PHYS 2211 Recitation 01 *Solution*

May 30 – Jun 01

You should work in collaborative groups of 3–4, but each student must write up their own solution to the problem. Show all your work, and explain all your reasoning.

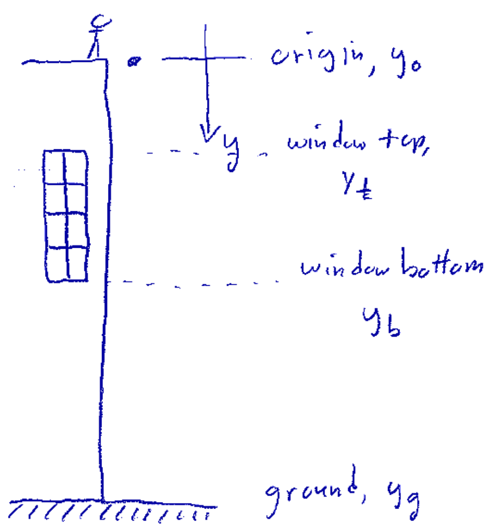
A ball is dropped from the roof of a building. Shortly thereafter, it passes by a window that is 1.5 m tall. An inhabitant times the ball as it passes, noting that it is visible passing the window for 0.15 s. The ball hits the ground 1.0 s later.

A second ball is thrown downward from the roof, with a speed equal to half the speed the first ball has as it hits the ground. If the two balls are to hit the ground at the same time, how much time after the first ball is dropped should the second ball be thrown?

TA Analysis: Each student should complete this portion of the worksheet individually, following along as the TA works the problem. The work you show here will be factored into your grade!

A. Decide what principle will be used to solve the problem. Make a labeled sketch, and define variables.

The balls are in free-fall, so this is a constant-acceleration kinematics problem, in which the acceleration is that due to gravity. I'll choose the origin at the top of the building, with positive y downward.



First Ball, "A"

Known

$y_{A0} = y_0 = 0\,\text{m}$

$y_{Ab} - y_{At} = +1.5\,\text{m}$

$v_{A0} = 0\,\text{m/s}$

$a_A = g = +9.8\,\text{m/s}^2$

$t_{Ab} - t_{At} = 0.15\,\text{s}$

unknown

y_{At} v_{At} t_{At}

y_{Ab} v_{Ab} t_{Ab}

y_{Ag} v_{Ag} t_{Ag}

Second Ball, "B"

Known

$y_{B0} = y_0 = 0\,\text{m}$

$v_{B0} = \frac{1}{2} v_{Ag}$

$a_B = g = +9.8\,\text{m/s}^2$

relevant unknowns

t_{B0}

B. Find the velocity of the first ball at the instant it reaches the top of the window.

Don't know velocity at bottom. Use $y_f = y_i + v_i \Delta t + \frac{1}{2}a(\Delta t)^2$ and solve for v_i

$$v_i = \frac{(y_f - y_i) - \frac{1}{2}a(\Delta t)^2}{\Delta t} = \frac{(y_{Ab} - y_{At}) - \frac{1}{2}g(t_{Ab} - t_{At})^2}{t_{Ab} - t_{At}}$$

$$= \frac{(+1.5m) - \frac{1}{2}(+9.8 m/s^2)(0.15s)^2}{0.15s} = 9.27 m/s = \boxed{9.3 m/s}$$

C. Find the total time required for the first ball to reach the ground.

With answer from pt B, can find time to reach top of window, t_{At}.

$$v_f = v_i + a\Delta t \Rightarrow \Delta t = \frac{v_f - v_i}{a} \Rightarrow t_{At} - t_{Ao} = \frac{v_{At} - v_{Ao}}{a}$$

$$t_{At} = \frac{v_{At} - v_{Ao}}{a} + t_{Ao} = \frac{9.27 m/s - 0 m/s}{+9.8 m/s^2} + 0s = 0.95s$$

Adding the time to pass the window (0.15s) and the time below the window (1.0s) yields $0.95s + 0.15s + 1.0s = 2.10s = \boxed{2.1s}$

Student Analysis: Complete the worksheet in collaborative groups of 3–4, with each student writing up their own solution to the problem. Show all your work, and explain all your reasoning.

D. The balls have the same acceleration, and must land at the same time. What else must be the same for the balls? Find its value.

The balls have the same final position, $y_{Ag} = y_{Bg}$.

From the time it takes the first ball to fall

$$y_f = y_i + v_i \Delta t + \frac{1}{2}a(\Delta t)^2 \Rightarrow y_{Ag} = y_{Ao} + v_{Ao}(t_{Ag} - t_{Ao}) + \frac{1}{2}g(t_{Ag} - t_{Ao})^2$$

$$y_{Ag} = 0m + (0 m/s)(2.1s - 0s) + \frac{1}{2}(+9.8 m/s^2)(2.1s - 0s)^2 = \boxed{21.5m}$$

E. The second ball is thrown with a speed equal to half the speed the first ball has as it hits the ground. Find those speeds.

