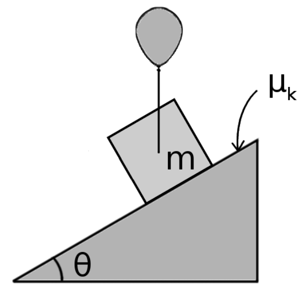
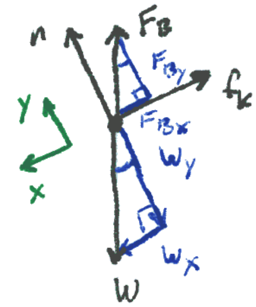
I. (16 points) A block with mass $m = 10.0\,\text{kg}$ is on a plane inclined $\theta = 30.0°$ to the horizontal, as shown. A balloon is attached to the block to exert a constant upward force $F_B = 9.8\,\text{N}$. If the block moves down the plane with a constant velocity, what is the coefficient of kinetic friction μ_k between the block and plane? (*On Earth.*)



Use Newton's Second Law. Sketch a Free Body Diagram of the block. There is a gravitational force W downward, a balloon force F_B upward, a normal force n up away from the plane, and a kinetic friction force f_k up along the plane. Choose a coordinate system. I'll choose x down the plane, and y up away from the plane. Write out Newton's Second Law for each axis.



I'll start with the x axis, showing signs explicitly, so symbols represent magnitudes. The velocity is constant, so the acceleration is zero.

$$\sum F_x = W_x - F_{Bx} - f_k = ma_x = 0 \qquad \Rightarrow \qquad mg\sin\theta - F_B\sin\theta = \mu_k n$$

So

$$\mu_k = \frac{(mg - F_B)\sin\theta}{n}$$

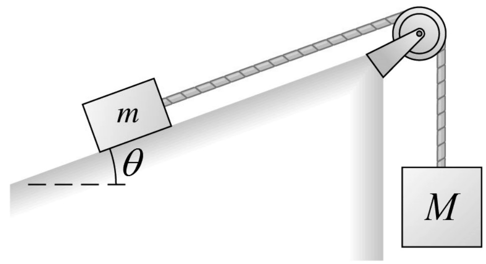
Now for the y axis.

$$\sum F_y = n + F_{By} - W_y = ma_y = 0 \qquad \Rightarrow \qquad n = mg\cos\theta - F_B\cos\theta = (mg - F_B)\cos\theta$$

Substitute this expression for n into the expression for μ_k.

$$\mu_k = \frac{(mg - F_B)\sin\theta}{(mg - F_B)\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \tan(30.0°) = 0.577$$

1. (6 points) The block of mass m is held at rest on the frictionless plane that makes an angle θ with the horizontal. An ideal rope passes over an ideal pulley, connecting it to a hanging block of mass M, as shown. When the block of mass m is released, it accelerates down the plane. Compare the resulting tension T in the rope with gravitational force on the block of mass M. (*On Earth.*)



. . . .

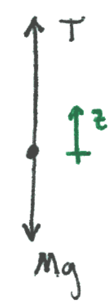
Since block m accelerates down the plane, the block M accelerates upward. The net force on block M must, therefore, be upward. There's an upward tension T acting on M, and a downward gravitational force Mg acting on it. For the net force to be upward,

$$T > Mg$$

II. (16 points) If $m = 7.7\,\mathrm{kg}$, $M = 2.2\,\mathrm{kg}$, and $\theta = 28°$, what is the acceleration magnitude of the block of mass m after release?

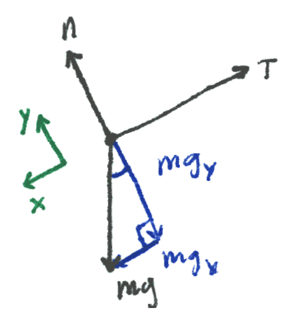
. . . .

