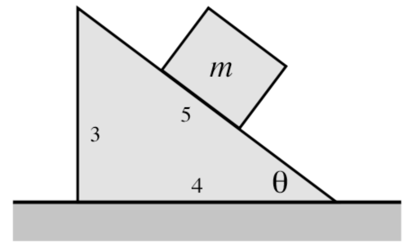
PHYS 2211 Recitation 04 *Solution*

Jun 13–15

You should work in collaborative groups of 3–4, but each student must write up their own solution to the problem. Show all your work, and explain all your reasoning.

A wooden block of mass m is placed on a wooden wedge, which rests on a horizontal tabletop. The wedge has the shape of a 3-4-5 triangle, so that the inclined surface of the wedge makes an angle θ relative to the horizontal, satisfying the trigonometric identities:

$$\sin\theta = 3/5 \qquad \cos\theta = 4/5 \qquad \tan\theta = 3/4$$

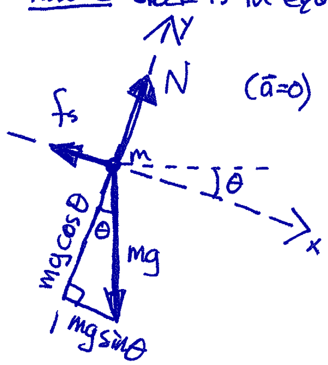


The coefficients of friction between the block and wedge are: $\mu_s = 0.500$ and $\mu_k = 0.200$.

TA Analysis: Each student should complete this portion of the worksheet individually, following along as the TA works the problem. The work you show here will be factored into your grade!

A. Will the block remain stationary, if placed on the wedge at rest? Justify your conclusion by constructing a free body diagram and applying Netwon's 2nd Law to the block.

Assume block is in equilibrium: $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$. Choose coord axes // and $\perp$ to wedge.



- Since block is not slipping along wedge, friction is static

$$\sum \vec{F}_x = 0 \rightarrow \langle +mg\sin\theta\rangle + \langle -f_s\rangle = 0$$

$$\boxed{f_s = mg\sin\theta = \tfrac{3}{5}mg}$$

$$\sum \vec{F}_y = 0 \rightarrow \langle +N\rangle + \langle -mg\cos\theta\rangle = 0$$

$$\boxed{N = mg\cos\theta = \tfrac{4}{5}mg}$$

Now, we note that N determines the maximum possible static friction

$f_{s,max} = \mu_s N = (0.5)(\tfrac{4}{5}mg) \rightarrow \boxed{f_{s,max} = \tfrac{2}{5}mg}$, for this block on this wedge

$\boxed{\text{BUT WAIT!}}$ We deduced that the actual static friction required in this case would be $f_s = \tfrac{3}{5}mg$ — that's more than the upper limit

$\Rightarrow$ Contradiction! Our starting assumption must be invalid

$\boxed{\text{NO — block will NOT remain stationary, in equilibrium}}$

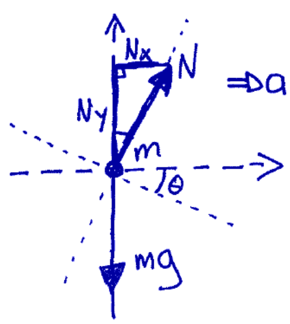
B. A friend suggests that the block can be prevented from slipping down the wedge simply by pushing the wedge to the right at some constant speed v. He asserts that you can use the 2nd Law to determine the precise speed necessary. Is he correct? Why or why not?

"Constant speed" (and <u>direction</u>...) implies zero acceleration

→ this would <u>again</u> be an <u>equilibrium</u> situation, and the analysis above in Part A would again be applicable.

$\boxed{\text{This would not work}}$

C. You decide instead that the block can be prevented from slipping by giving the wedge — and by association, the block – a constant rightward *acceleration* of some magnitude a. Explain why this would work, *even if there were no friction whatsoever.*



Note that the acceleration would be horizontal, so the "best" coordinate system would be horizontal + vertical, fixed to the tabletop

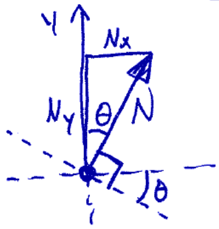
— In this coord system, <u>component</u> of N would cancel grav force ⇒ N would be bigger than it was in part A

— horizontal component N_x would be an unbalanced component, and could provide exactly the right amount of a_x to keep up with the wedge.

Result: No accel for block <u>relative to the wedge</u> (No slippage...)

