PHYS 2211 Recitation 03 *Solution*

Jun 06–08

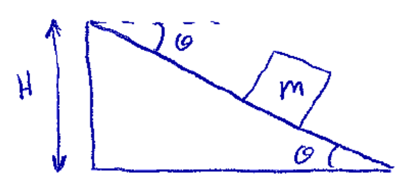
You should work in collaborative groups of 3–4, but each student must write up their own solution to the problem. Show all your work, and explain all your reasoning.

Warehouse Worries

You are working a Co-op assignment at a plant that manufactures goldfish bowls. Finished bowls are packed in crates of 20, each crate having a total weight (i.e. gravitational force) W. Due to the fragile contents, a crate cannot be allowed to impact any hard surface at a speed greater than $v_{\text{max}} = 1.00\,\text{m/s}$. Crates arrive at the loading bay on a conveyor belt that is at a height $H = 1.50\,\text{m}$ above the level of the shipping pallets onto which they are stacked for transport. You have been asked to find a way to lower crates onto the pallets without damaging the contents. You decide to construct a ramp, inclined at an angle $\theta = 16.0°$ below the horizontal. Consisting of wood covered by a slippery plastic polymer, you figure that you will be able to slide crates down the ramp without any difficulty.

TA Analysis: Each student should complete this portion of the worksheet individually, following along as the TA works the problem. The work you show here will be factored into your grade!

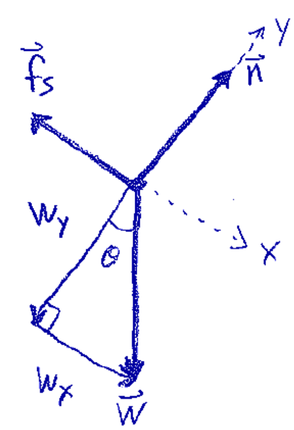
A. When you shove the first crate onto the ramp, it immediately stops. Identify the forces acting on the crate as it sits stationary, *after* you are no longer shoving it, and it has stopped. Make a sketch, then construct a free body diagram for the crate. Choose an appropriate coordinate system and resolve any forces not parallel to an axis into components.



↪ **Forces:**
Normal force $\vec{n}$ perpendicular to the ramp
Static friction force $\vec{f_s}$ parallel to the ramp
Weight $\vec{W}$, the Earth's downward gravitational force

Since the acceleration of the crate is zero, axes are chosen to minimize the number of forces that must be resolved into components.

$$W_x = |W| \sin\theta \qquad\qquad W_y = |W| \cos\theta$$



B. Write out Newton's 2nd Law for the crate, and calculate values for all the forces you identified in Part A. Express all magnitudes as numerical multiples of W to three decimal places.

↪ Signs indicating direction will be shown explicitly, so symbols represent magnitudes.

$$\sum F_x = W_x - f_s = ma_x = 0 \qquad \Rightarrow \qquad f_s = W\sin\theta = W\sin(16^\circ) = 0.276W$$

$$\sum F_x = n - W_y = ma_y = 0 \qquad \Rightarrow \qquad n = W\cos\theta = W\cos(16^\circ) = 0.961W$$

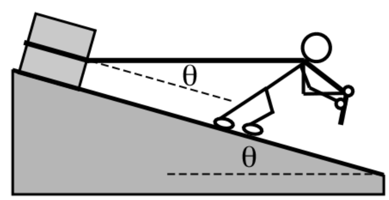
C. Determine the magnitude and direction of the *net* force exerted on the crate *by the ramp*.

↪ Since the acceleration of the crate is zero, the net force on the crate must also be zero. The only force on the crate that is *not* exerted by the ramp is the gravitational force. Therefore, the net force exerted by the ramp on the crate must be equal and opposite the gravitational force. Its magnitude is W, directed upward.

$$\vec{F}_{\text{ramp}} = -\vec{W}$$

Student Analysis: Complete the worksheet in collaborative groups of 3–4, with each student writing up their own solution to the problem. Show all your work, and explain all your reasoning.

In an attempt to get the crate moving, you tie a cable around it, and pull on the crate in a direction parallel to the *ground* (**NOT** the ramp). You find that if you pull with a force equal to *half* the crate's weight, you still cannot make it budge.