Apply Newton's Second Law to the hanging block. Sketch a Free Body Diagram. There is a gravitational force Mg downward and a tension T upward. Choose a coordinate system. I'll choose z upward, so a_z has a positive value. I'll write out Newton's Second Law showing signs explicitly, so symbols represent magnitudes.



$$\sum F_x = T - Mg = Ma_z \qquad \Rightarrow \qquad T = Mg + Ma_z$$

Now apply Newton's Second Law to the sliding block. Sketch a Free Body Diagram. There is a gravitational force mg downward, a normal force n up away from the plane, and a tension force T up along the plane. Choose a coordinate system. I'll choose x down the plane, so a_x has a positive value, and I'll choose y up away from the plane. Writing out Newton's Second Law for the x direction,



$$\sum F_x = mg_x - T = mg\sin\theta - T = ma_x$$

Substitute the expression for T, and note that, with my choice of coordinate systems, $a_x = a_z = a$.

$$mg\sin\theta - (Mg + Ma) = ma \quad \Rightarrow \quad mg\sin\theta - Mg = ma + Ma = (m + M)\,a$$

Solve for the acceleration.

$$a = \frac{(m\sin\theta - M)\,g}{m + M} = \frac{(7.7\,\mathrm{kg}\sin 28° - 2.2\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)}{7.7\,\mathrm{kg} + 2.2\,\mathrm{kg}} = 1.4\,\mathrm{m/s^2}$$

III. (16 points) Saturn's moon Titan requires 16 days to orbit the planet. The orbit is circular, with a radius of 1.222×10^6 km. What is the mass of Saturn?

. . . .

Apply Newton's Second Law to Titan. The only force on Titan is the gravitational force from Saturn. Titan's acceleration is centripetal.

$$\sum F_c = G\frac{M_S M_T}{r^2} = M_T a_c = M_T \frac{v^2}{r} \qquad \Rightarrow \qquad M_S = \frac{v^2 r}{G}$$

The speed of Titan is the distance it travels in a given time. We know the time T for one orbit, and the distance for one orbit is that orbit's circumference.

$$v = \frac{2\pi r}{T} \qquad \Rightarrow \qquad M_S = \frac{(2\pi r/T)^2 r}{G} = \frac{4\pi^2 r^3}{GT^2}$$

Remembering to convert the orbit's radius to meters, and the time to seconds

$$M_S = \frac{4\pi^2 \left(1.222 \times 10^9\,\text{m}\right)^3}{6.673 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2 \left(16\,\text{days} \times 24\,\text{hr/day} \times 60\,\text{min/hr} \times 60\,\text{s/min}\right)^2} = 5.6 \times 10^{26}\,\text{kg}$$

2. (6 points) What is the ratio of Titan's mass, M_T, to Saturn's mass, M_S?

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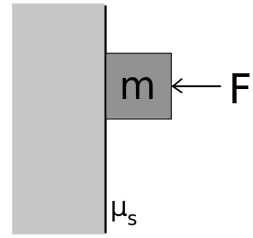
Since the gravitational force on Titan depends linearly on Titan's mass, the orbital data does not depend on Titan's mass (just as the free-fall acceleration of an object near the Earth's surface does not depend on its mass). Therefore, orbital data cannot reveal Titan's mass (just as the free-fall acceleration of an object near the Earth's surface cannot reveal its mass), and

M_T/M_S cannot be determined from the information provided.

3. (8 points) A horizontal external force $F = 150\,\mathrm{N}$ is applied to a $10\,\mathrm{kg}$ block against the wall. The coefficient of static friction between the wall and the block is $\mu_s = 0.8$. What is the magnitude of the friction force on the block? (*On Earth.*)



. . . .

Apply Newton's Second Law to the block. The downward force of gravity on the block is $mg = (10\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right) = 98\,\mathrm{N}$. The *maximum* force of static friction is $f_{\mathrm{s,max}} = \mu_s n = 0.8\,(150\,\mathrm{N}) = 120\,\mathrm{N}$. This is *more than enough* to balance the gravitational force, so the block is stationary (in particular, friction *doesn't* pull the block *up* the wall!). The force of static friction must *not* be its maximum value—instead, it must be just enough to balance the gravitational force, or $98\,\mathrm{N}$.

4. (8 points) A spherical object with radius r is dropped on Earth to reach its terminal velocity v_1, if another spherical object with the same mass and drag coefficient, but radius $2r$ is dropped, what is its terminal velocity in terms of v_1? (*Do NOT neglect drag!*)

. . . .

