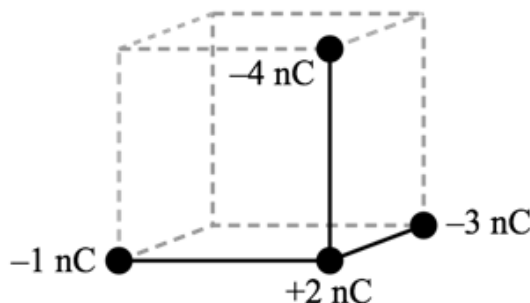


- I. (17 points) Four charges (-1.0 nC , $+2.0\text{ nC}$, -3.0 nC , and -4.0 nC), each with mass 0.10 g , are brought together and fixed at adjacent corners of a cube of edge length 1.0 mm , as shown. The -4.0 nC charge breaks free and accelerates away. How fast does it ultimately end up going?

Let the charges be $q_1 = -1.0\text{ nC}$, $q_2 = +2.0\text{ nC}$, etc. Use the Work-Energy Theorem.

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}}$$



Let the four charges constitute the system. No external forces do work on this system. No internal non-conservative forces change other forms of energy into thermal energy. The potential energy change can be related to the change in potential at the location of q_4 as it moves.

$$0 = \Delta K + Q\Delta V + 0 \quad \Rightarrow \quad 0 = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + q_4 (V_f - V_i)$$

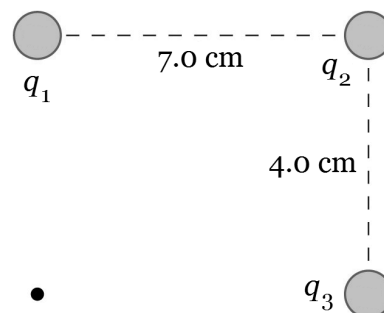
The initial velocity of q_4 is zero. As q_4 “ultimately” ends up infinitely far away, if we let $V = 0$ at infinite distance, then $V_f = 0$ and $V = KQ/r$ is a valid expression for the potential due to a point charge. The potential at the location of q_4 is the sum of the potentials due to the other three charges.

$$0 = \left(\frac{1}{2}mv_f^2 - 0\right) + q_4 (0 - V_i) \quad \Rightarrow \quad q_4 V_i = \frac{1}{2}mv_f^2 \quad \Rightarrow \quad q_4 \left(\frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} + \frac{Kq_3}{r_3} \right) = \frac{1}{2}mv_f^2$$

Let the edge length of the cube be ℓ and solve for v_f .

$$\begin{aligned} \frac{1}{2}mv_f^2 &= q_4 \left(\frac{Kq_1}{\ell\sqrt{2}} + \frac{Kq_2}{\ell} + \frac{Kq_3}{\ell\sqrt{2}} \right) \\ v_f &= \sqrt{\frac{2Kq_4}{m\ell} \left(\frac{q_1}{\sqrt{2}} + q_2 + \frac{q_3}{\sqrt{2}} \right)} \\ &= \sqrt{\frac{2(8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(-4.0 \times 10^{-9} \text{ C})}{(0.10 \times 10^{-3} \text{ kg})(1.0 \times 10^{-3} \text{ m})} \left(\frac{-1.0 \times 10^{-9} \text{ C}}{\sqrt{2}} + 2.0 \times 10^{-9} \text{ C} + \frac{-3.0 \times 10^{-9} \text{ C}}{\sqrt{2}} \right)} \\ &= 0.77 \text{ m/s} \end{aligned}$$

- II. (16 points) Three point charges, $q_1 = -3.5 \mu\text{C}$, $q_2 = +12 \mu\text{C}$, and $q_3 = +4.5 \mu\text{C}$, are situated at three corners of a rectangle as shown. What is the electric potential at the free corner where there is no charge?
Announced during quiz: Potential is zero at infinite distance.



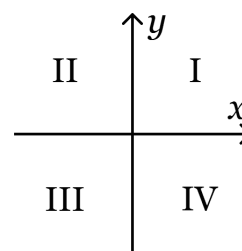
The potential at the free corner is the sum of the potentials due to the individual point charges.

$$\begin{aligned}
 V &= V_1 + V_2 + V_3 = \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} + \frac{Kq_3}{r_3} \\
 &= (8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \left(\frac{-3.5 \times 10^{-6} \text{ C}}{0.040 \text{ m}} + \frac{12 \times 10^{-6} \text{ C}}{\sqrt{(0.040 \text{ m})^2 + (0.070 \text{ m})^2}} + \frac{4.5 \times 10^{-6} \text{ C}}{0.070 \text{ m}} \right) \\
 &= 1.1 \times 10^6 \text{ V}
 \end{aligned}$$

1. (5 points) In the problem above, in what quadrant, if any, is the direction of the potential at the free corner?

