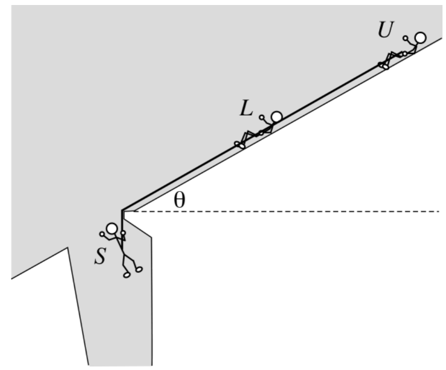
PHYS 2211 Recitation 05 *Solution*

Jun 20–22

You should work in collaborative groups of 3–4, but each student must write up their own solution to the problem. Show all your work, and explain all your reasoning.

Mountain climbers Sven, Lars, and Ulaf (having identical masses m) are navigating down a steep snowfield inclined at $\theta = 30°$ below the horizontal, when a crevasse opens beneath Sven. Fortunately, he is tethered to Lars 5.0 m upslope, who is in turn tethered to Ulaf, a further 5.0 m upslope. Both Lars and Ulaf fall to the ground, and begin to slip downslope (with coefficient of kinetic friction $\mu_k = 0.15$), as Sven drops downward while suspended by his tether. All three climbers gain speed as Lars and Ulaf slip toward the brink of the crevasse! Are these intrepid mountain climbers doomed?



TA Analysis: Each student should complete this portion of the worksheet individually, following along as the TA works the problem. The work you show here will be factored into your grade!

A. Sketch a free body diagram for Sven, and use the diagram to explain *qualitatively* whether the tension in his tether cord is greater than, equal to, or less than his gravitational weight, mg.

let tension in cord be T_1 → acts upward on Sven:

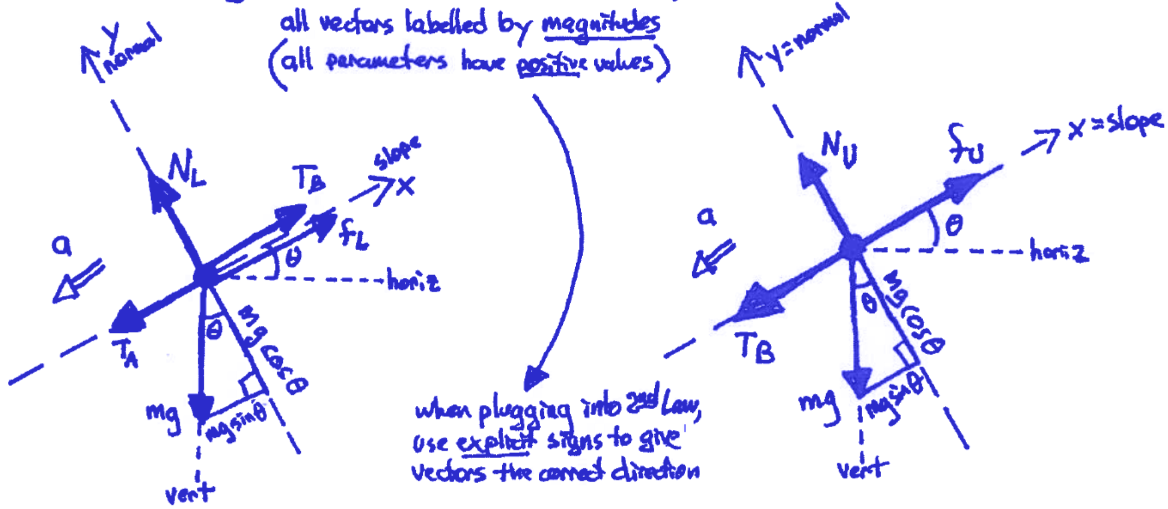
- Since Sven is accelerating downward, he must be experiencing a net downward force

⇒ upward pull of tension must be LESS THAN the downward pull of gravity $T_1 < mg$

Sven is accelerating downward

B. Sketch free body diagrams for both Lars and Ulaf, choosing appropriate coordinate axes and decomposing all force vectors as appropriate. Write out *five* Newton's Law equations for the three climbers, and identify *seven* quantities in those equations that are currently unknown. Identify another idea that we can invoke to get a 6th and 7th equation, to be able to compute all unknowns. (Here, the mass m of the climbers is presumed to be a known, but unspecified, quantity.)