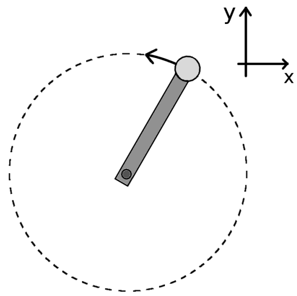
The terminal velocity is the constant velocity that comes about when the upward drag force on a falling object balances the downward gravitational force.

$$mg = \tfrac{1}{2} C\rho A v^2 \qquad \Rightarrow \qquad v = \sqrt{\frac{2mg}{C\rho A}}$$

If the radius of a sphere is doubled, then the *cross-sectional* area increases by a factor of 4.

$$v' = \sqrt{\frac{2mg}{C\rho A'}} = \sqrt{\frac{2mg}{C\rho\, 4A}} = \frac{1}{2}\sqrt{\frac{2mg}{C\rho A}} = v_1/2$$

5. (8 points) A 10 kg spherical object is attached to a 10 m massless rod to form a uniform vertical circular motion with an angular speed $\omega = 1\,\mathrm{rad/s}$ as shown. At the top of the loop, what is the magnitude and direction of the force exerted by the rod on the object? (*On Earth.*)



. . . .

Apply Newton's' Second Law to the object at the top of the loop. Sketch a Free Body Diagram. There's a gravitational force mg downward, but it isn't clear whether the stick will be pushing the object upward (a normal force) or pulling it downward (a tension). We need to assume something, so let's just call it a stick force S, and assume it's a normal force upward.

Choose a coordinate system. I'll choose the c axis downward, toward the center of the circle, which is the direction of the known acceleration. Write out Newton's Second Law. I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_c = mg - S = ma_c = mr\omega^2$$

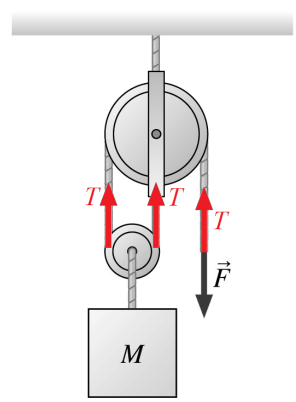
Solve for S.

$$S = mr\omega^2 - mg = (10\,\mathrm{kg})\left[(10\,\mathrm{m})\,(1\,\mathrm{rad/s})^2 - 9.8\,\mathrm{m/s^2}\right] = -2\,\mathrm{N}$$

As symbols were intended to represent magnitudes, the negative value for the stick force tells us that our assumed direction is incorrect. The stick actually provides a downward tension,

$$-2\,\hat{y}\,\mathrm{N}$$

6. (8 points) A block of mass M is being raised at a constant velocity by the system of ideal ropes and pulleys, as shown. With what force magnitude F is the rope being pulled? (*On Earth.*)



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The tension in an ideal rope is the same everywhere, and away from its contact and connection points.

Apply Newton's Second Law to the block and its attached pulley. There is a downward gravitational force Mg on it, and *two* upward tension forces, as shown.

$$\sum F_{\mathrm{up}} = 2T - Mg = Ma_{\mathrm{up}} = 0 \qquad \Rightarrow \qquad T = Mg/2$$

At the end of the rope, the tension must balance the pulling force, so

$$F = T = Mg/2$$

7. (8 points) Imagine a planet with with three times the mass of Earth, and twice the Earth's radius. What would the gravitational acceleration, g_{planet}, be on this planet's surface?

. . . .

Gravitational acceleration is due to gravitational force.

$$mg = G\frac{Mm}{r^2} \qquad \Rightarrow \qquad g = G\frac{M}{r^2}$$

So

$$g_{\text{planet}} = G\frac{M_{\text{planet}}}{r^2_{\text{planet}}} = G\frac{3M}{(2r)^2} = \frac{3}{4}\,G\frac{M}{r^2} = \frac{3}{4}\left(9.8\,\text{m/s}^2\right) = 7.4\,\text{m/s}^2$$