Physics 2212G/J Test form **433** Name Solutions

Fall 2016

Test 3 Recitation Section (see back of test):_________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

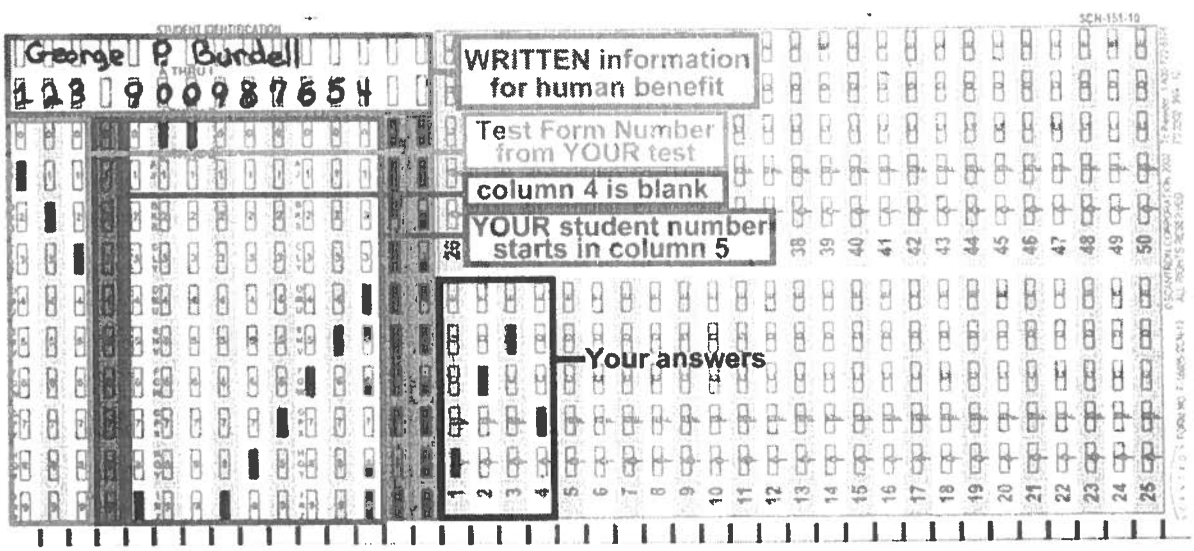
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}$	$e = 1.60\times10^{-19}\ \mathrm{C}$	$m_e = 9.11\times10^{-31}\ \mathrm{kg}$
$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$	$g = 9.81\ \mathrm{m/s^2}$	$m_p = 1.67\times10^{-27}\ \mathrm{kg}$

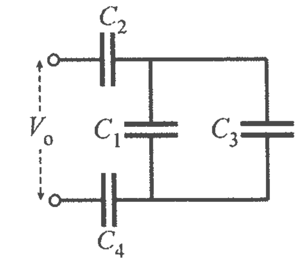
Your test form is: 433



Our next test will be on Tuesday, November 29

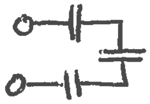
The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* Four capacitors $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are arranged in the network at right. A potential difference of magnitude V_o is applied across the terminals, and the capacitors are allowed to charge up to equilibrium. Determine the amount of charge stored on each capacitor. Express each answer in terms of the parameters C and V_o.



① C_1 and C_3 are in parallel $C_{13} = C_1 + C_3 = 4C$

→ revised network is:

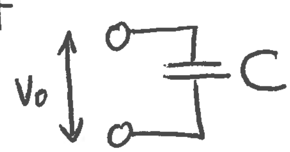


② All three are in series $C_{eq} = \left(\frac{1}{C_2} + \frac{1}{C_{13}} + \frac{1}{C_4}\right)^{-1} = \left(\frac{1}{2C} + \frac{1}{4C} + \frac{1}{4C}\right)^{-1}$

$= \left(\frac{1}{C}\right)^{-1}$

$\Rightarrow \boxed{C_{eq} = C}$ for entire network

③ Analyze equivalent



$Q_{TOT} = C_{eq}\,\Delta V = CV_o$

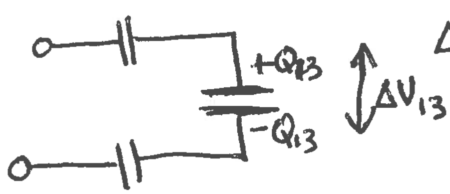
④ Rebuild original network

- $C_{13}\,C_2\,C_4$ in series — same stored charge, equal to total charge on equivalent

