Physics 2212G/J
Fall 2016
Test 1

Test form **411**

Name Solution Set

Recitation Section (see back of test):__________

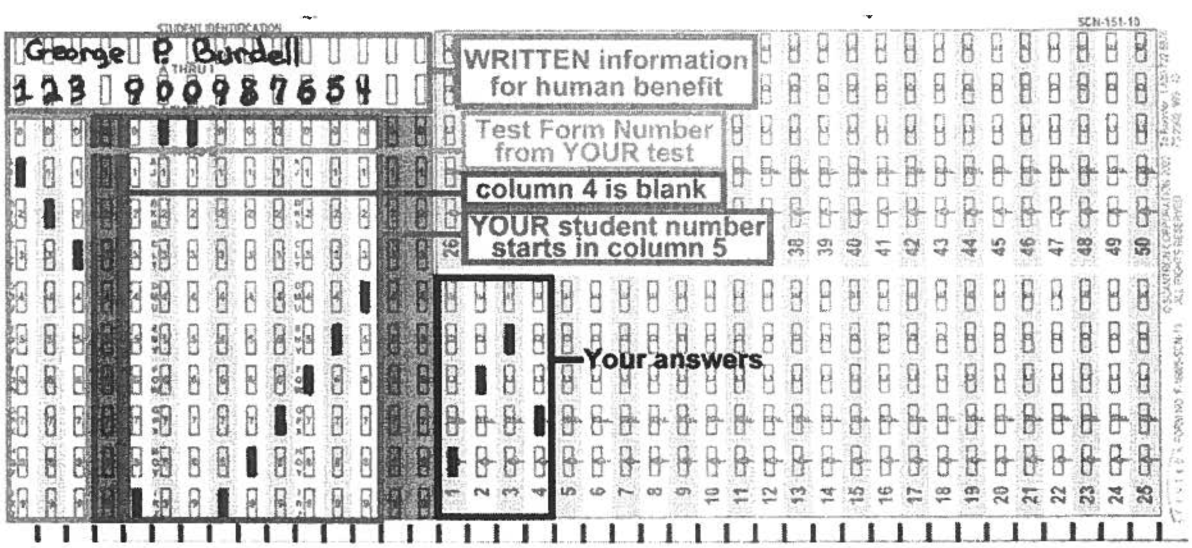
A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".
2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.
4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**
5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Numerical Constants:

$k = 8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}$	$e = 1.60\times10^{-19}\ \mathrm{C}$	$m_e = 9.11\times10^{-31}\ \mathrm{kg}$
$\varepsilon_o = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$	$g = 9.81\ \mathrm{m/s^2}$	$m_p = 1.67\times10^{-27}\ \mathrm{kg}$

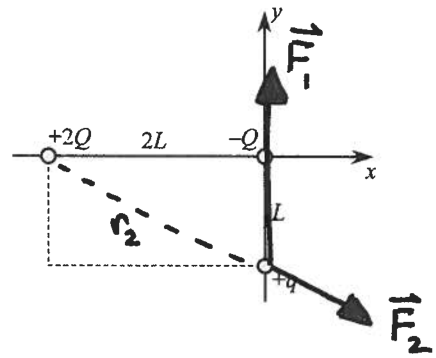
Your test form is: 411



Our next test will be on Tuesday, October 4

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* The figure at right displays point charges placed at three corners of a $2L \times L$ rectangle. Determine the direction of the net electric force on charge $+q$. Express your answer as a numerical direction angle measured relative to one of the coordinate axes.



