Physics 2212G/J | Test form $\boxed{422}$ | Name **Solutions**

Fall 2016

Test 2 | Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

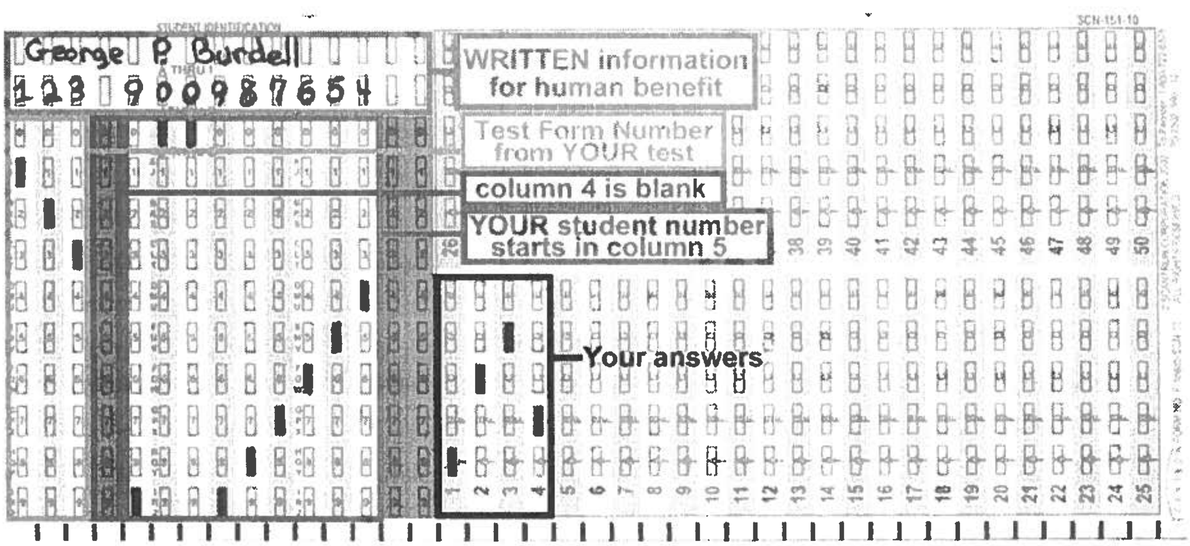
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99\times10^{9}\ \mathrm{N{\cdot}m^2/C^2}$	$e = 1.60\times10^{-19}\ \mathrm{C}$	$m_e = 9.11\times10^{-31}\ \mathrm{kg}$
$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$	$g = 9.81\ \mathrm{m/s^2}$	$m_p = 1.67\times10^{-27}\ \mathrm{kg}$

Your test form is: $\boxed{422}$

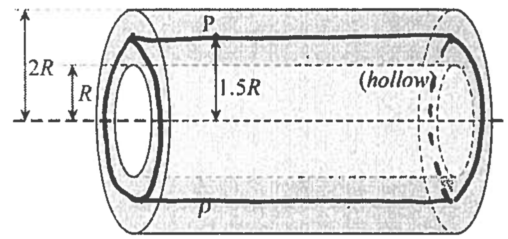


Our next test will be on Tuesday, November 1

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[I] *(20 points)* The figure at right shows a cutaway view of a finite sub-length of a very long cylindrical non-conducting pipe (inner radius R, outer radius $2R$) A uniform charge per unit volume ρ has been placed on the pipe.

Use Gauss' Law to calculate the magnitude of the electric field at the point P, which is a distance $1.5R$ from the central axis of the pipe. Be sure to specifiy the direction of the field.



① Consider gassian surface = cylinder of radius $r = \frac{3}{2}R$, that includes point P (and generic length ΔL)

⇒ Field is constant and radial on surface, so:

$$\oint \vec{E} \cdot \delta \vec{A} \longrightarrow \oint E(r)\, \delta A = E(r) \oint dA = E(r) \cdot 2\pi r \cdot \Delta L$$

