
I. (16 points) Two carts are in concentric circular tracks. The first is a distance 2.5 m from the center and the second is a distance 7.5 m from the center. They can both travel at a constant speed of 0.75 m/s. If they both pass $\theta = 0$ at time $t = 0$, what is the first time after that when they both pass through $\theta = 0$ together again?

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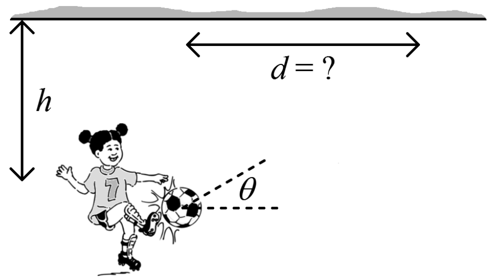
Since the circumference of a circle is proportional to its radius, the circumference of the larger circle is $7.5\,\mathrm{m}/2.5\,\mathrm{m} = 3$ times the circumference of the smaller circle. As the cars travel at the same speed, it will take the car on the larger circle 3 times as long to complete one revolution as the car on the smaller circle.

Therefore, the cars will pass through $\theta = 0$ together when the outer car has made one revolution and the inner car has made three. The time for the outer car to make one revolution is

$$\Delta s = v_t\,\Delta t \qquad \Rightarrow \qquad \Delta t = \frac{\Delta s}{v_t} = \frac{2\pi\,R_{\mathrm{outer}}}{v_t} = \frac{2\pi\,(7.5\,\mathrm{m})}{0.75\,\mathrm{m/s}} = 63\,\mathrm{s}$$

II. (16 points) Rebecca practices soccer kicks—in her living room! When the ball leaves her foot, it is travelling 16 m/s at an angle $\theta = 35°$ above the horizontal, as shown. The ceiling is a height $h = 1.8\,\mathrm{m}$ above the ball at that instant. At what horizontal distance d from that point does the ball strike the ceiling? (*On Earth.*)



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The ball undergoes projectile motion, which is a special case of constant-acceleration kinematics where the horizontal acceleration is zero and the vertical acceleration is g.

Choose a coordinate system. I'll put the origin at the point where the ball leaves Rebecca's foot, with x horizontal to the right, and y vertically upward. Consider the vertical motion to find the time at which the ball hits the ceiling.

$$y = y_0 + v_{0y}\,\Delta t + \tfrac{1}{2}a_y\,(\Delta t)^2 \qquad \Rightarrow \qquad \tfrac{1}{2}a_y\,(\Delta t)^2 + v_{0y}\,\Delta t + (y_0 - y) = 0$$

which is quadratic in Δt, where

$$a_y = -g = -9.81\,\mathrm{m/s^2} \qquad v_{0y} = v_0 \sin\theta = (16\,\mathrm{m/s})\sin(35°) = 9.18\,\mathrm{m/s} \qquad y_0 = 0\,\mathrm{m} \qquad y = +1.8\,\mathrm{m}$$

So

$$\Delta t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad \text{where} \qquad A = \tfrac{1}{2}a_y = \left(-9.81\,\mathrm{m/s^2}\right)/2 = -4.91\,\mathrm{m/s^2}$$

$$B = v_{0y} = 9.18\,\mathrm{m/s}$$

$$C = (y_0 - y) = (0\,\mathrm{m} - 1.8\,\mathrm{m}) = -1.8\,\mathrm{m}$$

which yields two results for Δt, 0.223 s and 1.65 s. If there were no ceiling, the ball would be 1.8 m above the starting point twice, once on the way up and once on the way back down. Since it hits the ceiling on the way up, $\Delta t = 0.223\,\mathrm{s}$. The ball must travel horizontally for this same time.

$$x = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2}a_x\,(\Delta t)^2 \qquad \Rightarrow \qquad x = v_0 \cos\theta\,\Delta t$$

as the horizontal acceleration a_x is zero and the ball starts at the origin ($x_0 = 0\,\mathrm{m}$). Therefore

$$d = x = v_0 \cos\theta\,\Delta t = (16\,\mathrm{m/s})\cos(35°)(0.223\,\mathrm{s}) = 2.9\,\mathrm{m}$$

1. (6 points) At what angle ϕ above the horizontal is the ball traveling at the instant it strikes the ceiling? *Hint:* Think about this before beginning calculations!

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When the ball strikes the ceiling, it is still travelling upward, which excludes $\phi = -42°$. It must be travelling at an angle less than its initial launch angle, which excludes $\phi = 52°$ and $\phi = \theta = 35°$. It does not travel in a straight line from its launch point to the place it strikes the ceiling, which excludes $\phi = \tan^{-1}(h/d)$. The only remaining choice is

$$\phi = 28°$$

Note that this could be calculated by finding the ball's velocity at the instant it strikes the ceiling

$$v_x = v_{0x} = v_0 \cos\theta \qquad \text{and} \qquad v_y = v_{0y} + a_y\,\Delta t = v_0 \sin\theta + a_y\,\Delta t \qquad \text{then} \qquad \tan\phi = \frac{v_y}{v_x}$$

III. (16 points) A person is using a rope to pull a large stone of mass M along the level floor of a quarry. After initially pulling hard to get the stone moving, the person has pull with a constant tension force T_1 to keep the stone moving at a constant speed. How much force T_2 would the person have to exert to pull the same stone out of the quarry at the same constant speed, on a ramp that makes an angle θ with the horizontal? Express T_2 in terms of other parameters defined in the problem, and physical or mathematical constants. You may assume the ramp is made of the same material as the ground. (*On Earth.*)

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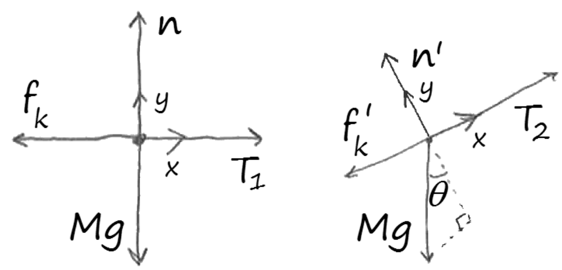
Use Newton's Second Law. Sketch a Free Body Diagram for the situation in which the stone is pulled along level ground. The gravitational force Mg will be downward, a normal force n will be upward, a tension force T_1 will be horizontal (I've chosen it to the right), and a kinetic friction force f_k will be horizontal in the other direction.

