

- I. (14 points) A $2.0\ \mu\text{g}$ dust particle, that has a charge of $q = +3.0\ \text{nC}$, leaves the ground with an upward initial speed of $v_0 = 1.0\ \text{m/s}$. It encounters a $E = 400.0\ \text{N/C}$ electric field which is slanted $\theta = 30.0^\circ$ from the vertical, as shown. What maximum height above the ground does the particle reach? (Neglect gravity and drag.)

Use constant-acceleration kinematics. Let the particle start at the origin, and the $+y$ direction be upward.

$$y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2 \quad \text{and} \quad v_{yf} = v_{iy} + a_y \Delta t$$

where i and f denote initial and final states. At maximum height, the vertical component of the particle's velocity is zero. As the time to maximum height is neither known nor asked, eliminate it from the constant-acceleration kinematic equations.

$$0 = v_{iy} + a_y \Delta t \quad \Rightarrow \quad \Delta t = \frac{-v_{iy}}{a_y}$$

so

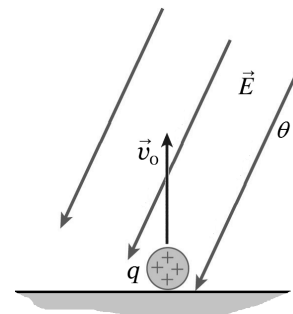
$$y_f = 0 + v_{iy} \left(\frac{-v_{iy}}{a_y} \right) + \frac{1}{2} a_y \left(\frac{-v_{iy}}{a_y} \right)^2 = \frac{-v_{iy}^2}{a_y} + \frac{v_{iy}^2}{2a_y} = \frac{-v_{iy}^2}{2a_y} = \frac{-v_0^2}{2a_y}$$

Using Newton's Second Law and the definition of electric field,

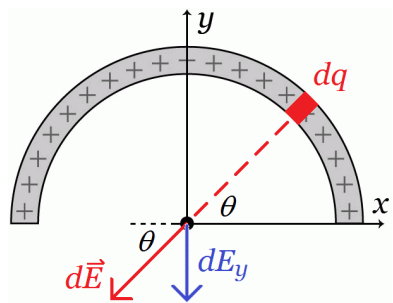
$$a_y = \frac{F_y}{m} = \frac{qE_y}{m} = \frac{-qE \cos \theta}{m}$$

Substituting this into the expression for the maximum height, and remembering to convert the particle's mass to kilograms,

$$y_f = \frac{-v_0^2}{2(-qE \cos \theta/m)} = \frac{v_0^2 m}{2qE \cos \theta} = \frac{(1.0\ \text{m/s})^2 (2.0 \times 10^{-9}\ \text{kg})}{2(3.0 \times 10^{-9}\ \text{C})(400.0\ \text{N/C}) \cos 30.0^\circ} = 9.6 \times 10^{-4}\ \text{m} = \mathbf{0.96\ \text{mm}}$$



- II. (18 points) Positive charge Q is uniformly distributed along a thin, flexible rod of length L . The rod is then bent into the semicircle as shown in the figure. Find an expression for the electric field vector at the center of the semicircle. Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.



A point-like element of charge, dq , produces an element of field, $d\vec{E}$, at the origin. If one adds up (integrates) the contributions to the field at the origin due to all the elements of charge, one can see from symmetry that the x components will cancel and the y components all point in the negative direction. The magnitude of the field is

$$E = \int dE_y = \int dE \sin \theta$$

Since the elements of charge are point-like,

$$dE = \frac{K dq}{r^2} \Rightarrow E = \int \frac{K dq}{r^2} \sin \theta$$

The linear charge density λ of the rod is uniform, and every point on the rod is the same distance R from the center.

$$\lambda = \frac{Q}{L} = \frac{dq}{ds} \Rightarrow dq = \lambda ds \quad \text{and} \quad 2\pi R = 2L \Rightarrow R = L/\pi$$

