Physics 2212G/J
Fall 2016
Test 4

Test form 446

Name Solutions

Recitation Section (see back of test): ________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

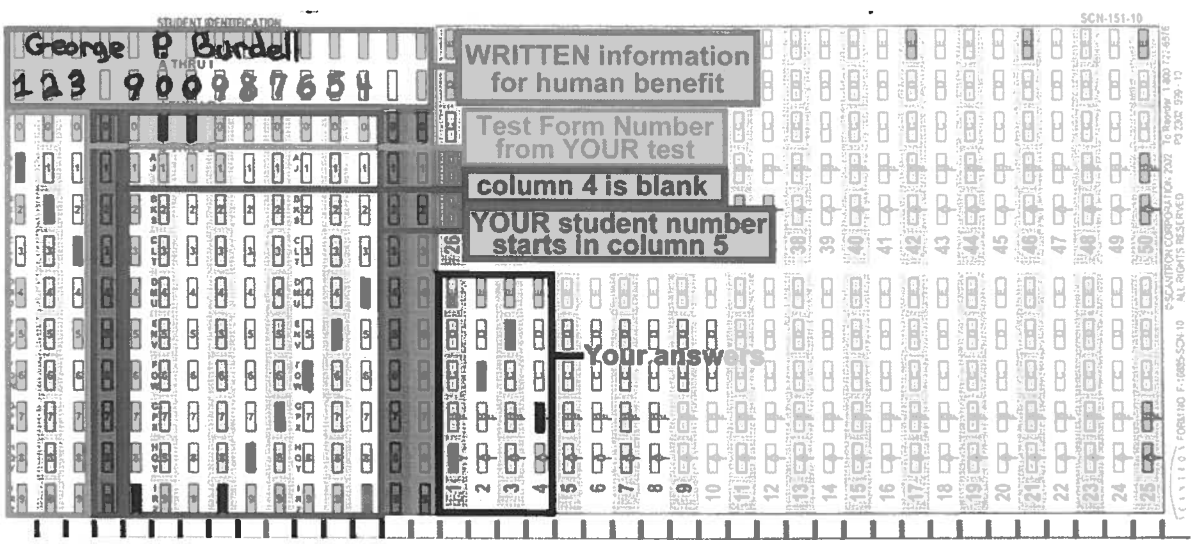
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}$	$e = 1.60\times10^{-19}\ \mathrm{C}$	$m_e = 9.11\times10^{-31}\ \mathrm{kg}$
$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$	$g = 9.81\ \mathrm{m/s^2}$	$m_p = 1.67\times10^{-27}\ \mathrm{kg}$

Your test form is: 446

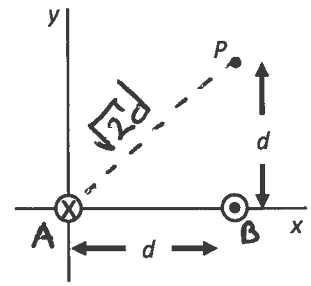


Our Final Exam will be held during Period 19
Monday, December 12 at 6:00 pm

The following problem will be hand-graded. *__Show all your work for this problem__.* *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* In the figure at right, two long straight wires carry current perpendicular to the xy-plane. The left wire, located at $x = 0$ carries current I into the page; the right wire, located at $x = d$ carries current $2/3\, I$ out of the page.

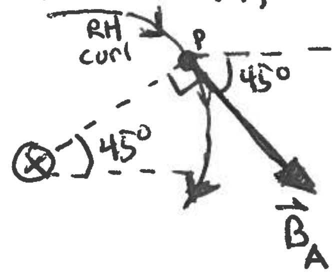
Calculate the magnetic field at the point P, located at coordinates $(x,y) = (d,d)$. Express your answer as a Cartesian vector (using $\hat{\imath}$ and $\hat{\jmath}$), in terms of the parameters I and d, along with symbols for any necessary physical constants (i.e. do not substitute any numerical values).