$\Rightarrow Q_2 = Q_{13} = Q_4 = Q_{TOT} = CV_o$

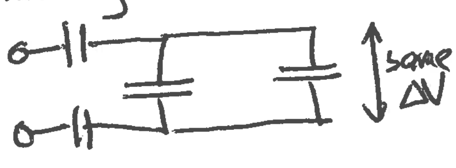
So $\boxed{Q_2 = CV_o}$ $\boxed{Q_4 = CV_o}$ two down, two to go!

- Knowing Q_{13}, we can deduce ΔV_{13}



$\Delta V_{13} = \frac{Q_{13}}{C_{13}} = \frac{CV_o}{4C} = V_o/4$

- Knowing ΔV_{13}, we know $\Delta V_1 = \Delta V_3 = \Delta V_{13}$ (parallel config)

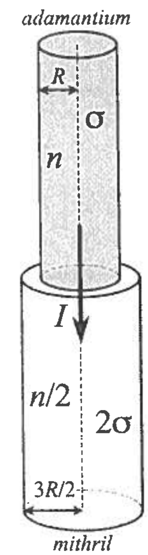


then $Q_1 = C_1 \Delta V_1 = \frac{CV_o}{4}$ $\boxed{Q_1 = \frac{CV_o}{4}}$

$Q_3 = C_3 \Delta V_3 = 3C\frac{V_o}{4}$ $\boxed{Q_3 = \frac{3CV_o}{4}}$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* Adamantium is a metal having conductivity σ and charge carrier density n. Mithril is a metal having conductivity 2σ and charge carrier density n/2. In the figure at right, an adamantium wire of radius R is spliced onto a mithril wire of radius $3R/2$, and an electric current I is is driven through the junction.



(i) Compare the electric fields in the two halves of the wire, by determining the electric field magnitude E_A in adamantium as a fraction (or multiple) of E_M, the electric field magnitude in mithril.

(ii) Compare the drift speeds of electrons in the two halves of the wire, by determining the drift speed v_A in adamantium as a fraction (or multiple) of v_M, the drift speed in mithril.

(i) use $\vec{J} = \sigma \vec{E}$ and $|\vec{J}| = \frac{I}{A} = \frac{I}{\pi R^2}$

$\rightarrow$ at junction, $I_A = I_M$

$$J_A \cdot \cancel{\pi} R_A^2 = J_M \cancel{\pi} R_M^2$$

$$\sigma_A E_A R_A^2 = \sigma_M E_M R_M^2$$

$$\cancel{\sigma} E_A \cancel{R}^2 = (2\cancel{\sigma}) E_M \left(\tfrac{3}{2}\cancel{R}\right)^2$$

$$E_A = 2 \cdot \tfrac{9}{4} E_M \rightarrow \boxed{E_A = \tfrac{9}{2} E_M}$$

(ii) use $I = ei = enAv_d$

again, $I_A = I_M$

$$\cancel{e} n_A \cancel{\pi} R_A^2 v_A = \cancel{e} n_M \cancel{\pi} R_M^2 v_M$$

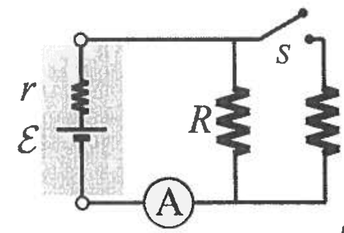
$$\cancel{n} \cancel{R}^2 v_A = \left(\tfrac{1}{2}\cancel{n}\right)\left(\tfrac{3}{2}\cancel{R}\right)^2 v_M$$

$$v_A = \tfrac{1}{2} \cdot \tfrac{9}{4} v_M$$

$$\boxed{v_A = \tfrac{9}{8} v_M}$$

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[III] *(20 points)* A real battery having known emf $\mathcal{E}$ and unknown internal resistance r is attached across a pair of identical load resistors R as show at right. When switch S is open, ammeter A measures a current I_o. When the switch is closed, the ammeter reads a current $I_c = 7/4\ I_o$. Determine the internal resistance r of the battery. Express your answer as a fraction or multiple of the resistance R.