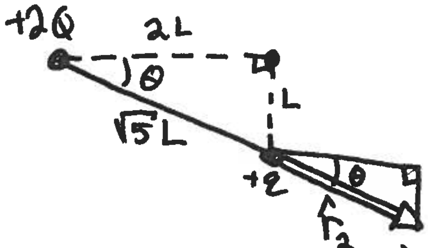
Charge #1 = $-Q$ at origin

$\Rightarrow$ force on $+q$ is $\left|\frac{kQq}{r_1^2}\right|(+\hat{j})$ (opposite charges attract)

since $r_1 = L$ $\quad \boxed{\vec{F}_1 = \frac{kQq}{L^2}(+\hat{j})}$

Charge #2 at $x = -2L$ $\rightarrow$ distance to $+q$ is $r_2 = \sqrt{(2L)^2 + (L)^2} = \sqrt{5}\,L$

$\Rightarrow$ force on $+q$ has magnitude $\frac{k(2Q)q}{5L^2} = \boxed{\frac{2}{5}\frac{kQq}{L^2}}$



(not to scale)

direction of repulsive force is down/right at angle θ

$\hat{r}_2 = (+\cos\theta)\hat{i} + (-\sin\theta)\hat{j} = +\frac{2}{\sqrt{5}}\hat{i} + \frac{-1}{\sqrt{5}}\hat{j}$

$\Rightarrow$ from position triangle, $\cos\theta = \frac{2L}{\sqrt{5}L} = \frac{2}{\sqrt{5}}$

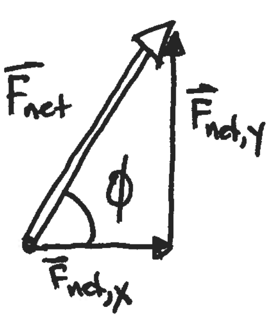
$\sin\theta = \frac{L}{\sqrt{5}L} = \frac{1}{\sqrt{5}}$

So, $\boxed{\vec{F}_2 = \frac{2kQq}{5L^2}\left[+\frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}\right]} = \frac{kQq}{L^2}\cdot\frac{2}{5}\left[\frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}\right]$

Adding these, to get net Force

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = \frac{kQq}{L^2}\left\{[+\hat{j}] + \frac{2}{5}\left[+\frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}\right]\right\}$$

$$= \boxed{\frac{kQq}{L^2}\left[\frac{4}{5\sqrt{5}}\hat{i} + \left(1 - \frac{2}{5\sqrt{5}}\right)\hat{j}\right]}$$



So direction, as angle above $+x$ axis, is:

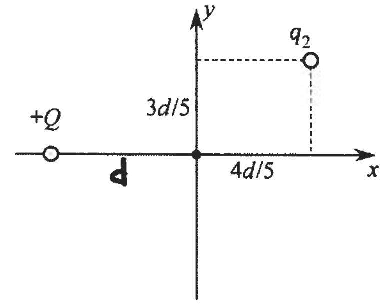
$$\phi = \tan^{-1}\left[\frac{\left(1 - \frac{2}{5\sqrt{5}}\right)}{4/5\sqrt{5}}\right] = \tan^{-1}\left(\frac{5\sqrt{5} - 2}{4}\right) = \boxed{66.5^\circ \text{ above } +x \text{ axis}}$$

The following problem will be hand-graded. <u>*Show all your work for this problem*</u>. *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* A positive source charge $+Q$ is placed on the negative x-axis, a distance d from the origin. A second source charge q_2 having unknown sign and magnitude is placed at $\langle x, y\rangle = \langle 4d/5\ , 3d/5\rangle$. It is found that the net electric field at the origin has <u>no</u> x-component, i.e. $\vec{E}_{\text{net},x} \equiv 0$.

(i) Determine the magnitude and sign of charge q_2. Express your answer as a multiple of Q.

(ii) Find the net electric field at the origin. Express your answer as a Cartesian vector (i.e. $\hat{\imath}$ and $\hat{\jmath}$ terms) involving the symbols k, Q, and d.



① If q_2 is tive, field $\vec{E}_2$ at origin is <u>toward</u> q_2 : up and to the right

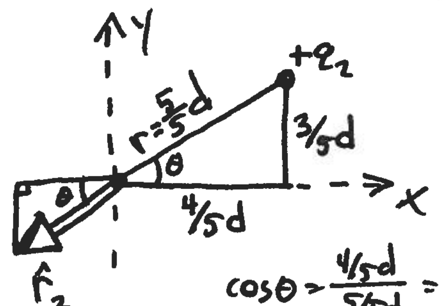
② If q_2 is positive, field $\vec{E}_2$ at origin is <u>away from</u> q_2 : down and to the left