Field of a long straight wire: $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{t}$

where $\hat{t}$ is a unit vector tangent to a right-handed circle around wire

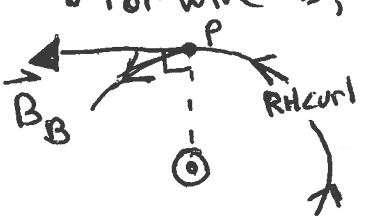
- For wire A, into page: $r = \sqrt{2}\,d$ $\hat{t}$ = unit vector at 45° down-angle $= \left(+\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}\right)$



so $\vec{B}_A = \frac{\mu_0 I}{2\pi\sqrt{2}\,d}\left(\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}\right)$

$$\boxed{\vec{B}_A = \frac{\mu_0 I}{4\pi d}(\hat{\imath} - \hat{\jmath})} = \frac{\mu_0 I}{2\pi d}\left(+\frac{1}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right)$$

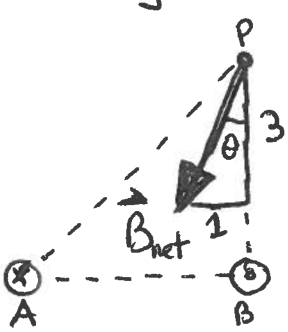
- For wire B, out of page: $r = d$, $\hat{t} = -\hat{\imath}$



$$\vec{B}_B = \frac{\mu_0\left(\frac{2}{3}I\right)}{2\pi d}(-\hat{\imath}) = \boxed{-\frac{\mu_0 I}{3\pi d}\hat{\imath}}$$

Then $\vec{B}_{net} = \frac{\mu_0 I}{\pi d}\left[\left(+\frac{1}{4}\hat{\imath} - \frac{1}{4}\hat{\jmath}\right) + \left(-\frac{1}{3}\hat{\imath}\right)\right]$

noting that $\frac{1}{4} - \frac{1}{3} = \frac{3}{12} - \frac{4}{12} = \frac{-1}{12}$, we get:



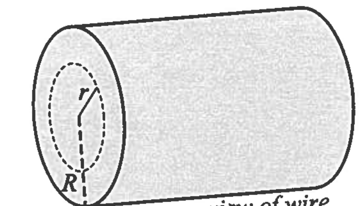
$$\boxed{\vec{B}_{net} = \frac{\mu_0 I}{12\pi d}\left[-\hat{\imath} - 3\hat{\jmath}\right]}$$

with some basic trig, we'd also find that

$|\vec{B}_{net}| = \frac{\sqrt{10}\,\mu_0 I}{12\pi d}$ at angle $\theta = 18.4°$ left of downward

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* A long straight conducting cylinder of radius R carries a steady, non-uniform current. The current density is given by the expression $J(r) = a\,r$, where r is the distance from the center of the wire and a is a positive constant.

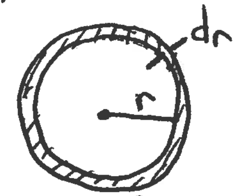


cutaway view of wire

(*i*) Find an expression for the total current flowing in the wire. Express your answer symbolically in terms of R and a, along with any other required mathematical constants.

(*ii*) Determine the magnitude of the magnetic field at a distance $d = 3/5\,R$ from the center of the wire.

(i) Sum total current by considering thin rings: radius $= r$, thickness $= dr$



current through such a sub-ring is

$$dI = J(r)\,dA = J(r)\cdot(2\pi r\,dr)$$

circumference $= 2\pi r$
thickness $= dr$
$\Rightarrow$ area $= 2\pi r\,dr$

$$\Rightarrow I_{TOT} = \int_{r=0}^{r=R} [a r]\cdot 2\pi r\,dr$$

$$= 2\pi a \int_0^R r^2\,dr = 2\pi a \left[\frac{r^3}{3}\right]_0^R$$

$$\longrightarrow \boxed{I_{TOT} = \tfrac{2}{3}\pi a R^3}$$

(ii) Ampere's Law

consider loop of radius $d = \frac{3}{5}R$

$$\oint_{loop} \vec{B}\cdot d\vec{s} = \oint_{loop} B(d)\cdot ds = B(d)\oint ds = B\cdot 2\pi d$$

Meanwhile, $I_{Through\ loop}$ = all current at radii $r \le d$

$\rightarrow$ borrowing from above, $I_{Through} = \int_0^d J(r)\,dA = 2\pi a \int_0^d r^2\,dr$

so $I_{Through} = \frac{2}{3}\pi a d^3$

Putting it together:
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{Through} \longrightarrow B\cdot \cancel{2\pi}\cancel{d} = \mu_0 \frac{2\cancel{\pi}}{3} a d^{\cancel{3}\,2}$