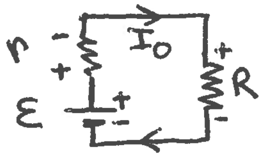


[Hint: when the switch is closed the two load resistors can be replaced by their single equivalent resistance.]

↳ no ΔV across an ideal ammeter!

Switch OPEN – no current through resistor on right

⇒ simple loop!



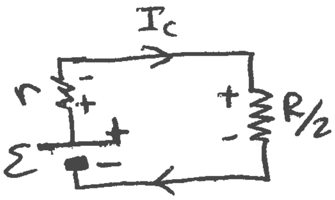
$$+\mathcal{E} - I_o r - I_o R = 0$$

$$\boxed{\mathcal{E} = I_o(r+R)}$$

Switch CLOSED both R's are in parallel

$$R_{eq} = \left(\frac{1}{R}+\frac{1}{R}\right)^{-1} = \left(\frac{2}{R}\right)^{-1} = R/2$$

⇒ with R_{eq}, simple loop again



$$+\mathcal{E} - I_c r - I_c \frac{R}{2} = 0$$

$$\boxed{\mathcal{E} = I_c\left(r+\frac{R}{2}\right)}$$

Compare equations

$$I_c\left(r+\frac{R}{2}\right) = \mathcal{E} = I_o(r+R)$$

$$\left(\frac{7}{4}\cancel{I_o}\right)\left(r+\frac{R}{2}\right) = \cancel{I_o}(r+R)$$

$$\frac{7}{4}r + \frac{7}{8}R = r + R$$

$$\frac{3}{4}r = \frac{1}{8}R$$

$$\boxed{r = \frac{R}{6}}$$

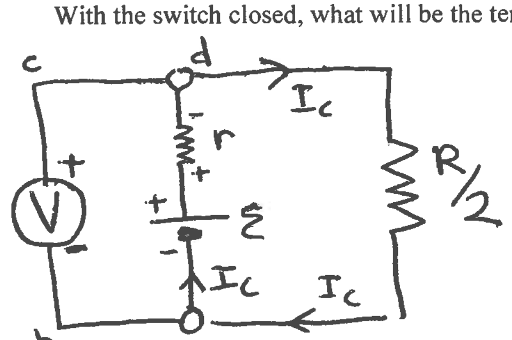
then $I_o = \frac{\mathcal{E}}{r+R} = \frac{\mathcal{E}}{R/6+R} = \frac{6\mathcal{E}}{7R}$

$I_c = \frac{\mathcal{E}}{r+R/2} = \frac{\mathcal{E}}{R/6+R/2} = \frac{3\mathcal{E}}{2R}$

will be useful for Bonus Question, below

[Bonus: 4 points]

With the switch closed, what will be the terminal potential across the battery? Express your answer as a fraction of $\mathcal{E}$.



no I through voltmeter

"Terminal Potential" = voltage measured by an ideal voltmeter attached across terminals a,d

→ look at loop abcda

$$\Delta V_{loop} = \underbrace{(+V_{Term})}_{\text{voltmeter}} + \underbrace{(+I_c r)}_{\text{internal resistance}} + \underbrace{(-\mathcal{E})}_{\text{internal emf}} = 0$$

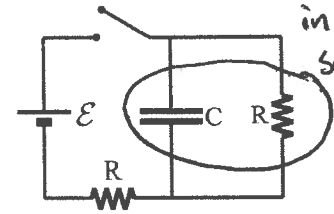
$$\Rightarrow V_{Term} + \left(\frac{3\mathcal{E}}{2R}\right)\left(\frac{R}{6}\right) - \mathcal{E} = 0$$