D. Calculate the magnitude of acceleration a_0 that prevents the block from slipping along a *frictionless* wedge.

In free body diagram above, we see that components of $\vec{N}$ are:



$\vec{N}_x = \langle +N\sin\theta \rangle$

$\vec{N}_y = \langle +N\cos\theta \rangle$

θ = angle between surface of wedge and horizontal and between normal to wedge and vertical

so, 2nd law for y-axis (true vertical) gives:

$$\sum \vec{F}_y = m\vec{a}_y = 0$$

$$\langle +N\cos\theta \rangle + \langle -mg \rangle = 0 \rightarrow N = \frac{mg}{\cos\theta} = \frac{5}{4}mg$$

→ see? Toldya the normal force would be greater than the gravitational force!!

For x-axis (true horizontal) — keep in mind that the block is accelerating relative to the Earth. [Being "stationary" relative to an accelerating wedge does not mean "$\vec{a}=0$" — this block is NOT in equilibrium]

$$\sum \vec{F}_x = m\vec{a}_x \quad \text{where } \vec{a}_x = \langle +a_0 \rangle$$

$$\rightarrow \langle +N\sin\theta \rangle = m\langle +a_0 \rangle$$

$N\sin\theta = ma_0$ (dropping vector formalism)

$\left(\frac{5}{4}mg\right)\left(\frac{3}{5}\right) = ma_0$ (using knowledge about N, $\sin\theta$)

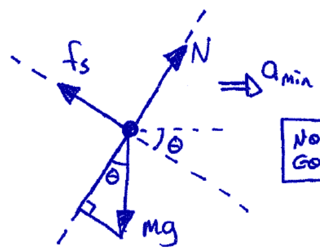
$\boxed{a_0 = \frac{3}{4}g}$, after cancelations (Note how block's mass is irrelevant!)

Student Analysis: Complete the worksheet in collaborative groups of 3–4, with each student writing up their own solution to the problem. Show all your work, and explain all your reasoning.

E. Allowing for friction, it should be possible to give the block and wedge an acceleration *smaller* than a_0, while still not allowing the block to slip. In such a case, what will be the direction of the friction force? Is it *static* or *kinetic* friction?

When $\vec{a}$ was zero (Part A) block slid down incline. When $\vec{a}$ was $\langle +a_0 \rangle \hat{i}$, block was in equilibrium. This is between those acceleration limits so the tendency is for block to slide down wedge. If its not actually sliding, it must be that $\boxed{\text{static friction is present, pointing upslope}}$

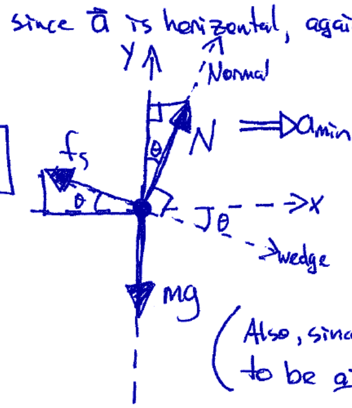
F. Construct a free body diagram for the block, in the situation where the minimum possible "safe" acceleration, a_{min}, is being imparted to both objects. (*Hint:* choose your coordinate axes based on the direction of $\vec{a}$.)



NOT GOOD | Good

This looks good, but note that $\vec{a}$ is not along either of the tilted axes

Since $\vec{a}$ is horizontal, again choose horizontal/vertical axes



Components are:

$\vec{N}_x = \langle +N\sin\theta \rangle$

$\vec{N}_y = \langle +N\cos\theta \rangle$

$\vec{f}_{s,x} = \langle -f_s\cos\theta \rangle$

$\vec{f}_{s,y} = \langle +f_s\sin\theta \rangle$

(Also, since we're almost slipping, f_s is assumed to be at the limit: $f_s = f_{s,max} = \mu_s N$)

G. Apply Newton's 2nd Law to determine the value for a_{min}. Express your answer as a numerical multiple of g.

Vertical: $\langle +N\cos\theta \rangle + \langle +f_s\sin\theta \rangle + \langle -mg \rangle = 0$

$$N\cos\theta + (\mu_s N)\sin\theta = mg \longrightarrow \boxed{N = \frac{mg}{\cos\theta + \mu_s\sin\theta}}$$

horizontal: $\langle +N\sin\theta \rangle + \langle -f_s\cos\theta \rangle = m\langle +a_{min} \rangle$ (don't forget — there IS horizontal accel, here!)