Since the first ball falls for 2.1 s, it lands with speed

$v_f = v_i + a\Delta t \Rightarrow v_{Ag} = v_{A0} + g(t_{Ag} - t_{A0})$

$= 0\,m/s + (9.8\,m/s^2)(2.10\,s - 0\,s) = 20.5\,m/s$

$= \boxed{21\,m/s}$

So the second ball is thrown at

$v_{B0} = \frac{v_{Ag}}{2} = \frac{20.5\,m/s}{2} = 10.3\,m/s = \boxed{10\,m/s}$

F. Write an expression in which the only unknown is the time the second ball is in the air. Solve for that time, and answer the original question.

$y_f = y_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2 \Rightarrow y_{Bg} = y_{B0} + v_{B0}\Delta t_B + \frac{1}{2}g(\Delta t_B)^2$

$\frac{1}{2}g(\Delta t_B)^2 + v_{B0}\Delta t + (y_{B0} - y_{Bg}) = 0$ ← A quadratic in Δt_B

$\Delta t_B = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ where
$A = \frac{1}{2}g = \frac{1}{2}(9.8\,m/s^2) = 4.9\,m/s^2$
$B = v_{B0} = 10.3\,m/s$
$C = (y_{B0} - y_{Bg}) = (0\,m - 21.5\,m)$

so

$\Delta t_B = +1.30\,s$, or $-3.39\,s$ Select the positive root. $\boxed{\Delta t_B = +1.3\,s}$

If ball B is in the air for 1.3 s, and ball A is in the air for 2.1 s, but they land at the same time, ball B must be thrown $2.1\,s - 1.3\,s = \boxed{0.8\,s}$ after ball A is dropped

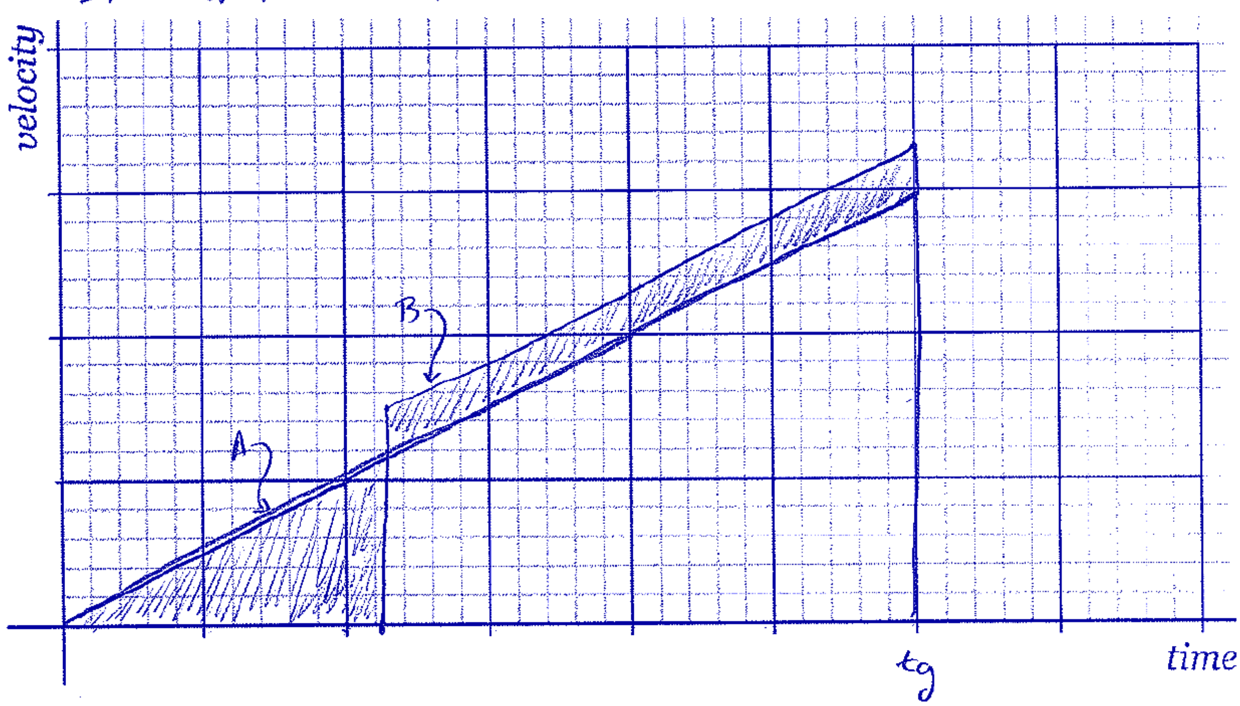
Checkpoint: Before continuing further, have the TA review your group's work so far.

G. In part F, you found two times for the second ball to be in the air, but rejected one. What minor change to the original question would change which time you rejected? (I.e., what similar question would be answered by selecting the time you rejected?)

If ball B were thrown upward, it would be in the air for 3.4s before landing. Therefore it would have to be thrown 2.1s − 3.4s = −1.3s "after" ball A is dropped (i.e., +1.3s before ball A is dropped).

H. Sketch velocity-time graphs for both balls on the grid below. Identify clearly which curve is associated with which ball. What must be the same for the two curves?

Same slope (acceleration). Same area (distance traveled).



Since total areas and unshaded areas are the same, the shaded areas must be the same, as well!