$$V_{Term} + \frac{\mathcal{E}}{4} - \mathcal{E} = 0$$

$$\boxed{V_{Term} = \frac{3}{4}\mathcal{E}}$$

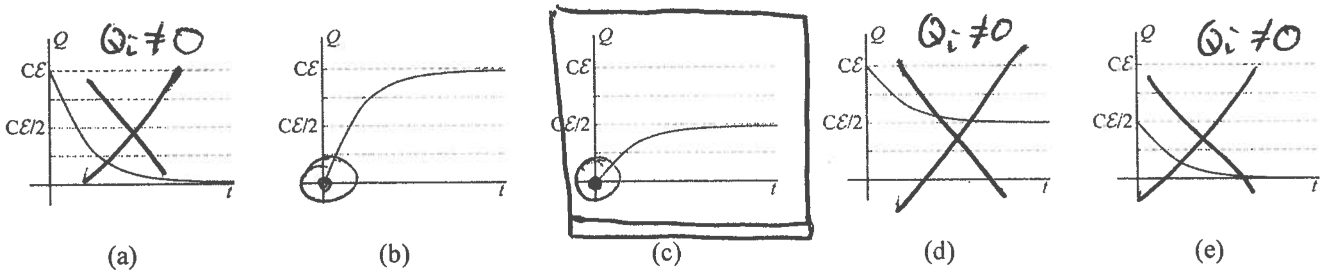
Question value 8 points

(1) In the circuit at right, the switch has been open for a very long time. If the switch is suddenly closed, which of the graphs below best depicts the potential charge stored on the capacitor as a function of time? Assume that $t = 0$ corresponds to the the moment the switch is closed.



in parallel! same ΔV

Hint: What current flows in each branch after the switch has been closed for a long time?



① with switch open, C will discharge through resistor on right

→ "open for a long time" means $\boxed{Q_i = 0}$

② Close switch → after another long time: C = fully charged → no current in center branch

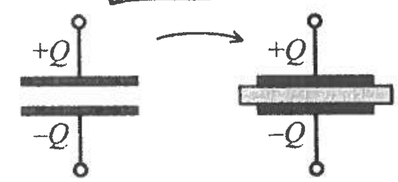
⇒ all current around outer loop $+\mathcal{E} - I_f R - I_f R = 0$ $\quad I_f = \mathcal{E}/2R$

③ $\Delta V_R = I_f R = \mathcal{E}/2$ for either resistor

→ $\Delta V_{c,f} = \Delta V_{R,f} = \mathcal{E}/2 \rightarrow Q_f = C\Delta V_f = \boxed{C\mathcal{E}/2}$

Question value 8 points

(2) An isolated capacitor has a total charge Q placed upon it. A dielectric material is carefully inserted into the space between the capacitor plates, without disturbing the charge on the capacitor. What will happen to the magnitude of the electric field E between the capacitor plates, and to the energy U stored in the capacitor?



(a) E will increase and U will remain the same.
(b) E will decrease and U will increase.
(c) E will remain the same and U will decrease.
(d) E will decrease and U will decrease.
(e) E will increase and U will increase.

Q = fixed, but C, ΔV

$C_\kappa = \kappa C_0$ → $\boxed{\text{capacitance increases}}$

then $\Delta V = \frac{Q}{C}$ → $\boxed{\Delta V \text{ decreases}}$

since plate separation does not change,

$\Delta V = |\vec{E}|d \rightarrow |\vec{E}| = \frac{\Delta V}{d}$

$\boxed{E \text{ decreases}}$

energy: $U_c = \frac{1}{2}Q\Delta V$

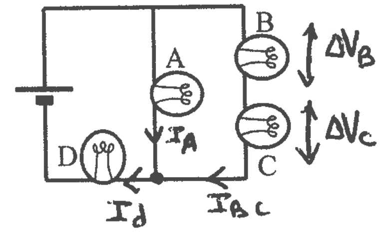
$= \frac{1}{2}$(something fixed) × (something that decreases)

$\boxed{U_c \text{ decreases}}$

[You could also use $U = \frac{1}{2}C\Delta V^2$ or $U = \frac{Q^2}{2C}$]

Question value 8 points

(3) Four identical bulbs are placed in the circuit shown at right. Rank the four bulbs in brightness, from greatest to least.