$$N\sin\theta - (\mu_s N)\cos\theta = m\,a_{min}$$

$$\left(\frac{\cancel{m}g}{\cos\theta + \mu_s\sin\theta}\right)\left[\sin\theta - \mu_s\cos\theta\right] = \cancel{m}\,a_{min}$$

$$a_{min} = \left[\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}\right] g$$

(Note that stuff in brackets all pure numbers (dimensionless) so both sides clearly have units of acceleration)

$$a_{min} = \left(\frac{\frac{3}{5} - \frac{1}{2}\cdot\frac{4}{5}}{\frac{4}{5} + \frac{1}{2}\cdot\frac{3}{5}}\right) g = \frac{1/5}{11/10}\, g = \boxed{\frac{2}{11}g}$$

Checkpoint: Before continuing further, have the TA review your group's work so far.

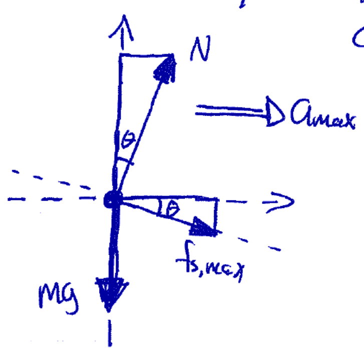
H. It should *also* be possible to give the block and wedge an acceleration greater than a_0 while still not allowing the block to slip. How does the friction force in this situation differ from the friction force in Parts E–G? Can you think of a "quick and easy" way to convert your expression for $a_{\min}$ into an expression for $a_{\max}$, the *greatest* possible acceleration for which the block will not slip?

For accel greater than a_0, the component of the normal force $\vec{N}_x$ is not "strong enough" by itself so block would tend to slip "back", or "up the ramp"
In limit where its almost slipping, f_s would be at its maximum value, pointing downslope : $\vec{f}_s$ reverses direction as compared to Part G.
Note everywhere $\vec{f}_s$ appears, it is expressed as $\mu_s N$ (because we're at slipping limit)
$\Rightarrow$ replacement of every "μ_s" by "$-\mu_s$" would automatically reverse the direction of $\vec{f}_s$. Nothing else about problem changes, except $a_{\min} \to a_{\max}$

so: $a_{\min} = \left[\dfrac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}\right] g \xrightarrow[\text{expect}]{\text{we}} a_{\max} = \left[\dfrac{\sin\theta - (-\mu_s)\cos\theta}{\cos\theta + (-\mu_s)\sin\theta}\right] g = \left[\dfrac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right] g$

I. Construct a free body diagram in the situation where the acceleration is $a_{\max}$ and use the 2nd Law to find an expression for $a_{\max}$. Does this expression match the "quick and easy" expression that you found in the previous step?

Okay — so you don't trust yourself with using that hand-wavy shortcut



Components are: $\vec{N}_x = \langle +N\sin\theta \rangle$ $\vec{N}_y = \langle +N\cos\theta \rangle$
$\vec{f}_{s,x} = \langle +f_s\cos\theta \rangle$ $\vec{f}_{s,y} = \langle -f_s\sin\theta \rangle$
(where $f_s = f_{s,max} = \mu_s N$)

$\sum \vec{F}_y = 0$
$\langle +N\cos\theta \rangle + \langle -f_s\sin\theta \rangle + \langle -mg \rangle = 0$
$N\cos\theta - (\mu_s N)\sin\theta = mg$
$\boxed{N = \dfrac{mg}{\cos\theta - \mu_s\sin\theta}}$

see? Same as in Part G, except we've switched $\mu_s \to -\mu_s$

(Note, too: N is bigger here than it was in part G vertical part must cancel gravity plus some of the friction)

$\sum \vec{F}_x = m\vec{a}_x$
$\langle +N\sin\theta \rangle + \langle +f_s\cos\theta \rangle = m\langle +a_{max} \rangle$
$N\sin\theta + (\mu_s N)\cos\theta = m a_{max}$

$m a_{max} = N(\sin\theta + \mu_s\cos\theta) = \left[\dfrac{mg}{\cos\theta - \mu_s\sin\theta}\right](\sin\theta + \mu_s\cos\theta)$

$a_{max} = \left[\dfrac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right] g = \left[\dfrac{\frac{3}{5} + \frac{1}{2}\cdot\frac{4}{5}}{\frac{4}{5} - \frac{1}{2}\cdot\frac{3}{5}}\right] g = \dfrac{5/5}{5/10}\, g = \boxed{2g}$

See! Amazing, isn't it! Same as what we predicted in Part H.

so: block will not slip for any acceleration in range $\frac{2}{11} g \le a \le 2g$