}{d^5}\hat{i}$

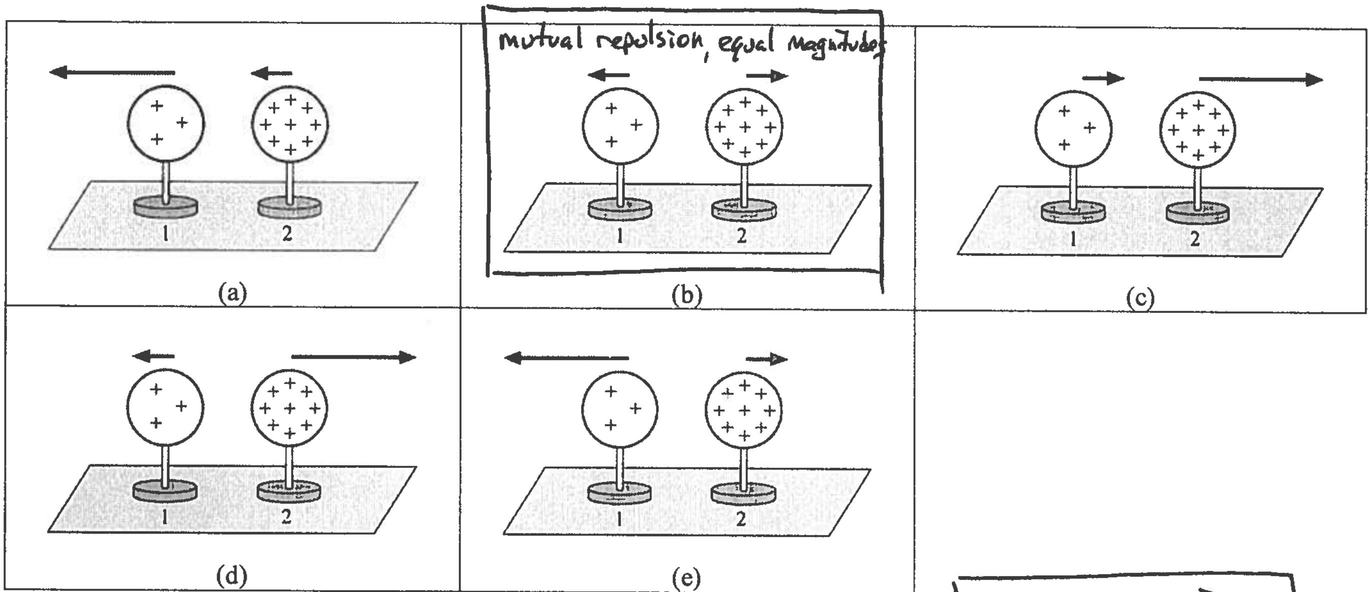
Finally, force on charge Q, by dipole field at its location, is

$$\vec{F}_{\text{dipole, on }Q} = Q\vec{E}_{\text{dipole, at }Q} = \boxed{\frac{2\alpha k^2 Q^2}{d^5}\hat{i}}$$

Force between a charge and a polarized neutral body drops off as: $\dfrac{1}{(\text{distance})^5}$

Question value 8 points

(1) Two uniformly charged spheres on insulating stands are attached to pucks on a frictionless air table, with charge Q on sphere 1 and charge $3Q$ on sphere 2. Which diagram below correctly depicts the magnitude and the direction of the electrostatic forces on the two spheres?

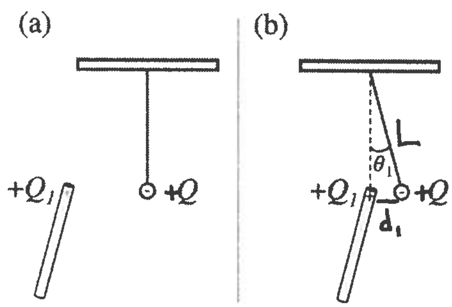


This is really just a Third Law question... we require $\boxed{\vec{F}_{1\,on\,2} = -\vec{F}_{2\,on\,1}}$

$\Rightarrow$ For both spheres, $|\vec{F}| \sim \frac{kQ\cdot 3Q}{r^2}$

Question value 8 points

(2) A small sphere carrying charge $+Q$ is attached to an insulating string and subject to Earth's gravity. An insulating rod has a charge $+Q_1$ at its very tip (figure a). Both charges are small enough to be considered pointlike. The tip of the rod is slowly brought to the starting position of the charged sphere, causing the sphere itself to be deflected by an angle θ_1 (figure b). The rod is removed and more charge is added at the very tip, so that it now carries a charge $Q_2 = 2Q_1$. What can we say about the deflection angle θ_2, if the tip of the rod is again moved to the starting position of the sphere?