But: At origin, field $\vec{E}_1$ due to $+Q$ is away from $+Q$: $\boxed{\vec{E}_1 \text{ is definitely to the right}}$

$\Rightarrow$ If $\vec{E}_{\text{net},x} = 0$, $\boxed{\vec{E}_{2x} \text{ must be to the left}}$ $\rightarrow$ case ② above must hold: $\boxed{q_2 \text{ is positive}}$

Now, visualize and compute <u>both</u> $\vec{E}$s

$$\boxed{\vec{E}_1 = \frac{kQ}{d^2}(+\hat{\imath})}$$ wow, that was hard...



$\cos\theta = \frac{4/5 d}{5/5 d} = \frac{4}{5}$

$\sin\theta = \frac{3/5 d}{5/5 d} = \frac{3}{5}$

To find $\vec{E}_2$, note that

A) $r_2 = \sqrt{(4/5 d)^2 + (3/5 d)^2} = \sqrt{\frac{25}{25} d^2} = \frac{5}{5} d = d$

[we actually have a 3-4-5 triangle]

B) $\hat{r}_2 = (-\cos\theta)\hat{\imath} + (-\sin\theta)\hat{\jmath} = (-\frac{4}{5})\hat{\imath} + (-\frac{3}{5})\hat{\jmath}$

so $$\boxed{\vec{E}_2 = \frac{k(+q_2)}{d^2}\left[-\frac{4}{5}\hat{\imath} - \frac{3}{5}\hat{\jmath}\right]}$$

$$\vec{E}_{net} = \frac{k}{d^2}\left[\left(+Q - \frac{4}{5}q_2\right)\hat{\imath} + \left(-\frac{3}{5}q_2\right)\hat{\jmath}\right] \Longrightarrow$$

require $\vec{E}_{net,x} \equiv 0$

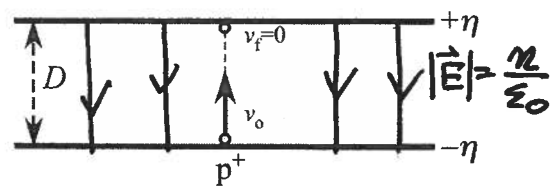
$Q - \frac{4}{5}q_2 = 0$ $\boxed{q_2 = +\frac{5}{4}Q}$

knowing q_2, we are then left with:

$$\vec{E}_{net} = \vec{E}_{net,y} = \frac{k}{d^2}\left(\frac{-3}{5}\cdot\frac{5}{4}Q\right)\hat{\jmath} = \boxed{\frac{-3}{4}\frac{kQ}{d^2}\hat{\jmath}}$$

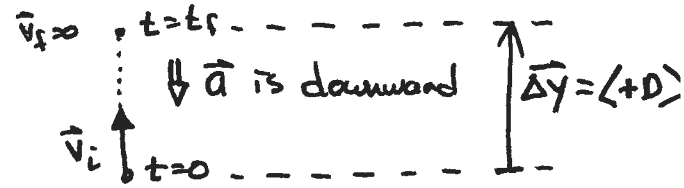
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* A capacitor consists of two large parallel plates separated by a distance D, carrying equal and opposite surface charge densities, $\pm\eta$. From the negative plate, a proton (mass m, charge $+e$) is fired directly toward the positive plate with a speed v_0. It is observed to stop momentarily just as it reaches the plate, after which it falls back toward the negative plate. (Assume that gravity is negligible.)



Find an expression for the magnitude of the surface charge density η on either plate. Express your answer in terms of the other parameters given in this problem, along with any necessary fundamental constants.