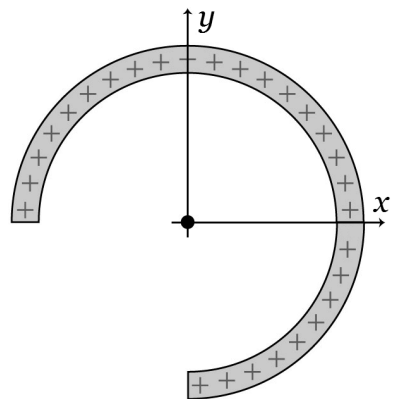
Relating an element of arc length to an element of angle and evaluating the integral

$$\begin{aligned} E &= \int \frac{K \lambda ds}{R^2} \sin \theta = \int_0^\pi \frac{K \lambda R d\theta}{R^2} \sin \theta = \frac{K \lambda}{R} \int_0^\pi \sin \theta d\theta = \frac{K \lambda}{R} [-\cos \theta]_0^\pi \\ &= \frac{-K \lambda}{R} [\cos \pi - \cos 0] = \frac{-K \lambda}{R} [-1 - 1] = \frac{2K \lambda}{R} = \frac{2K (Q/L)}{L/\pi} = \frac{2\pi K Q}{L^2} \end{aligned}$$

Since we've already determined that the electric field points in the $-y$ direction,

$$\vec{E} = \frac{-2\pi K Q}{L^2} \hat{j}$$

1. (5 points) Suppose we add another bent rod to that in the previous problem, with the same linear charge density, to make three-quarters of a circle as shown. If E_0 is the magnitude of the electric field at the origin for the semicircular rod above, what is the magnitude of the electric field at the origin now? *Hint:* You can answer this question without integration, and without having answered the problem above.



Let each quarter of the rod produce field magnitude E' at the center. Since the contributions of the upper right and upper left quarters make total field magnitude E_0 in the previous problem, then $E' = E_0/\sqrt{2}$.

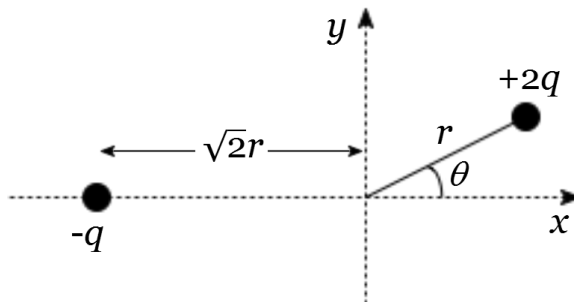
When all three quarters are present, the contributions to the field at the center due to the upper left and lower right quarters will cancel. The net field in the center is just that due to the upper right quarter. We have determined that the field due to a single quarter is

$$E_0/\sqrt{2}$$

- III. (18 points) A particle with positive charge $+2q$ can be positioned anywhere on a circle of radius r around the origin, making an angle θ with respect to the $+x$ axis. A particle with negative charge $-q$ is located on the x -axis at $x = -\sqrt{2}r$. In terms of q , r , θ and fundamental physical and mathematical constants as needed, what is the magnitude of the net electric field at the origin?

Let the particle with charge $-q$ be “1”, and the particle with charge $+2q$ be “2”. The components of the field at the origin will be

$$\begin{aligned} E_x &= E_{1x} + E_{2x} = -\frac{K|q_1|}{r_1^2} - \frac{K|q_2|}{r_2^2} \cos \theta \\ &= -\frac{Kq}{(\sqrt{2}r)^2} - \frac{K(2q)}{r^2} \cos \theta = \frac{-Kq}{r^2} \left[\frac{1}{2} + 2 \cos \theta \right] \end{aligned}$$



and

$$E_y = E_{1y} + E_{2y} = 0 - \frac{K|q_2|}{r_2^2} \sin \theta = -\frac{K(2q)}{r^2} \sin \theta = \frac{-Kq}{r^2} [2 \sin \theta]$$

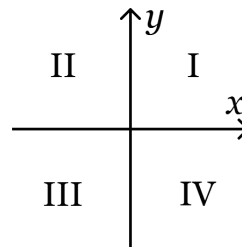
so

$$\begin{aligned} E &= \sqrt{E_x^2 + E_y^2} = \sqrt{\left(\frac{-Kq}{r^2} \left[\frac{1}{2} + 2 \cos \theta \right] \right)^2 + \left(\frac{-Kq}{r^2} [2 \sin \theta] \right)^2} \\ &= \frac{Kq}{r^2} \sqrt{\left[\frac{1}{2} + 2 \cos \theta \right]^2 + [2 \sin \theta]^2} = \frac{Kq}{r^2} \sqrt{\frac{1}{4} + 2 \left(\frac{1}{2} \right) 2 \cos \theta + 4 \cos^2 \theta + 4 \sin^2 \theta} \\ &= \frac{Kq}{r^2} \sqrt{\frac{1}{4} + 2 \cos \theta + 4 (\cos^2 \theta + \sin^2 \theta)} = \frac{Kq}{r^2} \sqrt{\frac{1}{4} + 2 \cos \theta + 4} = \frac{Kq}{r^2} \sqrt{\frac{17}{4} + 2 \cos \theta} \end{aligned}$$

2. (5 points) If it is possible, in which quadrant should the particle with charge $+2q$ be positioned for the net electric field to be in the $+y$ direction?

Since the particle with charge $-q$ makes field in the $-x$ direction, the particle with charge $+2q$ must make field with positive x and positive y components. Electric field points away from a positive charge, so that particle would have to be in

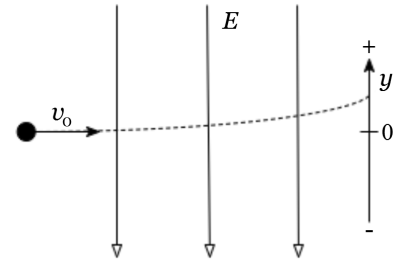
In quadrant III.



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3. (5 points) Several particles, each with the same initial horizontal initial velocity, enter a region with uniform vertical electric field. While traveling some distance, the particles are deflected by a vertical amount y . The particles are

- an electron (charge $-e$, mass m_e)
- a proton (charge $+e$, mass $1000m_e$)
- a muon (charge $-e$, mass $200m_e$)
- a neutron (charge 0, mass $1000m_e$).

The trajectory of the electron is sketched in the figure. Rank the particles according to where they arrive on the y axis, from topmost (most positive y) to bottommost (most negative y). (Gravity may be neglected.)



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The neutron is uncharged, so there is no electric force on it, and it is undeflected. The proton has the opposite charge from the electron, and so is deflected the opposite way (down the page). The muon has the same charge as the electron, so is deflected the same way as the electron, up the page. Because it is more massive, however, it has a lower acceleration, and is deflected less. The particles arrive in this order from top to bottom:

electron, muon, neutron, proton

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4. (5 points) Particles A and B have charges $-4q$ and $-2q$, respectively. In which diagram do the arrows best represent the force on each particle?

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From Newton's Third Law, we know the forces on the two particles must be equal and opposite.



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5. (5 points) A thin ring has a non-uniform linear charge density

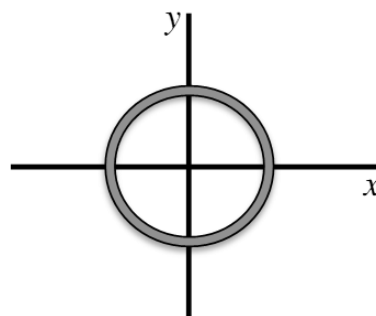
$$\lambda(\theta) = \lambda_0 \cos(\theta) \quad \text{for } 0 \leq \theta < 2\pi$$

where θ is an angle measured counter-clockwise from the positive x-axis, and λ_0 is a positive constant. What is the direction of the electric field at the center of the ring?

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The given charge distribution is symmetric about the x -axis. The electric field at the center can have no y component. The right half of the ring is positively charged, and the left half of the ring is negatively charged. As electric field points away from positive charge and toward negative charge, at the center of the ring

The field is in the $-x$ direction.