(a) $A = D > B > C$

(b) $A = D > B = C$

(c) $D > A > B > C$

(d) $A > B > C > D$

(e) $D > A > B = C$

① All current flows through D

$\boxed{\text{D is brightest}}$

② Resistance of B+C branch > resistance of A branch

⇒ current through A branch > B+C branch

$\boxed{\text{A is brighter than B or C}}$

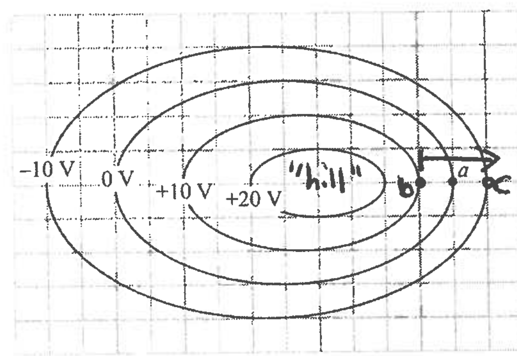
③ Same current flows through B and C

$\boxed{\text{B and C are equally bright}}$

$|P| = I^2R = \frac{\Delta V^2}{R}$

Question value 8 points

(4) The figure at right displays a series of equipotential curves. The grid spacing is 1.0 cm per square. What is the magnitude and direction of the electric field at point *a*?



(a) 500 N/C, to the left.

(b) The field is zero at *a*.

(c) 1000 N/C, to the left.

(d) 1000 N/C, to the right.

(e) 500 N/C, to the right.

"gradient" $\vec{E}_x = \langle -\frac{\Delta V}{\Delta X} \rangle$

• from b to c : $\vec{\Delta X} = +2.0\text{cm} = 2\times10^{-2}\text{m}$

$V_b = +10V,\ V_c = -10V$, so $\Delta V = V_c - V_b = -20V$

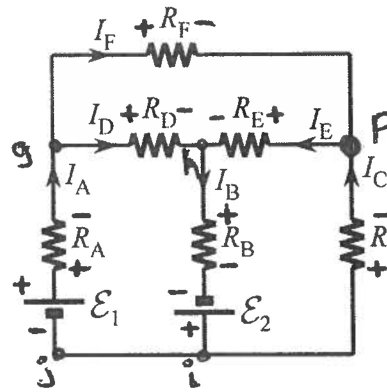
then $\vec{E}_x = -\frac{(-20V)}{(+2\times10^{-2}m)} = -(-1000\ V/m) = +1000\ V/m$

recall: "V/m" is the same as "N/c" so

$$\boxed{\vec{E}_x = \langle +1000\ N/c \rangle}$$

↳ to the right

The next two questions both involve the following situation:
Two ideal emfs and six resistances are arranged in the circuit shown at right. In each branch of the circuit, the assumed direction of current flow is also indicated.



Question value 4 points

(5) Which of the equations below is a valid application of the Loop Rule to the circuit?

$\boxed{\text{(a)} \quad -I_D R_D - I_B R_B + \mathcal{E}_2 + \mathcal{E}_1 - I_A R_A = 0}$

(b) $+\mathcal{E}_1 - I_A R_A - I_F R_F \quad I_C R_C = 0$

~~(c)~~ $\cancel{+\mathcal{E}_2 + I_B R_B + I_D R_D - I_F R_F = 0}$ Not even an actual loop!

~~(d)~~ $\cancel{+I_D R_D + I_E R_E - I_B R_B = 0}$ Not even an actual loop!

(e) $-I_C R_C \quad I_F R_F - I_D R_D - I_B R_B + \mathcal{E}_2 = 0$

① Use direction of current to indicate high potential (current in) and low potential (current out) for each resistor

② Loop ghijg: $-I_D R_D - I_B R_B + \mathcal{E}_2 + \mathcal{E}_1 - I_A R_A = 0$

(down through R_D) (down through R_B) (up through $\mathcal{E}_2$) (up through $\mathcal{E}_1$) (down through R_A)

Question value 4 points

(6) Which of the equations below is a valid application of the Junction Rule to the circuit?

~~(a)~~ $\cancel{I_A + I_D + I_F + I_C = I_B + I_E}$ not a junction

(b) $I_A = I_B + I_C$

$\boxed{\text{(c)} \quad I_E = I_C + I_F}$

~~(d)~~ $\cancel{I_A + I_D = I_C + I_B}$ not a junction

~~(e)~~ $\cancel{I_F + I_E = I_D}$ not a junction

at point P in circuit: I_c, I_F flow in, I_E flows out

$\rightarrow \boxed{I_c + I_F = I_E}$