① Use kinematics to determine $\vec{a}$



- quick way —

  "speed equation" says

$$V_f^2 = V_i^2 + 2\vec{a}\cdot\vec{\Delta y}$$

$$0 = V_0^2 + 2(-a)\cdot(+D)$$

$$\longrightarrow \boxed{a = V_0^2/2D}$$

[here, "a" is the magnitude of downward acceleration
$\vec{a} = \langle -a \rangle \hat{j}$ is the acceleration]

- longer way — displacement equation: $\vec{\Delta y} = \vec{V}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$$\langle +D \rangle = \langle +V_0 \rangle \Delta t + \tfrac{1}{2}\langle -a \rangle \Delta t^2$$

velocity equation: $\vec{V}_{yf} = \vec{V}_{yi} + \vec{a}\,\Delta t$

$$0 = \langle +V_0 \rangle + \langle -a \rangle \Delta t \longrightarrow \boxed{\Delta t = \frac{V_0}{a}}$$

Plug back into displacement equation

$$\langle +D \rangle = \langle +V_0 \rangle \left(\frac{V_0}{a}\right) + \tfrac{1}{2}(-a)\frac{V_0^2}{a^2}$$

$$D = \frac{1}{2}\frac{V_0^2}{a} \longrightarrow \boxed{a = V_0^2/2D}$$ same as above

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- given accel, we know magnitude of force:

$$|\vec{a}| = \frac{|\vec{F}|}{m} \longrightarrow |\vec{F}| = ma = \frac{mV_0^2}{2D}$$

- given force, we can determine field between plates (proton = "test charge")

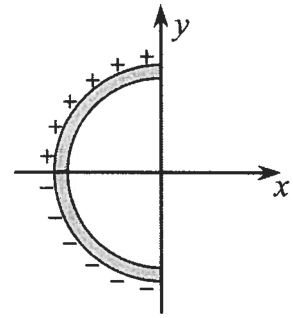
$$|\vec{E}| = \left|\frac{\vec{F}}{q}\right| = \frac{\frac{mV_0^2}{2D}}{e} = \frac{mV_0^2}{2eD}$$

- knowing field strength between capacitor plates, we recall

$$E = \frac{Q/A}{\varepsilon_0} = \frac{\eta}{\varepsilon_0} \quad \text{so} \quad \boxed{\eta = \varepsilon_0 E = \frac{\varepsilon_0 m V_0^2}{2eD}}$$

The next two questions both involve the following situation:

A thin plastic rod is bent in to a semicircle of radius R as shown at right. The upper-left portion (in Quadrant II) has uniform charge per unit length $+\lambda$, and the lower-left portion (in Quadrant III) has uniform charge per unit length $-\lambda$.



Question value 4 points

(1) What is the direction of the net electric field at the origin?

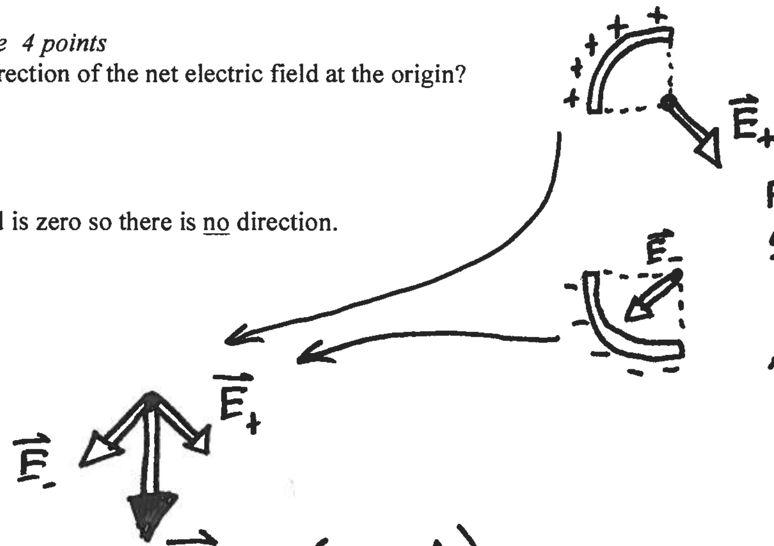
(a) $+\hat{\imath}$

(b) $+\hat{\jmath}$

(c) The field is zero so there is <u>no</u> direction.