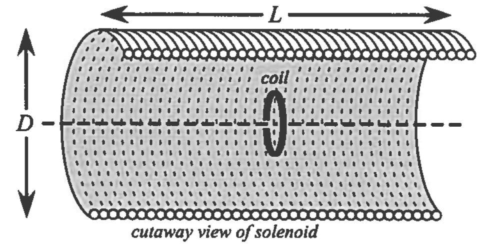
$$\boxed{B = \frac{\mu_0 a d^2}{3} = \frac{3\mu_0 a R^2}{25} = \frac{9\mu_0 I_{TOT}}{50\pi R}}$$

The following problem will be hand-graded. <u>Show all your work for this problem</u>. Make no marks and leave no space on your answer card for it.

[III] *(20 points)* A solenoid of diameter D and length L has N_S total windings of wire along its axis. A sinusoidal current is driven through the solenoid, given by the expression:

$$I(t) = I_o \sin(\omega t)$$

Assume that a *positive* value for $I(t)$ corresponds to a clockwise flow of current around the circumference of the solenoid, when viewed from the left. A circular coil of diameter $D/3$ having N_C total windings is placed coaxially within the solenoid.



cutaway view of solenoid

(*i*) Use Faraday's Law to find an expression for the induced emf in the coil as a function of time.

(*ii*) Use Lenz's Law to determine the <u>direction</u> of the induced current in the coil at time $t = 0$.

Sign convention means that when I = pos, $\vec{B}_{solenoid}$ is rightward in solenoid

magnitude is $B(t) = \mu_0 n I(t) = \mu_0 \frac{N_S}{L}\left[I_o \sin(\omega t)\right]$

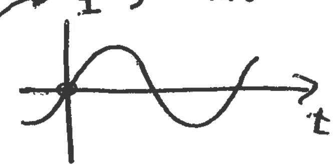
$\Rightarrow$ flux through one winding of coil is $\Phi_1 = BA_{coil} = B \cdot \pi \frac{diam^2}{4} = B\pi \frac{(D/3)^2}{4}$

Total flux through N_c windings of coil is

$$\Phi_{TOT} = N_c \Phi_1 = N_c \cdot \mu_0 \frac{N_S}{L} I_o \sin(\omega t) \cdot \pi \frac{D^2}{36}$$

$$= \frac{\mu_0 \pi N_c N_S D^2 I_o}{36L} \sin(\omega t)$$

$\Phi(t) \sim \sin(\omega t)$



induced emf is then $\mathcal{E} = -\frac{d\Phi}{dt} = \boxed{-\frac{\mu_0 \pi N_c N_S D^2 I_o \omega}{36L} \cos(\omega t)}$

sign is not critical here

but note: $\mathcal{E}(t=0)$ is negative

To get direction, note:

at $t=0$ solenoid current is changing from neg to pos

$\Rightarrow$ $\vec{B}_{sol}$ is changing from leftward to rightward

$\Rightarrow$ Flux through coil is changing from leftward to rightward

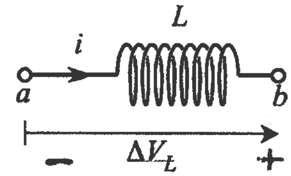
coil self-generates leftward flux = leftward self-induced $\vec{B}$-field

From sign convention, leftward $\vec{B}$ means negative $I_{induced}$ consistent!

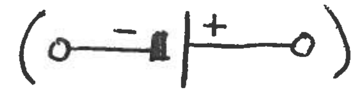
$\Rightarrow$ $\boxed{\text{at } t=0,\ I_{ind} = \text{ccw around coil}}$ (seen from left)

Question value 8 points

(1) A solenoid having self-inductance $L = .48$ H has a rightward-directed current flowing through it. At a particular moment in time, the potential difference across the inductor, moving from left to right, is $\Delta V_L = +24$ V. What can be said about the current flowing through the inductor at this moment?



(a) The current is 2.0 A at this moment, but is increasing as time passes.
(b) The current is 50 amps and ~~constant at this moment~~
(c) The current is decreasing at a rate of 50 amps per second.
(d) The current is increasing at a rate of 50 amps per second.
(e) The current is 0.02 A at this moment, but is decreasing as time passes.



Note: at this moment, inductor is acting as an emf that would sustain a rightward current!