Potential is a scalar!

The potential has no direction.



- III. (17 points) An infinitely long, straight non-conducting cylinder (radius R) has a uniform charge per unit volume ρ . Use Gauss' Law to find an expression for the electric field at a point outside the cylinder, a distance r from the cylinder's axis. Express your answer as a function of the variable r and the fixed parameters R and ρ (as well as any necessary constants such as ϵ_0 and π).

Choose a Gaussian Surface that matches the symmetry of the field and that passes through the point at which the field magnitude is to be determined. This would be a finite cylinder of radius r and arbitrary length L , concentric with the cylinder of charge, since symmetry considerations require the field to be radial (in the $\hat{r}$ direction). Then

$$\epsilon_0 \Phi = q_{\text{in}} \quad \Rightarrow \quad \epsilon_0 \oint \vec{E} \cdot d\vec{A} = \rho V \quad \Rightarrow \quad \epsilon_0 \left[\int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} \right] = \rho \pi R^2 L$$

There is no flux through the ends of the Gaussian Surface, as the field there is tangent to the surface. The field on the curved side of the cylindrical Gaussian Surface is uniform in magnitude, and parallel to $d\vec{A}$ at every point.

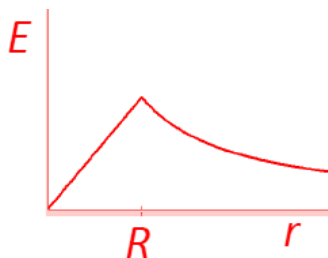
$$\epsilon_0 \left[0 + \int_{\text{side}} E \cos 0 dA \right] = \epsilon_0 E \int_{\text{side}} dA = \epsilon_0 E A_{\text{side}} = \epsilon_0 E 2\pi r L = \rho \pi R^2 L$$

Solve for the magnitude of the field, E . Note that it does not depend on L . Remembering the direction,

$$\vec{E} = \frac{\rho R^2}{2\epsilon_0 r} \hat{r}$$

2. (5 points) Suppose one wanted to graph the magnitude of the electric field as a function of r , for all values of r (that is, including points inside the charged cylinder). Which of the following sketches best represents the situation?

Whether the cylinder's surface is approached from inside or outside, the field there must approach the same value. From symmetry considerations, the field at the center of the cylinder must be zero. Only one of the offered choices satisfies both of these requirements.

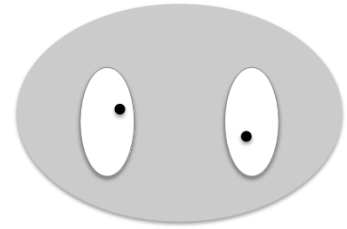


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3. (5 points) A chunk of metal has two hollow cavities (see figure). The metal itself carries net charge of -5 nC . The right cavity contains a point charge $+2\text{ nC}$, while the left cavity contains a point charge -3 nC . What is the net charge on the outer surface of the metal?

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Consider a Gaussian Surface that lies in the bulk of the metal, but that includes both cavities within it. The field at every point on this Surface is zero, as metals are conductors, so the flux through the Surface is zero. From Gauss' Law, the net charge within the Surface must also be zero. The cavities contain a net charge of -1 nC , so there must be a net charge of $+1\text{ nC}$ on the surfaces of the cavities. Since the metal has a net charge of -5 nC and charge must be conserved, the charge on the outer surface of the metal must be

-6 nC



-
4. (5 points) Two positive charges, with masses $m_1 = 1.0\text{ g}$ and $m_2 = 2.0\text{ g}$, are held a distance d apart. They are released simultaneously and accelerate away from each other, ending up with speeds v_1 and v_2 , respectively. How are v_1 and v_2 related?

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With no net external forces on the system consisting of the two charges, momentum must be conserved. Before the charges are released, the momentum is zero. After they are released, it must remain zero. Therefore, the momenta of the two charges must be equal and opposite. As $\vec{p} = m\vec{v}$, the particle with greater mass must have a lower speed.

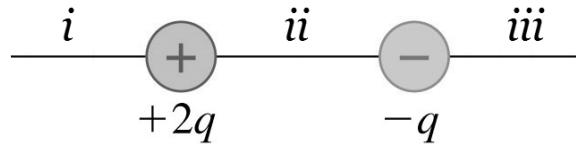
$v_1 > v_2$

-
5. (5 points) Where on the horizontal axis, other than at $\pm$ infinity, is the potential equal to zero? Define zero potential as the potential at infinity.

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The potential due to a point charge is $V = Kq/r$ with respect to zero at infinity. The potential at a location due to the two point charges is the scalar sum of the potential due to each point charge individually. For the sum of the two potentials due to these particular point charges to be zero at a location, the location must be nearer the charge with smaller magnitude. This happens

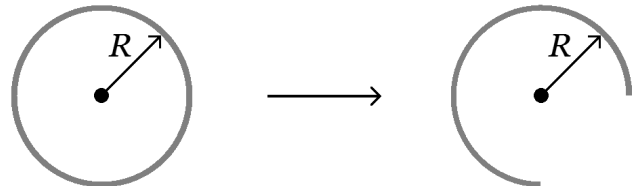
At a point in region *ii* and at a point in region *iii*.



-
6. (5 points) The potential at the center of a uniformly charged ring is V_0 (relative to zero at infinity). If 1/4 of the ring is removed, as shown below, what will be the new potential at the same position?

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All the charge is the same distance from the center. Since potential is a scalar, *how* it's distributed spatially at that distance doesn't matter. With only three-quarters as much charge present after 1/4 of the ring is removed, the potential will be

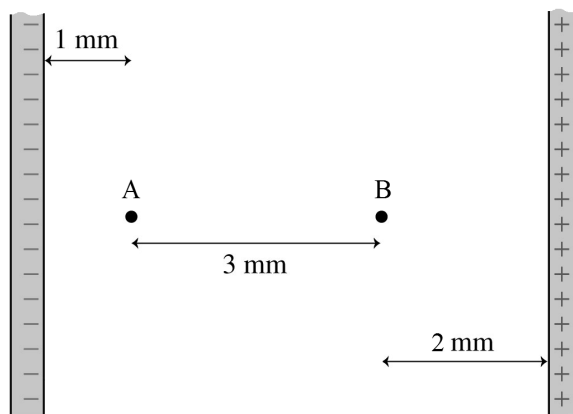


$3V_0/4$

7. (5 points) Two effectively infinite parallel plates, 6 mm apart, form a capacitor with the negative plate on the left, as shown. Point A is 1 mm from the negative plate and point B is 2 mm from the positive plate. Letting the potential be zero at the negative plate, what is V_A/V_B , the ratio of the potential at point A to that at point B ?

Potential increases linearly from the negative plate to the positive plate in an ideal parallel-plate capacitor. Point A is only one-quarter the distance from the negative plate that point B is.

$$V_A/V_B = 1/4$$



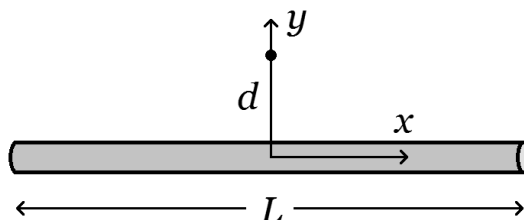
8. (5 points) A thin rod of length L lies on the x axis with its center at the origin. It has a linear charge density λ that varies with position x according to

$$\lambda = \lambda_0 \frac{x}{L}$$

where λ_0 is a constant. What is the electric potential due to the rod, with respect to zero at infinity, at a point located at d on the $+y$ axis?

With this particular charge distribution, for every element of charge dq at position $+x$ making an element of potential dV at the point, there is an element of charge $-dq$ at position $-x$ making an element of potential $-dV$. Each pair of such potential elements produces a net potential of zero, so the potential due to the entire rod is

zero



9. (5 points) A positively charged particle and a negatively charged particle, each having charge magnitude q , lie on the axis of a cylindrical surface, equidistant from the ends, as shown. Rank the flux through the top, side, and bottom of the cylinder, from greatest to least. Remember that electric flux can be positive or negative.

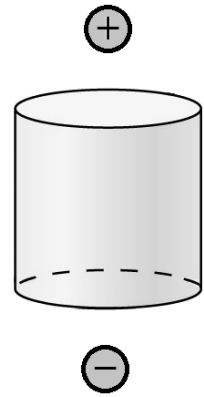
Field lines start on positive charges and end on negative charges. The area vectors for the cylinder surfaces point outward, so the angle between the field vectors and area vectors is between 90° and 180° on the top surface, and between 0° and 90° on the bottom surface. As flux is

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E \cos \theta dA$$

the flux through the top surface is negative, while that through the bottom surface is positive.

Because the charged particles are symmetrically placed, any field line that passes outward through the side of the cylinder as it heads away from the positive charge, must also pass inward through the side of the cylinder as it heads toward the negative charge. The flux through the side is zero.

$$\Phi_{\text{bottom}} > \Phi_{\text{side}} > \Phi_{\text{top}}$$



10. (5 points) A positively charged particle lies on the axis of a cylindrical surface, a distance d from one end, as shown. The cylinder has height h . Compare the *magnitude* of the electric flux through the top and bottom surfaces of the cylinder.

Consider all the field lines that pass through the top surface. Regardless of the relative values of h and d , some continue on to pass through the side, so not all of them will pass through the bottom.

$$|\Phi_{\text{top}}| > |\Phi_{\text{bottom}}|$$