$\boxed{\text{(d)} \quad -\hat{\jmath}}$

(e) $-\hat{\imath}$



Field vectors due to each subregion can be inferred via <u>symmetry</u>

Also – <u>magnitudes</u> of $\vec{E}_+$ and $\vec{E}_-$ are <u>equal</u>

$$\vec{E}_{net} = (-E_0\,\hat{j})$$

Question value 4 points

(2) Let E_0 be the magnitude of the electric field at the origin due to the semicircular rod. If the positively charged half is removed, what is the (new) magnitude of the electric field at the origin? *Hint: You can answer this question without doing an integral.*

(a) $2E_0$

(b) $E_0/2$

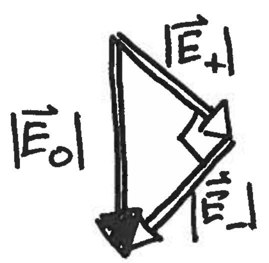
$\boxed{\text{(c)} \quad E_0/\sqrt{2}}$

(d) E_0

(e) $\sqrt{2}\,E_0$

Note that $\vec{E}_+$, $\vec{E}_-$ are equal in magnitude, and are directed 90° relative to each other

– we have a 45-45-90 triangle!



Pythagorean Theorem:

$$E_0^2 = E_+^2 + E_-^2$$

but $E_+ = E_-$

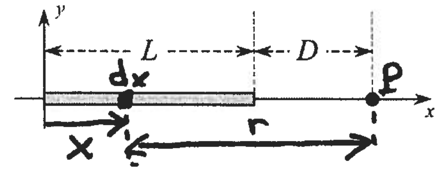
so $E_0^2 = (E_-)^2 + E_-^2 = 2E_-^2$

$$E_0 = \sqrt{2}\,E_-$$

$$\boxed{E_- = \frac{1}{\sqrt{2}} E_0}$$

Question value 8 points

(3) A uniformly charged thin plastic rod of length L lies on the positive x-axis with one end at the origin. The rod has total charge $+Q$. Which of the following expressions gives the correct magnitude for the electric field, at a point on the x-axis that is a distance D from the right end of the rod?



(a) $E = \int_{x=0}^{\cancel{L+D}} \frac{k\,Q\,dx}{L\,\cancel{x^2}}$

(b) $\boxed{E = \int_{x=0}^{L} \frac{k\,Q\,dx}{L\,(L+D-x)^2}}$

(c) $\cancel{E = \frac{k\,Q}{(D+L/2)^2}}$ NOT a point charge!

(d) $E = \int_{x=0}^{L} \frac{k\,Q\,dx}{L\,\cancel{(D+x)}^2}$

(e) $E = \int_{x=0}^{\cancel{x}} \frac{k\,Q\,dx}{L\,\cancel{(D+x)}^2}$

let small with dx be at position x

→ distance from field evaluation point (P) is $r = (D+L-x)$

→ charge on dx (= charge "at x") is $\delta Q = \lambda\,dx = \frac{Q}{L}dx$

So $d\vec{E} = \frac{\delta Q}{4\pi\varepsilon_0 r^2}\hat{r} \rightarrow \frac{Q/L\,dx}{4\pi\varepsilon_0 (D+L-x)^2}\hat{\imath}$ = field due to just "dx"

→ sum over all x values, range $x=0$ to $x=L$

magnitude of $\vec{E}$ is $\boxed{E = \int_{x=0}^{x=L} \frac{kQ}{L}\frac{dx}{(D+L-x)^2}}$

Question value 8 points

(4) The electric field strength 2.0 cm from the surface of an 11 cm-diameter metal sphere is 54,000 N/C. What is the magnitude of the charge on the ball?

(a) 18.2 nC

(b) 450 nC

(c) 2.40 nC

(d) 33.8 nC

(e) 101 nC

s = distance from surface to field point

R = distance from center to surface

"r" in pseudo-Coulomb's Law formula is $\underline{r = s + R = s + D/2} = 0.075\,\text{m}$

then $E = \frac{kQ}{r^2} = \frac{kQ}{(s+D/2)^2}$

$\boxed{Q = \frac{E(s+D/2)^2}{k}} = \frac{(5.4\times10^4\,\text{N/C})(7.5\times10^{-2}\,\text{m})^2}{(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)}$

$= 33.75\times10^{-9}\,\text{C}$

$Q = \underline{33.8\,\text{nC}}$

Question value 8 points

(5) A positive charge $+q$ is used as a test charge to detect the electric field created by a set of unknown source charges, $\{Q_i\}$. When placed at a particular point P, the charge experiences a force $\vec{F}_1$, which allows us to determine an electric field value $\vec{E}_1$. If, instead we were to use a larger, *negative* test charge, $-2q$, to probe the field, what would we measure for the force and field when the new test charge is placed at the same point P?