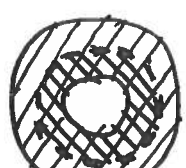
since this GS has radius $r = \frac{3}{2}R$, we get

$$\Phi = E(\tfrac{3}{2}R) \cdot 2\pi (\tfrac{3}{2}R)\Delta L$$

$$\boxed{\Phi_{GS} = E \cdot 3\pi R \Delta L}$$

② Now, determine charge found inside this surface

end view: charge is only found in cross-shaded region, between R and $\frac{3}{2}R$



$$\to Q_{in} = \rho \cdot V_{in} = \rho \cdot A_\perp \Delta L$$

$\hookrightarrow A_\perp$ = cross-sectional area between R and $\frac{3}{2}R$

$$= \rho \pi \left[(\tfrac{3}{2}R)^2 - (R)^2 \right] \Delta L = \rho\pi \left[\tfrac{9}{4}R^2 - R^2 \right] \Delta L$$

$$\boxed{Q_{in} = \tfrac{5}{4} \rho \pi R^2 \Delta L}$$

③ Substitute both expressions above into Gauss' Law:

$$\Phi_{gs} = \frac{1}{\varepsilon_0} Q_{in} \longrightarrow E \cdot 3\cancel{\pi}\cancel{R}\cancel{\Delta L} = \frac{1}{\varepsilon_0} \rho \frac{5}{4} \cancel{\pi} R^{\cancel{2}} \cancel{\Delta L}$$

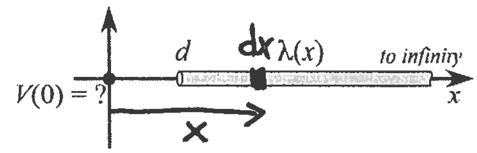
$$\boxed{E = \frac{5\rho R}{12\varepsilon_0}}$$

(magnitude)

if ρ = positive, $\vec{E}$ = radially outward
if ρ = negative, $\vec{E}$ = radially inward

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[II[*(20 points)* A *half-infinite* rod lies along the x-axis, extending from the finite position $x = +d$ to the infinite position $x = +\infty$. Charge is distributed non-uniformly along the rod, with a linear density given by the expression:



$$\lambda(x) = \frac{A}{x^2}$$

where A is a positive constant having units of [charge]×[length].

(*i*) Find an algebraic expression for the total charge Q on the rod. (Yes, Q is finite, despite the rod's infinite length!) Express your answer for Q in terms of A and d.

(*ii*) Find an algebraic expression for the electric potential V at the origin ($x = 0$), relative to the value $V = 0$ at infinity. Express your answer in terms of Q (that you just found), d, and the permittivity constant, ε_0.

① Given $\lambda(x)$, charge on a small subsegment of length dx is $\delta Q = \lambda(x)dx$

$\rightarrow \delta Q = \frac{A}{x^2}dx$ where $0 \le x < \infty$

② Total charge on rod is found by summing over all allowed x-values

$$Q = \int \delta Q = \int_{x=d}^{x=\infty} A\frac{dx}{x^2} = A\int_d^\infty x^{-2}dx = A\left[\frac{x^{-1}}{-1}\right]_d^\infty = A\left[\frac{-1}{x}\right]_d^\infty$$

$$Q = A\left[-\frac{1}{\infty} - \frac{-1}{d}\right] \Rightarrow \boxed{Q = \frac{A}{d}} \quad \text{or } A = Qd$$

③ Using same expression for "pointlike" charge δQ, potential is

$$V = \int \frac{\delta Q}{4\pi\varepsilon_0 r}$$ where r = distance from δQ to evaluation point

$\rightarrow$ here, for δQ at x, eval point at origin, we simply have $r = x$

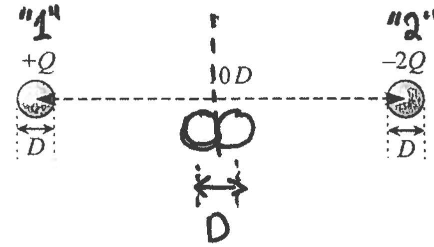
$$V(0) = \int_{x=d}^{x=\infty} \frac{\frac{A}{x^2}dx}{4\pi\varepsilon_0 x} = \frac{A}{4\pi\varepsilon_0}\int_d^\infty x^{-3}dx = \frac{A}{4\pi\varepsilon_0}\left[\frac{x^{-2}}{-2}\right]_d^\infty$$

$$= \frac{A}{4\pi\varepsilon_0}\left[\frac{-1}{2(\infty)^2} - \frac{-1}{2d^2}\right] = \frac{A}{8\pi\varepsilon_0 d^2} = \frac{A/d}{8\pi\varepsilon_0 d} = \frac{Q}{8\pi\varepsilon_0 d}$$ (from above)

$$\boxed{V(\text{origin}) = \frac{Q}{8\pi\varepsilon_0 d}}$$

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* A pair of identical small spheres having mass m and diameter D are given charges $+Q$ and $-2Q$, respectively. They are separated by a center-to-center separation distance $10D$, and released from rest. Determine the speeds of each of the two spheres when they impact one another. You may assume that the electric charge remains uniformly distributed on the spheres as they move.



① Charges have same mass, feel forces of equal magnitude, and thus same magnitude of acceleration

$\Rightarrow$ end result: they will meet at the exact midpoint between their initial positions

- their final center-to-center separation is $r_f = 2R = D$
- they both have the same final speeds $V_{1f} = V_{2f} = V_f$

② Energy is conserved: $\cancel{K_i}^{\text{zero}} + U_i = K_f + U_f$

$\rightarrow$ electrostatic PE is $U = \frac{kq_1q_2}{r}$ or $\frac{q_1q_2}{4\pi\varepsilon_0 r}$ where $r_i = 10D$, $r_f = D$

$$\rightarrow U_i = \frac{kQ(-2Q)}{10D} = -\frac{kQ^2}{5D}$$

$$U_f = \frac{kQ(-2Q)}{D} = -\frac{2kQ^2}{D}$$

$$K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = 2\left[\tfrac{1}{2}mv_f^2\right] = mV_f^2$$

so:

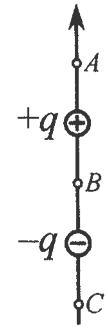
$$0 + \frac{-kQ^2}{5D} = mV_f^2 + \frac{-2kQ^2}{D}$$

$$mV_f^2 = \frac{+10kQ^2}{5D} - \frac{kQ^2}{5D} = \frac{9kQ^2}{5D}$$

$$\boxed{V_f = \sqrt{\frac{9kQ^2}{5mD}} \quad \text{or} \quad V_f = \sqrt{\frac{9Q^2}{20\pi\varepsilon_0 mD}}}$$

Question value 8 points

(1) Two point charges having equal magnitudes but opposite signs are placed as shown at right. Compare the electric potentials at positions A, B, and C.



(a) $V_A = V_B = V_C$

(b) $V_C > V_B > V_A$

(c) $\boxed{V_A > V_B > V_C}$

(d) $V_A = V_C > V_B$

(e) $V_B > V_A = V_C$

$V_{TOT} = \sum_i V_i$

where $V_i = \frac{kQ_i}{r_i}$

$\Rightarrow V_{TOT} = \frac{+kQ}{r_+} + \frac{-kQ}{r_-} = \boxed{kQ\left[\frac{1}{r_+} - \frac{1}{r_-}\right]}$

point A: $r_+ < r_-$ so $\frac{1}{r_+} - \frac{1}{r_-}$ is positive : $\boxed{V_A > 0}$

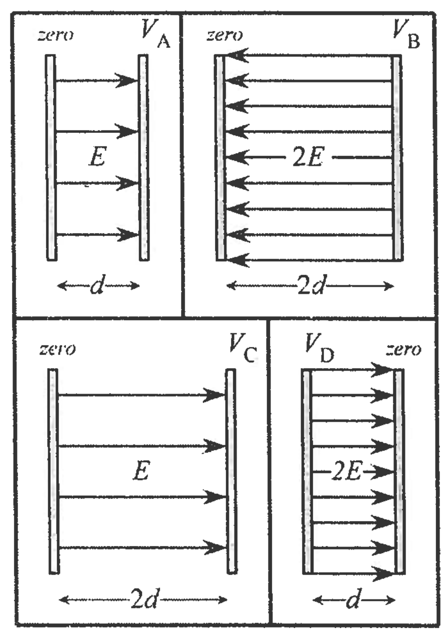
point B: $r_+ = r_-$ so $\frac{1}{r_+} - \frac{1}{r_-} = 0$: $\boxed{V_B = 0}$

point C: $r_+ > r_-$ so $\frac{1}{r_+} - \frac{1}{r_-}$ is negative : $\boxed{V_C < 0}$

$$\boxed{V_A > V_B > V_C}$$

Question value 8 points

(2) In the diagrams at right, four charged capacitors each have one plate specified as being at "zero volts". The field strength and plate separation within each capacitor is indicated. Rank, from highest (most positive) to lowest (most negative), the potentials V_A through V_D at the second plate of each capacitor.



(a) $V_B = V_D > V_A = V_C$

(b) $\boxed{V_B > V_D > V_A > V_C}$

(c) $V_B > V_C = V_D > V_A$

(d) $V_C > V_A > V_D > V_B$

(e) $V_B > V_A > V_D = V_C$

$\Delta V = -\vec{E} \cdot \vec{\Delta s}$

in each case, start at plate labelled "zero"

A: $V_A - \cancel{V_0}^{\text{zero}} = -\langle +E\rangle \cdot \langle +d\rangle = -Ed$

B: $V_B - \cancel{V_0}^{\text{zero}} = -\langle -2E\rangle \cdot \langle +2d\rangle = +4Ed$

C: $V_C - \cancel{V_0}^{\text{zero}} = -\langle +E\rangle \cdot \langle +2d\rangle = -2Ed$

D: $V_D - \cancel{V_0}^{\text{zero}} = -\langle +2E\rangle \cdot \langle -d\rangle = +2Ed$

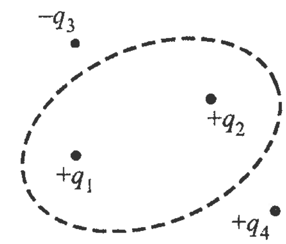
coord system: right = positive (for all four cases)

$+4Ed > +2Ed > -Ed > -2Ed$

$V_B > V_D > V_A > V_C$

Question value 8 points

(3) In the figure shown, the dashed line represents the cross-section of a three-dimentional Gaussian Surface. The four charges shown have magnitudes given by the symbols q_i, and signs as indicated explicitly in the figure. Thus, charge 3 is negative, while charges 1, 2, and 4 are positive. Let Φ_i represent the flux through the indicated Gaussian Surface due to charge "i" *only*. Which of the following flux comparisons is valid?



(a) $\cancel{\Phi_4 > \Phi_3}$

(b) $\boxed{\Phi_1 + \Phi_2 > \Phi_3 + \Phi_4}$

(c) $\cancel{\Phi_1 + \Phi_2 + \Phi_4 = \Phi_3}$

(d) $\cancel{\Phi_4 > 0}$

(e) $\cancel{\Phi_1 + \Phi_2 = \Phi_4 - \Phi_3}$

Gauss' Law

$$\Phi_{gs} = \frac{Q_{in}}{\varepsilon_0}$$

→ $+q_1$ and $+q_2$ are inside, so

$$\Phi_1 = \frac{+q_1}{\varepsilon_0} \quad \Phi_2 = \frac{+q_2}{\varepsilon_0} \quad \text{(both are positive)}$$

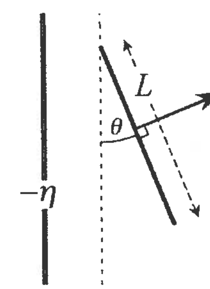
→ $-q_3$ and $+q_4$ are outside, so

$$\boxed{\Phi_3 = 0 \quad \Phi_4 = 0}$$

only statement consistent with this fact is $\boxed{\Phi_1 + \Phi_2 > \Phi_3 + \Phi_4}$ ("positive" > "zero")

Question value 8 points

(4) A very large charged surface has a uniform area charge density $-\eta$. A small, square plastic sheet of length L on a side is held near the surface as shown at right. (In the figure, we send an *edge view* of the plastic sheet; it extends a distance L directly into the page.) The sheet is oriented with its surface tilted at angle θ away from being parallel with the larger surface. The normal direction for the plastic sheet is chosen to be "up and to the right", as indicated in the figure. What is the electric flux through the plastic sheet?



(a) $\Phi = 0$, because plastic is an insulating material.

(b) $\Phi = -\frac{\eta L^2}{2\varepsilon_0}\sin\theta$

(c) $\Phi = +\frac{\eta L^2}{2\varepsilon_0}\sin\theta$

(d) $\boxed{\Phi = -\frac{\eta L^2}{2\varepsilon_0}\cos\theta}$

(e) $\Phi = +\frac{\eta L^2}{2\varepsilon_0}\cos\theta$

① Flat sheet in uniform field:

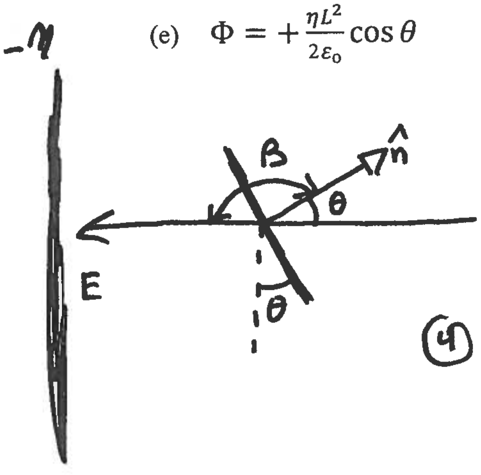
$$\Phi = \vec{E}\cdot\vec{A} = \vec{E}\cdot(L^2\hat{n}) = EL^2\cos\beta$$

where $\boxed{\beta = \text{angle between } \vec{E} \text{ and } \hat{n}}$

② Sheet has negative charge, so $\vec{E}$ = toward sheet = to the left

magnitude is $E = E_{sheet} = \frac{\eta}{2\varepsilon_0}$

③ $\hat{n}$ makes angle θ relative to rightward horizontal so $\beta = 180° - \theta$

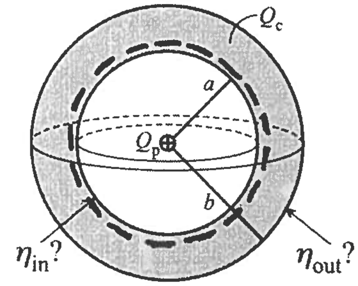


④ $\Phi = \left(\frac{\eta}{2\varepsilon_0}\right)(L^2)\cos(180-\theta) = \frac{\eta L^2}{2\varepsilon_0}\left[-\cos\theta\right]$

$$\boxed{\Phi = -\frac{\eta L^2}{2\varepsilon_0}\cos\theta}$$

The next two questions both involve the following situation:

A point charge Q_p = +2.4 nC is placed at the center of a thick conducting shell with inner radius a = 12.0 cm and outer radius b = 18.0 cm. The conductor carries a net charge Q_c = +3.6 nC. What is the charge density on the outer surface of the conductor?



Question value 4 points

(5) What is the charge density on the inner surface of the conductor?

(a) $\eta_{in} = -13$ nC/m^2

(b) $\eta_{in} = +2.4$ nC/m^2

(c) $\eta_{in} = 0$

(d) $\eta_{in} = -19$ nC/m^2

(e) $\eta_{in} = +5.9$ nC/m^2

negative is good

Take gaussian surface to be a sphere of radius r that is just barely larger than a : $E(r) = E(\text{in conductor}) = 0$

$\Rightarrow$ Therefore $\Phi_{gs} = 0$ and hence $Q_{in} = 0$

BUT: Q_{in} = (charge inside cavity) + (charge on inner surface)

$$\boxed{0 = +Q_p + Q_{inner}}$$

$$\boxed{Q_{inner} = -Q_p}$$ negative!

surface charge density: $\eta_{inner} = \dfrac{Q_{inner}}{A_{inner}} = \boxed{\dfrac{-Q_p}{4\pi a^2}} = -13.26\text{ nC/m}^2$ rounds to -13 nC/m^2

Question value 4 points

(6) What is the charge density on the outer surface of the conductor?

(a) $\eta_{out} = +5.9$ nC/m^2

(b) $\eta_{out} = +8.8$ nC/m^2

(c) $\eta_{out} = +2.9$ nC/m^2

(d) $\eta_{out} = +15$ nC/m^2

(e) $\eta_{out} = +11$ nC/m^2

$$Q_{inner} = -Q_p$$

$$Q_c = Q_{inner} + Q_{outer} = (-Q_p) + Q_{outer}$$

$$\Rightarrow Q_{outer} = Q_c + Q_p$$

$$\eta_{outer} = \frac{Q_{outer}}{A_{outer}} = \boxed{\frac{Q_c + Q_p}{4\pi b^2}} = +14.7\text{ nC/m}^2$$

rounds to $\boxed{+15\text{ nC/m}^2}$