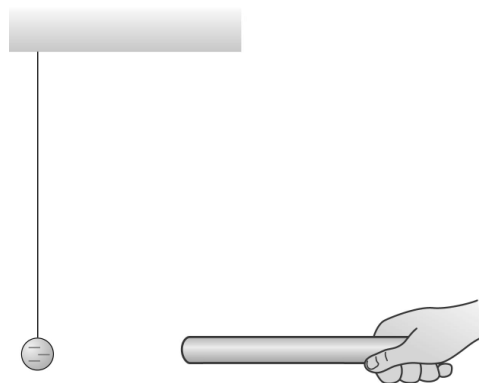


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6. (5 points) A lightweight negatively charged ball hangs from a thin insulating thread. If a neutral insulating rod is brought nearby, the ball ...

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moves toward the rod, as the rod becomes polarized.

No charge is transferred to or from the rod, so it remains neutral. However, insulators may certainly become polarized. In the non-uniform field of the ball, there is a net force attracting the polarized rod to the ball. By Newton's Third Law, there must also be a net force attracting the ball to the rod.

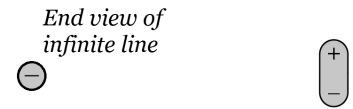


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7. (5 points) A dipole is held near a uniform infinite line of negative charge, oriented as shown. How, if at all, does the dipole move after it is released?

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The positive end of the dipole will be attracted to the rod, and the negative end will be repelled. The resulting torque will rotate the dipole counter-clockwise, putting the positive, attractive, end of the dipole in a stronger field than the negative, repulsive, end.

It rotates counter-clockwise and moves toward the line.

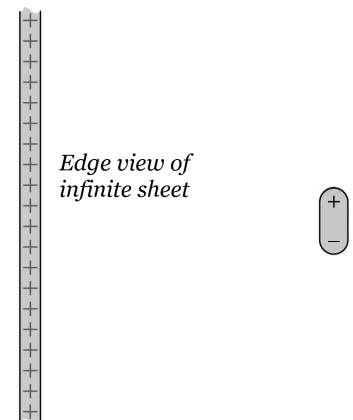


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8. (5 points) A dipole is held near a uniform infinite sheet of positive charge, oriented as shown. How, if at all, does the dipole move after it is released?

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The positive end of the dipole will be attracted to the sheet, and the negative end will be repelled. The resulting torque will rotate the dipole counter-clockwise, but because the field due to an infinite sheet is uniform, there will be no net force on the dipole.

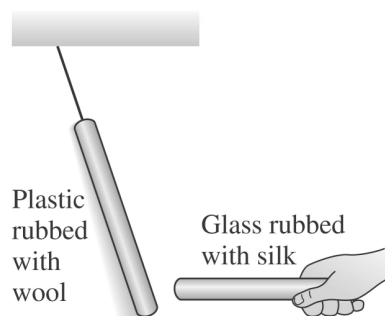
It just rotates clockwise.



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9. (5 points) A plastic rod that has been rubbed with wool is attracted to a glass rod that has been rubbed with silk. What test would indicate that a third object has the same sign charge as the glass rod?

As a neutral object can become polarized and attracted to any charged object, only a test involving a repulsive force can show that object is charged.

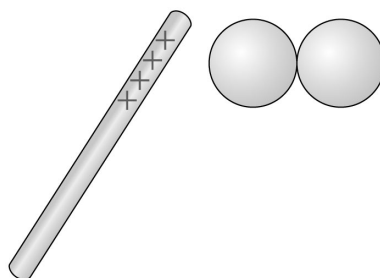
The object must have the same sign if it is repelled by the glass rod.



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10. (5 points) How could two identical conducting spheres be given equal charges?

Start with the spheres touching. Bring a charged object nearby. Then ...

As soon as the charged object is brought nearby, the combined two-sphere object is polarized, giving the spheres charges that are equal in magnitude but opposite in sign. Grounding the further sphere briefly removes its charge. When the charged object is removed, the charge on the nearer sphere will distribute itself equally between the spheres. So,



briefly connect the further sphere to ground, remove the charged object, and separate the spheres.