(a) $\boxed{\vec{F}_2 = -2\vec{F}_1 \text{ and } \vec{E}_2 = \vec{E}_1}$

(b) $\vec{F}_2 = 2\vec{F}_1$ and $\vec{E}_2 = -\vec{E}_1$

(c) $\vec{F}_2 = -\vec{F}_1$ and $\vec{E}_2 = 2\vec{E}_1$

(d) $\vec{F}_2 = \vec{F}_1$ and $\vec{E}_2 = \vec{E}_1$

(e) $\vec{F}_2 = 2\vec{F}_1$ and $\vec{E}_2 = -2\vec{E}_1$

$\vec{E}$ is created by sources $\{Q_i\}$
and independent of charge used as probe

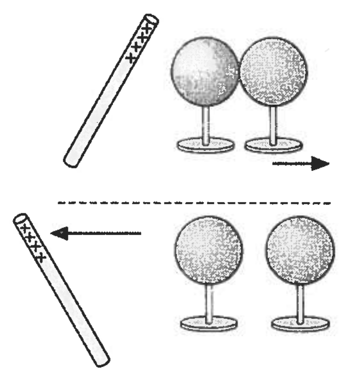
$\Rightarrow$ for both $+q$ and $-2q$, field is $\boxed{\vec{E}_2 = \vec{E}_1}$

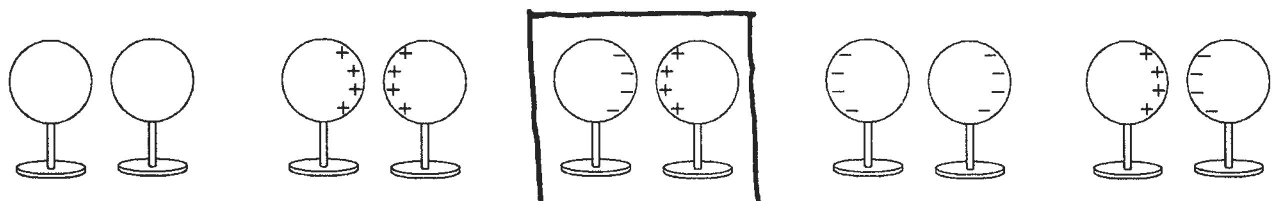
- when placed, $+q$ feels force $\vec{F}_1 = (+q)\vec{E}_1$
- similarly, $-2q$ feels force $\vec{F}_2 = (-2q)\vec{E}_2 = -2(q\vec{E}_2) = -2(q\vec{E})$

$\Rightarrow \boxed{\vec{F}_2 = -2\vec{F}_1}$

Question value 8 points

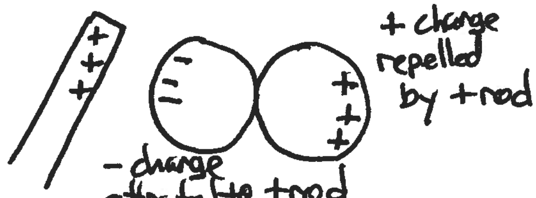
(6) Two solid uncharged conducting spheres are supported on insulating stands. With the spheres in direct contact, a positively-charged rod is brought close to the left-hand sphere, and *then* the right-hand sphere is pulled away (top figure). *After* separating the spheres, the rod is taken away (bottom figure). Which of the diagrams below best depicts the final distribution of electrical charge on the two spheres, after the rod has been removed?





(a) (b) (c) (d) (e)

① Presence of rod polarizes spheres, while in contact (as if they were a single conductor)



② Separating spheres isolates charges on each conductor:

③ Once rod is removed, opposing charges on conductors are attracted to each other, so they rearrange: