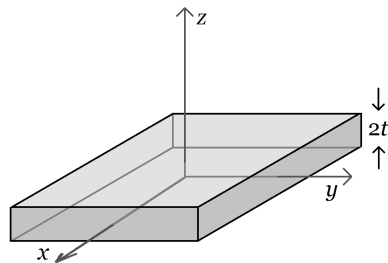
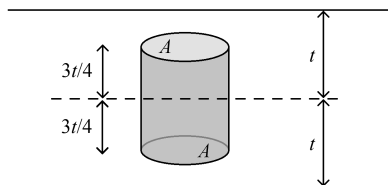


I. (18 points) A uniform infinite insulating slab of charge has a positive volume charge density ρ , and a thickness $2t$, extending from $-t$ to $+t$ in the z direction. In the x and y directions, it is infinite in extent. What is the magnitude of the electric field at a position $-3t/4$ inside the slab on the z axis? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.



Use Gauss' Law, $\epsilon_0 \Phi = q_{\text{in}}$. From symmetry, the electric field lines must point in the $\pm z$ directions. Choose a Gaussian Surface such as a cylinder that extends from $-3t/4$ to $+3t/4$, and has ends of area A . With that choice, the flux through each end is EA , and the flux through the curved side is zero.



$$\Phi = \oint \vec{E} \cdot d\vec{A} = 2EA$$

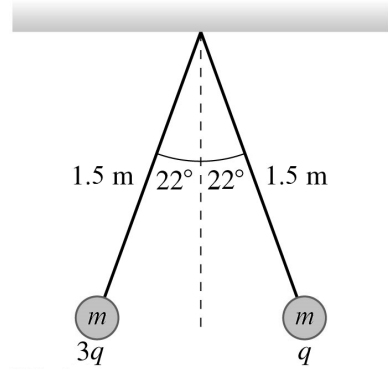
Since the charge density is uniform,

$$q_{\text{in}} = \rho V = \rho Ah = \rho A \left(\frac{6t}{4} \right)$$

So

$$\epsilon_0 \Phi = q_{\text{in}} \quad \Rightarrow \quad \epsilon_0 2EA = \rho A \left(\frac{6t}{4} \right) \quad \Rightarrow \quad E = \frac{3\rho t}{4\epsilon_0}$$

II. (16 points) Two point charges, each of mass $m = 4.0\text{ g}$, on threads 1.5 m long, repel each other after being charged to q and $3q$ as shown in the figure below. What is the charge q ? (On Earth. Do NOT neglect gravity.)



Use Newton's Second Law. Sketch a Free Body Diagram for one of the objects, such as that with charge q . There will be a gravitational force mg , a tension force T , and an electric force F_E on each. Choose a coordinate system. Write Newton's Second Law for each axis. I'll let symbols represent magnitudes, so signs are shown explicitly. Letting ℓ be the length of the threads, d be the distance between the charged particles, and θ be the given angle that the threads make with the vertical,

$$\sum F_y = T_y - mg = ma_y = 0$$

$$T \cos \theta = mg \quad \Rightarrow \quad T = \frac{mg}{\cos \theta}$$

and

$$\sum F_x = F_E - T_x = ma_x = 0$$

$$K \frac{(3q)q}{d^2} = T \sin \theta = \left(\frac{mg}{\cos \theta} \right) \sin \theta = mg \tan \theta$$

The distance d can be expressed in terms of ℓ ,

$$\sin \theta = \frac{d/2}{\ell} \quad \Rightarrow \quad d = 2\ell \sin \theta$$

So

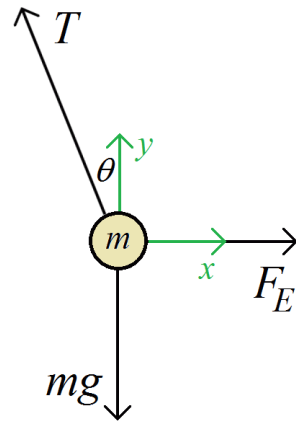
$$3K \frac{q^2}{(2\ell \sin \theta)^2} = mg \tan \theta$$

Solve for q .

$$q^2 = \frac{mg}{3K} \tan \theta (2\ell \sin \theta)^2 \quad \Rightarrow \quad q = \pm 2\ell \sin \theta \sqrt{\frac{mg}{3K} \tan \theta}$$

$$= \pm 2 (1.5\text{ m}) \sin 22^\circ \sqrt{\frac{(4.0 \times 10^{-3}\text{ kg})(9.81\text{ N/kg})}{3(8.988 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)} \tan 22^\circ}$$

$$= \pm 0.86\ \mu\text{C}$$



1. (5 points) Let the magnitude of the tension in the thread on the left, attached to the charge $3q$, be T_{left} . What is the magnitude of the tension in the thread on the right?

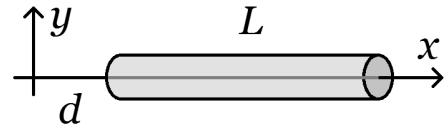
Even though the charges on the two particles aren't the same, all the forces are.

$$T_{\text{right}} = T_{\text{left}}$$

III. (16 points) A thin rod with cross-sectional area A and length L lies on the x axis. It has a non-uniform volume charge density ρ that depends on position x according to

$$\rho = \rho_0 \left(\frac{x}{L} \right)$$

What is the magnitude of the electric field at the origin? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.



Since the charge density varies with x , divide the rod into point-like elements of charge dq that are “small” in the x direction. Each of these elements has area A and length dx , and produces an element of field dE at the origin. As these elements of charge are point-like,

$$E = \int dE = \int K \frac{dq}{r^2}$$

Both dq and r should be expressed in terms of x so the integral may be evaluated.

$$r = x \quad \text{and} \quad dq = \rho dV = \rho A dx = \rho_0 \left(\frac{x}{L} \right) A dx$$

All the elements of charge from d to $d + L$ contribute to the field.

$$E = \int dE = \int_d^{d+L} K \frac{\rho_0 (x/L) A dx}{x^2} = \frac{K \rho_0 A}{L} \int_d^{d+L} \frac{dx}{x} = \frac{K \rho_0 A}{L} \ln x \Big|_d^{d+L} = \frac{K \rho_0 A}{L} \ln \frac{d+L}{d}$$

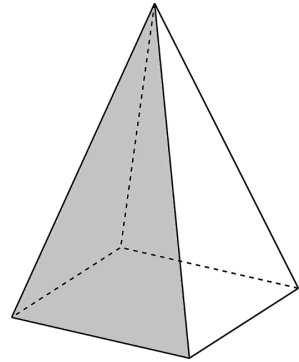
2. (5 points) What is the direction of the electric field at the origin?

Since x is positive, if ρ_0 were positive, all the charge on the rod is positive. A positive probe charge placed at the origin would be repelled from the rod. As the force on a positive probe charge is in the direction of the electric field, the electric field in this situation must be in the $-x$ direction.

However, if ρ_0 were negative, all the charge on the rod would be negative. A positive probe charge placed at the origin would be attracted toward the rod, and the electric field in this situation must be in the $+x$ direction. So, the electric field is

In the $-x$ direction if $\rho_0 > 0$, but in the $+x$ direction if $\rho_0 < 0$.

3. (5 points) A pyramid has a square base with $s = 14$ cm edges. The sides are isosceles triangles, with one edge of length s and two edges of length $2s$. The base is parallel to the ground, and a vertical uniform electric field of strength 285 N/C passes upward through the pyramid. What is the electric flux through the front (shaded) side?