(a) $\theta_2 = 2\theta_1$

$\boxed{\text{(b) } \theta_1 < \theta_2 < 2\theta_1}$

~~(c) $\theta_2 = \theta_1$~~

(d) $\theta_2 > \theta_1$

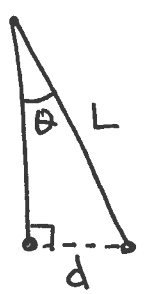
~~(e) $\theta_2 < \theta_1$~~

- with $Q_2 = 2Q_1$, force is obviously stronger, so there is more repulsion — hence, $\boxed{\theta_2 > \theta_1}$
- But: more repulsion means greater separation distance $d_2 > d_1$

$$F_2 = \frac{k(2Q_1)Q}{d_2^2} \text{ will be less than } 2\times F_1$$

- distance d_2 will be greater than d_1 but surely not $2\times d_1$

$$d_1 < d_2 < 2d_1$$



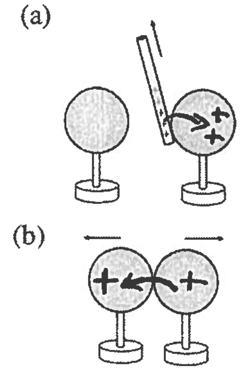
with some trig, one can then show

$$\theta_1 < \theta_2 < 2\theta_1$$

change is mobile, and easily transferable

Question value 4 points

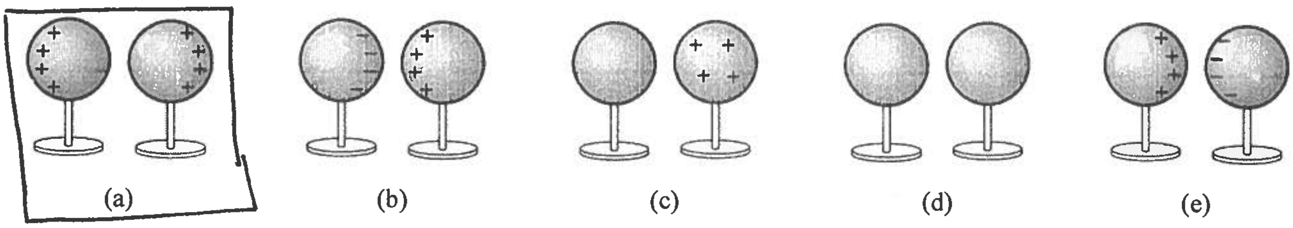
(3) Two $\boxed{\text{conducting}}$ spheres are mounted on insulating supports. A positively charged rod is brought between the two spheres and now touches the sphere on the right, (situation a). The rod is then removed and the two spheres allowed to briefly touch (situation b). Finally, the two spheres are separated. Which of the figures below best characterizes the final charge states of the two spheres?



in Figure (a): during contact, some + charge transfers to sphere on right

in Figure (b): both spheres will share excess + charge

when separated, both have + charge



note how + charges place themselves as far as possible from other sphere...

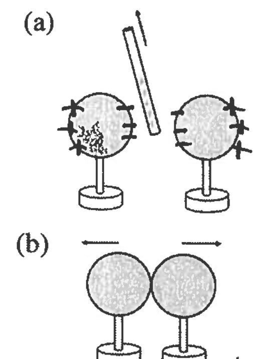
Question value 4 points

(4) Two conducting spheres are mounted on insulating supports. A positively charged rod is brought between the two spheres without touching them (situation a). The rod is then removed and the two spheres allowed to briefly touch (situation b). Finally the two spheres are separated. Which of the figures below best characterizes the final charge states of the two spheres?

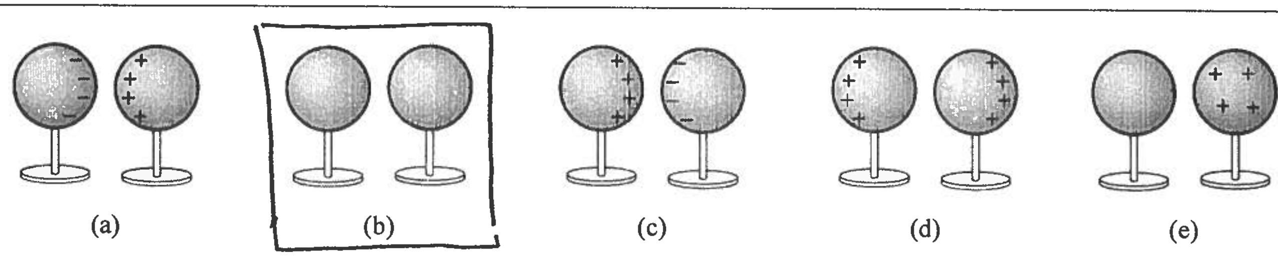
in Figure (a) spheres polarize, but stay neutral

when rod is removed, spheres de-polarize

in Figure (b) they are uncharged an unpolarized — no transfer will occur!

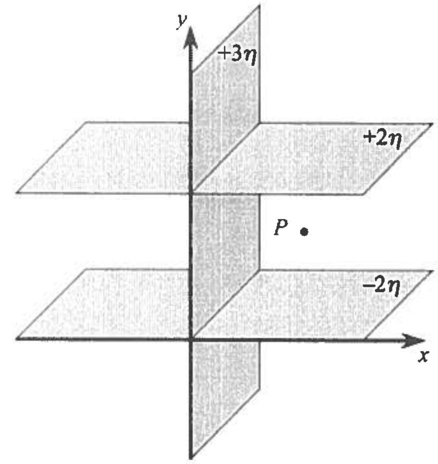


both are neutral



Question value 8 points

(5) The figure at right displays three uniformly charged sheets that can be assumed to be very large in extent. (That is, they extend well beyond the boundaries of the figure itself.) What is the magnitude of the electric field at the point P indicated?



(a) $\frac{3\eta}{2\epsilon_0}$

(b) $\frac{\sqrt{13}\,\eta}{2\epsilon_0}$

(c) $\frac{5\eta}{2\epsilon_0}$

(d) $\frac{7\eta}{2\epsilon_0}$

(e) $\frac{\sqrt{7}\,\eta}{2\epsilon_0}$

[1] Horizontal sheets $\pm 2\eta$ form a "capacitor configuration"

$\vec{E}$ = vertical $= 2\left[\frac{2\eta}{2\varepsilon_0}\right](-\hat{j})$

$$\boxed{\vec{E}_y = -\frac{4\eta}{2\varepsilon_0}\hat{j}}$$

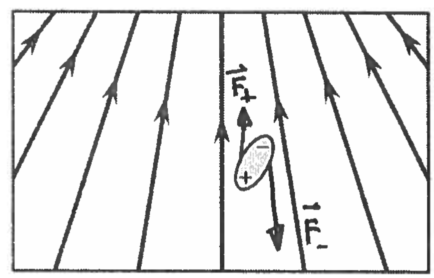
[2] Vertical sheet $+3\eta$ generates a horizontal field:

$$\boxed{\vec{E}_x = +\frac{3\eta}{2\varepsilon_0}\hat{i}}$$

Net field has magnitude $\sqrt{E_x^2+E_y^2} = \sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{4}{2}\right)^2}\,\frac{\eta}{\varepsilon_0} = \boxed{\frac{5}{2}\frac{\eta}{\varepsilon_0}}$

The next two questions both involve the following situation:

The figure at right depicts a permanent electric dipole that has been placed within a non-uniform electric field.



Question value 4 points

(6) What net force will the dipole experience, at the moment shown?

(a) A force directed *leftward.*
(b) No force at all.
(c) A force directed *upward.*
$\boxed{\text{(d) A force directed } \textit{downward.}}$
(e) A force directed *rightward.*

(−) side is in stronger part of the field: $\vec{F}_-$ is down, and "larger"

(+) side is in weaker part of the field: $\vec{F}_+$ is up, and "weaker"

⇒ Net force is down [right Now, but see below!]

Question value 4 points

(7) What net torque will the dipole experience, at the moment shown?

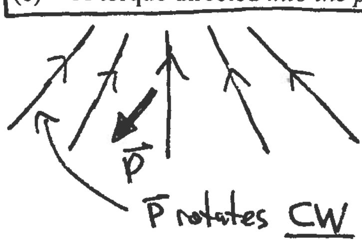
(a) A torque directed *upward.*
(b) A torque directed *out of the page.*
(c) No torque at all.
(d) A torque directed *downward.*
$\boxed{\text{(e) A torque directed } \textit{into the page.}}$

At moment shown, dipole moment $\vec{p}$ is down + leftish

Torque: $\vec{\tau} = \vec{p}\times\vec{E}$ → rotates $\vec{p}$ to make it $\parallel$ to $\vec{E}$

⟶ at moment shown, rotation would be clockwise

By Right hand Rule, $\vec{\tau}$ as a vector is $\boxed{\text{into the page}}$



[Note that after the dipole rotates, the force (previous question) will change from down to up...]