Inductor: when crossing through, in same direction as current flow, Faraday's law gives

$$\Delta V_L = -L\frac{dI}{dt}$$

① $\Delta V_L \neq 0$ means $\frac{dI}{dt} \neq 0 \rightarrow$ $\boxed{I \text{ is not constant}}$

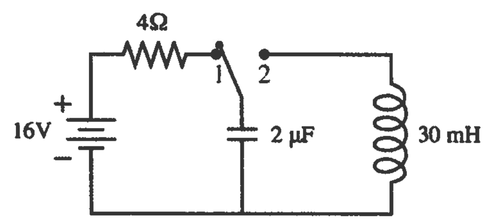
$$\Rightarrow \frac{dI}{dt} = \frac{-\Delta V_L}{L} = \frac{-24V}{0.48H} = \boxed{-50\,A/s}$$

consistency check: inductor sustaining a decreasing current!

negative sign implies a decreasing current

Question value 8 points

(2) The switch has been in position 1 for a long time. It is changed to position 2 at $t = 0$ s. What is the maximum current through the inductor?



(a) 7.63 mA
(b) 4.00 A
(c) 120 mA
(d) 131 mA
(e) 1.20 A

when at possition 2, LC circuit sustains oscillating current

→ Electromagnetic energy is conserved: $U_c + U_L = \text{constant}$

→ when $I = \text{max}$, all energy is in inductor

→ at $t=0$, all energy is in capacitor

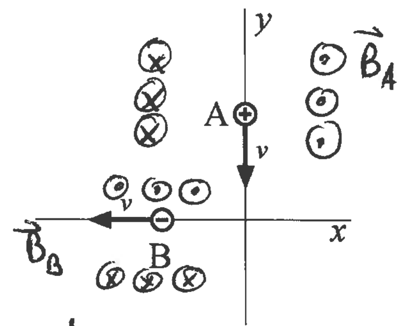
$$\cancel{U_{cf}} + U_{Lf} = U_{ci} + \cancel{U_{Li}}$$

0 0

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$$

$$\boxed{I_{max} = \sqrt{\frac{C}{L}}\,\mathcal{E} = 0.1306\text{ A}}$$

The next two problems involve the following situation:
Positive point charge A moves with constant velocity along the y-axis. Negative point charge B moves with constant velocity along the x-axis. Both charges are moving with identical speeds v.



Question value 4 points

(3) At the moment when A crosses the origin, what direction is the magnetic force (if any) exerted by A <u>on B</u>?

(a) $\vec{F}_{on\ B}$ is in the <u>negative</u> ~~x-direction~~.
(b) $\vec{F}_{on\ B}$ is zero.
(c) $\vec{F}_{on\ B}$ is in the <u>negative</u> y-direction.
(d) $\vec{F}_{on\ B}$ is in the <u>positive</u> y-direction.
(e) $\vec{F}_{on\ B}$ is in the <u>positive</u> ~~x-direction~~.

Magnetic field due to A:
- into page in left half-plane, and
- out of page in right half plane

$\rightarrow$ at charge B, $\vec{B}_A$ = into page = $-\hat{k}$
for charge B, $(q\vec{v})$ = to the right = $+\hat{i}$ (remember: q_B = negative!)
magnetic force: $q\vec{v}\times\vec{B} \sim \hat{i}\times(-\hat{k}) = +\hat{j}$

$\rightarrow$ $\boxed{\vec{F}_{on\ B} \text{ is in positive y-direction}}$

RHR confirms this

Question value 4 points

(4) At the moment when A crosses the origin, what direction is the magnetic force (if any) exerted by B <u>on A</u>?

(a) $\vec{F}_{on\ A}$ is zero.
(b) $\vec{F}_{on\ A}$ is in the <u>negative</u> x-direction.
(c) $\vec{F}_{on\ A}$ is in the <u>positive</u> ~~y-direction~~.
(d) $\vec{F}_{on\ A}$ is in the <u>negative</u> ~~y-direction~~.
(e) $\vec{F}_{on\ A}$ is in the <u>positive</u> x-direction.