- FBD's involving inclined planes require caution, and an abundance of space



Lars:

x: $\langle +T_B \rangle + \langle +f_L \rangle + \langle -T_A \rangle + \langle -mg\sin\theta \rangle = m\langle -a \rangle$

y: $\langle +N_L \rangle + \langle -mg\cos\theta \rangle = 0$

Ulaf:

x: $\langle +f_U \rangle + \langle -T_B \rangle + \langle -mg\sin\theta \rangle = m\langle -a \rangle$

y: $\langle +N_U \rangle + \langle -mg\cos\theta \rangle = 0$

don't forget poor Sven:

y only: $\langle +T_A \rangle + \langle -mg \rangle = m\langle -a \rangle$

"Unknowns"

- two friction forces, f_L & f_U (both kinetic)
- two normal forces, N_L & N_U
- two tensions, T_A & T_B
- acceleration magnitude, a

[Of course, the two normal forces will involve "easy" solutions, but right now, they are technically unknown]

Addional idea = kinetic friction:

$$f_L = \mu_K N_L$$

and

$$f_U = \mu_K N_U$$

C. Determine the tension in each of the two tether cords, expressed as a fraction or multiple of mg. Determine the magnitude of the acceleration with which Lars and Ulaf are slipping downslope, and with which Sven is plummeting vertically down into the crevasse.

For both Lars and Ulaf, equations for y-axis give $N_L = mg\cos\theta = N_U$

and hence, $f_L = \mu_k mg\cos\theta = f_U$

→ these substitutions leave us with three equations in the unknowns T_A, T_B, and a:

Lars: $T_B - T_A + \mu_k mg\cos\theta - mg\sin\theta = -ma$

Ulaf: $-T_B + \mu_k mg\cos\theta - mg\sin\theta = -ma$

Sven: $+T_A - mg = -ma$

add all three equations: $2\mu_k mg\cos\theta - mg[1+2\sin\theta] = -3ma$

$a = g[1+2\sin\theta - 2\mu_k\cos\theta]/3 \Rightarrow \boxed{a = 0.58g} = 5.68\ m/s^2$

technically, 5.7 m/s², since this is a two-digit situation

We can then find T_A from the equation for Sven: $T_A = +mg - ma = m(g-a) = m(0.42g)$

$\boxed{T_A = 0.42mg}$ less than mg, as predicted in Part A

Finally, equation for Ulaf gives us T_B

$T_B = \mu_k mg\cos\theta - mg\sin\theta + ma = mg[0.13 - 0.50 + 0.58]$ $\boxed{T_B = 0.21mg}$

D. What will be the speed of all three climbers at the moment Lars slips over the edge of the crevasse?

Simple kinematics! Let length of tether be $L = 5.0\,m$

motion along incline (Lars and Ulaf) or vertical motion (Sven)

Involves: $v_i = 0$, $v_f = ??$ a = known, Δs = known

⇒ use "speed equation":

$v_f^2 = v_i^2 + 2\vec{a}\,\vec{\Delta s}$ → ie keep track of signs!

for downward/downslope accel and displacement,

$v_f^2 = 0^2 + 2(-a)(-L)$

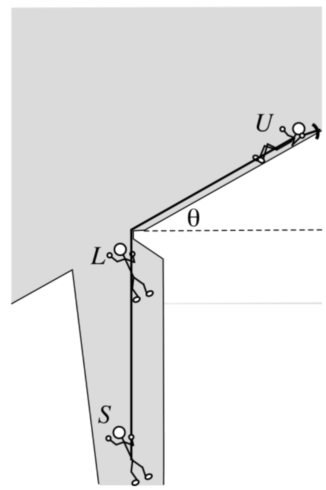
$v_f = \sqrt{2aL} = \boxed{7.5\ m/s}$

don't use a rounded value for a, here!

Student Analysis: Complete the worksheet in collaborative groups of 3–4, with each student writing up their own solution to the problem. Show all your work, and explain all your reasoning.

At the moment Lars slips over the edge, Ulaf manages to wrestle an ice-pick out of his pack. Digging the pick into the snow, he is able to generate a drag force D, that gradually slows him down. He comes to a complete stop a distance $s = 1.0\,\text{m}$ from the lip of the crevasse.