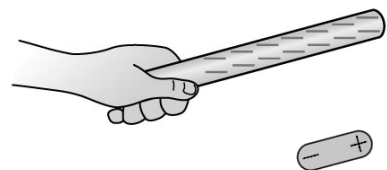


In a uniform field, there can be no net flux through the pyramid. The field at the bottom is into the pyramid and therefore produces negative flux. The flux through the triangular sides is positive and, by symmetry, each must have one-fourth the flux of the bottom.

$$\Phi_{\Delta} = \frac{-\Phi_{\text{base}}}{4} = \frac{(285 \text{ N/C})(0.14 \text{ m})^2}{4} = 1.4 \text{ N}\cdot\text{m}^2/\text{C}$$

4. (5 points) The dipole is held near the negatively-charged rod as shown. If gravity can be neglected, how, if at all, will the dipole move when it is released?

The positive end of the dipole will be attracted to the negative rod, and the negative end of the dipole will be repelled. As the field magnitude is greater nearer the rod, the attractive force on the positive end will now be greater than the repelling force on the negative end.



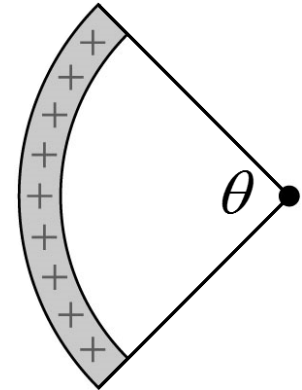
It rotates counter-clockwise and moves toward the rod.

5. (5 points) Thin rods of various lengths, but having the same uniform linear charge density λ , are bent into arcs with the same radius R . Since the rods are of various lengths, they will subtend various angles θ . For what angle θ will the magnitude of the electric field at the center of the arc be greatest?

.....

If all the rods are placed symmetrically about the horizontal axis, like the rod shown, then the vertical components of the field will cancel for every rod. For rods that subtend less than half a circle, additional bits of length contribute more horizontal electric field to the right. But for rods that subtend more than a half circle, additional bits of length contribute horizontal field to the left, canceling, in part, the rightward field created by the leftmost half-circle of rod. So, maximum field is achieved at an angle

$$\theta = \pi$$



6. (5 points) A thin insulating rod of length L lies on the y axis, with its center at the origin, as shown. The rod has a non-uniform linear charge density λ that depends on position y according to

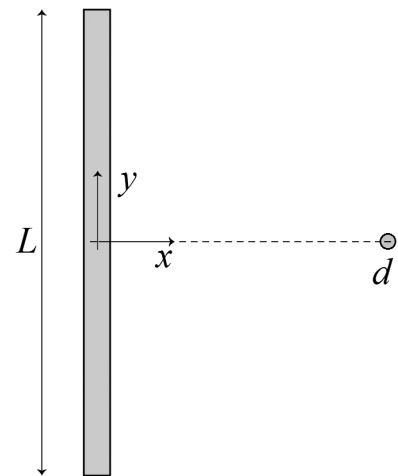
$$\lambda = \lambda_0 \cos\left(\pi \frac{y}{L}\right)$$

where λ_0 is a positive constant. What is the direction of the electric field, if any, at an *electron* located a distance d from the rod on the $+x$ axis?

.....

Since λ_0 is positive, and $\pi \frac{y}{L}$ extends from $-\pi/2$ at the bottom of the rod, to $+\pi/2$ at the top, every element of charge on the rod is positive. Each element of charge, then, would repel a positive probe charge placed a distance d from the rod on the $+x$ axis (the fact that an electron is actually there, is irrelevant). By symmetry, for every element of charge at a position $+y$, there is an element at position $-y$ with the same charge. The field due to an element of charge at $+y$ points down and to the right, while the field due to an element of charge at $-y$ points up and to the right. The y components of these field elements cancel, so

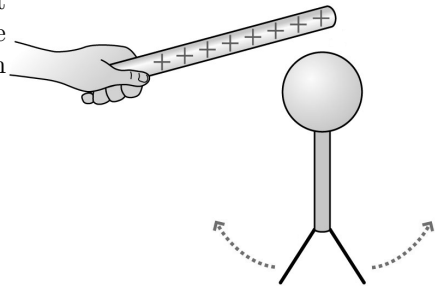
The electric field is in the $+x$ direction.