Magnetic field due to B:
- <u>into</u> page in bottom half-plane
- <u>out of</u> page in top half-plane
  (remember: $(q\vec{v})$ is <u>rightward</u>!)
- Zero <u>on</u> the x-axis

So, when A crosses the origin, it is at a location where $\vec{B}_B = 0$

$$\rightarrow \vec{F}_{on\ A} = (q\vec{v})_A \times (\vec{B}_B) = (q\vec{v})_A \times 0$$

$$\boxed{\vec{F}_{on\ A} = 0}$$

Question value 8 points

(5) The loop of wire placed in the uniform magnetic field shown at right will experience

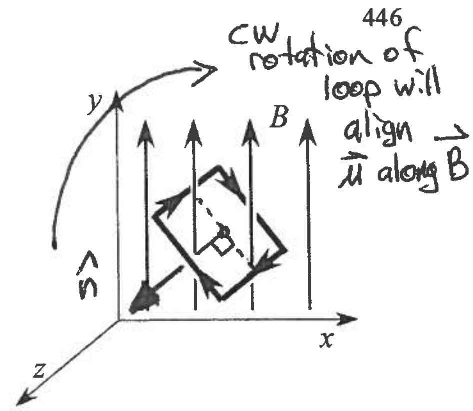
(a) a torque about the negative ~~x-axis.~~ ---> rotations around the x-axis will NOT align $\vec{\mu} \parallel$ to $\vec{B}$!

$\boxed{\text{(b) a torque about the negative } z\text{-axis.}}$

(c) a torque about the positive ~~x-axis.~~ --->

(d) *zero* torque, because the field is uniform.

(e) a torque about the positive *z*-axis.



RH curl: for current directed clockwise, from our point of view, normal direction $\hat{n}$ = down and to the ~~right~~ left (in xy plane)

⇒ loop has $\boxed{\text{magnetic moment } \vec{\mu}}$ that is $\boxed{\text{down and right}}$

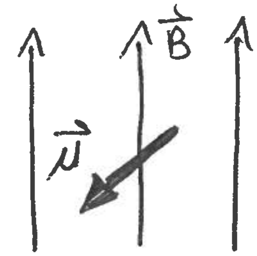
Dipole in a field feels a torque:

$\vec{\tau} = \vec{\mu} \times \vec{B}$

— aligns $\vec{\mu}$ parallel to $\vec{B}$

— tends to rotate loop CW in page

$\boxed{\text{— CW in the page means rotation around } -z \text{ axis}}$

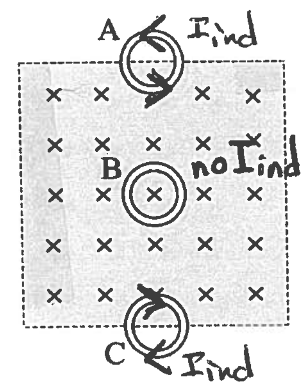


Right Hand Rule for $\vec{\mu} \times \vec{B}$:

$\swarrow \vec{\mu} \times \uparrow \vec{B} = \otimes$ into page

Question value 8 points

(6) In the figure at right, a copper ring is dropped vertically into a region containing a uniform magnetic field directed into the page. The ring enters the field at A and departs at C. What will be the direction of the magnetic force on the ring (if any) at positions A, B, and C (shown in the diagram)?



(a) $\vec{F}_A$ is up, ~~$\vec{F}_B$ is up~~, $\vec{F}_C$ is up.

(b) ~~$\vec{F}_A$ is zero~~, ~~$\vec{F}_B$ is up~~, $\vec{F}_C$ is zero.

$\boxed{\text{(c) } \vec{F}_A \text{ is up, } \vec{F}_B \text{ is zero, } \vec{F}_C \text{ is up.}}$

(d) ~~$\vec{F}_A$ is down~~, $\vec{F}_B$ is zero, $\vec{F}_C$ is up.

(e) ~~$\vec{F}_A$ is down~~, ~~$\vec{F}_B$ is up~~, ~~$\vec{F}_C$ is down~~

① entering: Flux into page increasing

→ ring self generates flux out of page via CCW current

→ bottom of ring (in field) has "rightward" current

$\vec{F} = I\vec{L} \times \vec{B}$ on bottom of ring = $\boxed{\text{upward force}}$

② within field: no change in flux → no induced current

→ $\boxed{\text{no magnetic force}}$

③ exiting: Flux into page decreasing

→ ring self-generates flux into page via CW current

→ top of ring (in field) has "rightward" current

$\vec{F} = I\vec{L} \times \vec{B}$ on top of ring = $\boxed{\text{upward force}}$