E. What is the magnitude of all three climbers' acceleration, as they all come to a stop?



Moving downslope, while slowing down: upslope acceleration

displacement is $\vec{\Delta x} = -(L-s) = -4.0\,m$

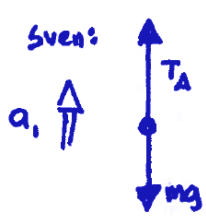
⇒ speed equation (using answer from G as v_i) gives:

$v_f^2 = v_i^2 + 2(+a_1)(-L+s)$, $v_f \to 0$

$a_1 = \frac{v_i^2}{2(L-s)} = 7.1\,m/s^2$ $(= 0.725g)$

F. Draw free body diagrams for Sven and Lars. Use qualitative arguments to compare the tension in Sven's cord to his weight mg, and to compare the tension in Lars's cord to their combined weight, $2mg$.

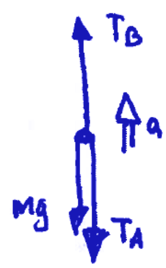
Sven:



upward accel for Sven: net force upward

$T_A > mg$

Lars:



again: upward accel implies that net force is upward

$T_B > (T_A + mg)$

but $T_A > mg$ implies

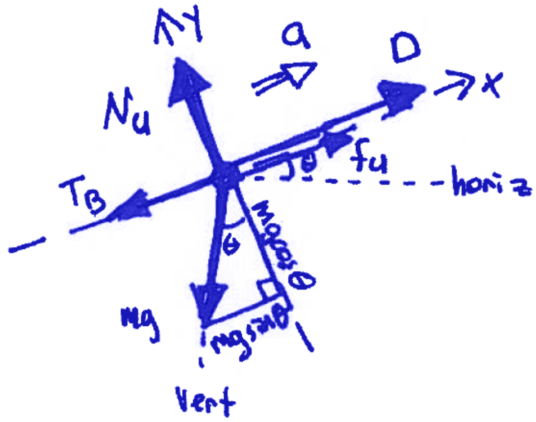
$T_B > (T_A + mg) > (mg + mg)$

$T_B > 2mg$

G. Sketch a free body diagram for Ulaf, indicating appropriate coordinate axes, and decomposing all forces as necessary.

This is pretty much the same as before — only difference is that the accel is now directed upslope, plus an extra upslope force, D!

[I'm guessing that D is large]



Checkpoint: Before continuing further, have the TA review your group's work so far.

H. Write out *four* Newton's Law equations for the three climbers, and identify the five "unknowns" that you will be able to determine, using these equations. (Recall that a fifth relationship is available, due to the nature of kinetic friction!)

Sven: $\langle +T_A \rangle + \langle -mg \rangle = m\langle +a_1 \rangle$

Lars: $\langle +T_B \rangle + \langle -T_A \rangle + \langle -mg \rangle = m\langle +a_1 \rangle$

Ulaf:

x: $\langle +D \rangle + \langle +f_u \rangle + \langle -mg\sin\theta \rangle + \langle -T_B \rangle = m\langle +a_1 \rangle$

y: $\langle +N_u \rangle + \langle -mg\cos\theta \rangle = 0$

→ "unknowns" are now T_A, T_B, D, f_u, and N_u [but again, finding N_u is pretty trivial]

additional relation is $f_u = \mu_k N_u$

I. Determine the tensions in the two tethers, and the magnitude of the drag force generated by the ice-pick. (Express all answers as fractions or multiples of mg.) In particular, verify that the two tensions match your qualitative expectations in Part F.

Y-direction for Ulaf immediately gives us $N_4 = mg\cos\theta$, so again $\boxed{f_4 = \mu_k mg\cos\theta}$

Our remaining equations are:

$$D - T_B + \mu_k mg\cos\theta - mg\sin\theta = ma_1$$
$$+T_B - T_A - mg = ma_1$$
$$T_A - mg = ma_1$$

$$D + \mu_k mg\cos\theta - mg\sin\theta - 2mg = 3ma_1 \quad \text{(adding all three equations)}$$
$$D = 3ma_1 + 2mg + mg\sin\theta - \mu_k mg\cos\theta$$
$$\Rightarrow D = mg\left[3(0.725) + 2 + 0.5 - 0.13\right]$$

$$\boxed{D = 4.5\,mg}$$ Yes - that is a rather large drag force!

then

$$T_A = m(g+a) = 1.725\,mg$$
$$\boxed{T_A = 1.7\,mg}$$ yup — greater than mg, as predicted!

and

$$T_B = T_A + mg + ma = 1.725\,mg + mg + 0.725\,mg$$
$$\boxed{T_B = 3.4\,mg}$$ yup — greater than 2mg, as predicted!