7. (5 points) An electroscope is originally neutral, then a positively-charged rod is brought nearby (but not touching) the sphere, as shown. Why do the leaves move apart?

.....

Since the rod does not touch the electroscope, the electroscope must remain neutral. Negative charges, however, are attracted toward the positive rod, polarizing the electroscope and leaving positive charge on the leaves.



The electroscope remains neutral, but is polarized.
The positively-charged leaves repel each other.

8. (5 points) When a particle of mass m and charge $-q$ is released from rest at the negative plate of a parallel-plate capacitor, it reaches the positive plate at a time Δt_1 . When a particle of mass $2m$ and charge $+q$ is released from rest at the positive plate of the same capacitor, it reaches the negative plate at a time Δt_2 . What is the ratio of times, $\Delta t_2/\Delta t_1$?

.....

Since the electric field in a parallel-plate capacitor is uniform, the force on (and thus the acceleration of) particles within it is constant. From constant-acceleration kinematics, and remembering that the particles start from rest,

$$\Delta s = \frac{1}{2}a(\Delta t)^2$$

Newton's Second Law ($F = ma$) and the relationship between electric force and field ($F = qE$) mean

$$\Delta s = \frac{1}{2} \left(\frac{qE}{m} \right) (\Delta t)^2$$

Each particle travels the same distance Δs between the plates. In terms of magnitudes

$$\Delta s_1 = \Delta s_2 \quad \Rightarrow \quad \frac{1}{2} \left(\frac{q_1 E}{m_1} \right) (\Delta t_1)^2 = \frac{1}{2} \left(\frac{q_2 E}{m_2} \right) (\Delta t_2)^2$$

As the magnitude of q_1 is the same as the magnitude of q_2 ,

$$\frac{1}{2} \left(\frac{qE}{m_1} \right) (\Delta t_1)^2 = \frac{1}{2} \left(\frac{qE}{m_2} \right) (\Delta t_2)^2 \quad \Rightarrow \quad \frac{(\Delta t_1)^2}{m_1} = \frac{(\Delta t_2)^2}{m_2} \quad \Rightarrow \quad \frac{\Delta t_2}{\Delta t_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{2m}{m}} = \sqrt{2}$$

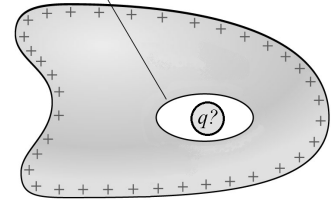
9. (5 points) A hollow conductor carries a net charge $Q = +5 \mu\text{C}$, but the charge on the outer surface is found to $+3 \mu\text{C}$. Describe the particle within the hollow.

.....
 If the conductor has a net charge of $+5 \mu\text{C}$, but only $+3 \mu\text{C}$ is on the outer surface, the other $+2 \mu\text{C}$ must be on the inner surface.

Consider a Gaussian Surface that lies entirely within the bulk of the conductor. The electric field is zero there, so there is no flux through the surface, and no net charge within the surface. For there to be no net charge within this Gaussian Surface even though there is a charge of $+2 \mu\text{C}$ on the inner surface,

The particle has a charge of $q = -2 \mu\text{C}$.

A hollow completely enclosed by the conductor



10. (5 points) A uniform solid sphere of charge has radius R and positive total charge Q . Which graph best represents the electric field it creates, as a function of distance r from the center?

.....
 From symmetry considerations, the field at the very center of the sphere must be zero. However, a Gaussian Surface anywhere inside the sphere will contain net charge, so the field anywhere other than the very center cannot be zero. Only one of the choices offered satisfies both of these conditions.