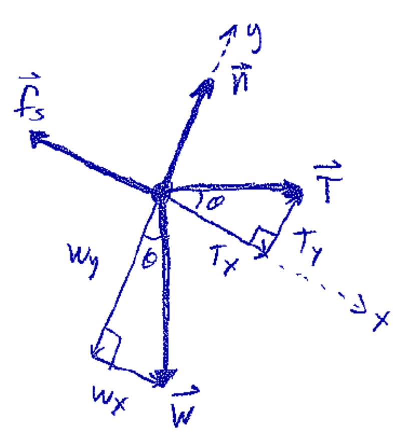
D. Construct a free body diagram for the crate, as you pull the cable.

↪ Like part A, **Forces:**
Normal force $\vec{n}$ perpendicular to the ramp
Static friction force $\vec{f_s}$ parallel to the ramp
Weight $\vec{W}$, the Earth's downward gravitational force
Tension force $\vec{T}$ from the rope

Components:

$$W_x = |W|\sin\theta \qquad W_y = |W|\cos\theta$$

$$T_x = |T|\cos\theta \qquad T_y = |T|\sin\theta$$



E. Apply Newton's 2nd Law to determine the magnitudes of the normal force acting on the crate and the friction force acting on the crate. Express your answers as numerical multiples of W.

↪ Like part B, signs indicating direction will be shown explicitly, so symbols represent magnitudes.

$$\sum F_x = T_x + W_x - f_s = ma_x = 0$$

So

$$f_s = T_x + W_x = T\cos\theta + W\sin\theta = \left(\frac{W}{2}\right)\cos\theta + W\sin\theta$$

$$= W\left(\frac{\cos\theta}{2} + \sin\theta\right) = W\left(\frac{\cos(16°)}{2} + \sin(16°)\right) = 0.756W$$

Note that the magnitude of f_s is *different* than it was in part B. f_s does *not* have a fixed value!

Then

$$\sum F_y = T_y - W_y + n = ma_y = 0$$

So

$$n = W_y - T_y = W\cos\theta - T\sin\theta = W\cos\theta - \left(\frac{W}{2}\right)\sin\theta$$

$$= W\left(\cos\theta - \frac{\sin\theta}{2}\right) = W\left(\cos(16°) - \frac{\sin(16°)}{2}\right) = 0.823W$$

Checkpoint: Before continuing further, have the TA review your group's work so far.

F. Notice the difference between the computed magnitude of the normal force in Parts B and E. Explain in words why there is a difference.

↪ There is a component of the tension perpendicular to the ramp surface in part E. This component reduces the force the ramp surface must exert to "balance" the perpendicular (y) component of the weight, compared to part B.

G. You find that with a slight extra nudge, the crate will begin moving down the ramp. When that happens, the cable goes slack, and you see that the crate slides with a constant acceleration $0.10g$ down the ramp. Is the crate in equilibrium? Explain.

$\hookrightarrow$ No, since the acceleration is now non-zero, the crate is not in equilibrium.

H. Construct a free body diagram for the crate, as it finally slides down the ramp. Determine the magnitudes of all forces acting on the crate, expressing your answers as multiples of W. Would your answers change if the crate moved down the ramp at a different speed?

$\hookrightarrow$ Now that the crate is moving, the friction force is kinetic, $\vec{f}_k$. Once again, signs indicating direction will be shown explicitly, so symbols represent magnitudes.

$$\sum F_x = n - W_y = ma_y = 0 \qquad \Rightarrow \qquad n = W\cos\theta = W\cos(16^\circ) = 0.961W$$

as in part B. But

$$\sum F_x = W_x - f_k = ma_x \neq 0\,!$$

So

$$\begin{aligned} f_k = W_x - ma_x &= W\sin\theta - m(0.10g) \\ &= W\sin(16^\circ) - 0.10W = (\sin(16^\circ) - 0.10)\,W = 0.176W \end{aligned}$$

This answer would not change if the crate's speed were different. Only the acceleration (change in velocity) matters, not the magnitude of the velocity itself.