Choose a coordinate system. I'll let the positive x direction be horizontal to the right, and the positive y direction be vertically upward. Write Newton's Second Law for each dimension. I'll show signs explicitly, so symbols represent magnitudes. Note that the coefficient of kinetic friction is not defined in the problem statement, but it will be the same for the two situations. The plan for a solution, then, is to find an expression for μ_k from level-ground situation, and then use it for the ramp.



$$\sum F_y = n - Mg = Ma_y = 0 \qquad \Rightarrow \qquad n = Mg$$

$$\sum F_x = T_1 - f_k = Ma_x = 0 \qquad \Rightarrow \qquad T_1 - \mu_k n = 0 \qquad \Rightarrow \qquad \mu_k = \frac{T_1}{n} = \frac{T_1}{Mg}$$

Now sketch a Free Body Diagram for the stone being pulled up the ramp. The gravitational force Mg will be downward, a new normal force n' perpendicular away from the ramp surface, a new tension force T_2 (the answer to the question) will be up along the ramp surface, and a new kinetic friction force f'_k will be down along the ramp surface.

Choose a coordinate system. I'll let the positive x direction be up along the ramp, and the positive y perpendicular away from the ramp surface. Write Newton's Second Law for each dimension. Once again, I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_y = n' - Mg\cos\theta = Ma_y = 0 \qquad \Rightarrow \qquad n' = Mg\cos\theta$$

$$\sum F_x = T_2 - f'_k - Mg\sin\theta = Ma_x = 0 \quad \Rightarrow \quad T_2 = f'_k + Mg\sin\theta = \mu_k n' + Mg\sin\theta$$

$$= \left(\frac{T_1}{Mg}\right)(Mg\cos\theta) + Mg\sin\theta = T_1\cos\theta + Mg\sin\theta$$

2. (6 points) Consider the situation when the person is hauling the stone up the inclined ramp with force T_2 at a constant speed. If the rope suddenly breaks, what is the **initial** acceleration of the stone? *Hint:* Think about the net force on the stone immediately *before* the rope breaks.

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Since the acceleration is zero before the rope breaks, the net force must be zero before the rope breaks. All other forces on the stone together must balance the tension T_2 by being equal and opposite it. By Newton's Second Law, when the tension is removed, those forces cause an acceleration

T_2/M down the ramp

3. (8 points) During the hammer throw Olympic event, athletes spin a large mass (called the hammer) at the end of a cable of radius r around in a circle, continuously accelerating rotationally until releasing the mass and letting it fly off on a tangent. During a typical throw the athlete starts at rest and then rotates through 4 complete rotations with a constant angular acceleration, α, before releasing the hammer with a final angular velocity of ω_1. If instead the athlete rotated through 8 complete rotations with the same angular acceleration α, what would the final angular velocity ω_2 be in terms of ω_1? (*On Earth.*)

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Use constant (angular) acceleration kinematics.

$$\theta = \theta_0 + \omega_0 \, \Delta t + \tfrac{1}{2}\alpha \, (\Delta t)^2 \qquad \text{and} \qquad \omega = \omega_0 + \alpha \, \Delta t$$

Eliminate Δt.

$$\Delta t = \frac{\omega - \omega_0}{\alpha} \qquad \Rightarrow \qquad \theta = \theta_0 + \omega_0 \left(\frac{\omega - \omega_0}{\alpha}\right) + \tfrac{1}{2}\alpha \left(\frac{\omega - \omega_0}{\alpha}\right)^2 \qquad \Rightarrow \qquad 2\alpha\,(\theta - \theta_0) = \omega^2 - \omega_0^2$$

With $\theta_0 = 0$, doubling $\Delta\theta$ will double ω^2. So

$$\omega_2 = \sqrt{2}\omega_1$$

4. (8 points) A painter is painting a horizontal ceiling. He paints at a constant speed with brushstrokes moving from left to right, pressing the brush into the ceiling with a force $\vec{F}_P$. Which of the following free body diagrams best represents all of the forces exerted on the brush? $\vec{f}_f$ is a friction force, and $\vec{F}_N$ is a normal force. (*On Earth.*)

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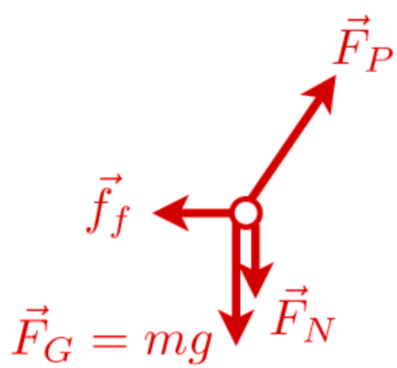
The normal force must be perpendicular away from the ceiling (that is, straight down). Only one of the offered choices satisfies this requirement.



5. (8 points) You are trying to move a 225 kg fridge by pulling on it with a rope at 15° above horizontal, but it won't budge. The coefficient of static friction between the fridge and the ground is $\mu_s = 0.70$ and the coefficient of kinetic friction is $\mu_k = 0.55$. You can pull on the rope with a maximum force of 1000 N. What is the force of friction on the fridge? (*On Earth.*)

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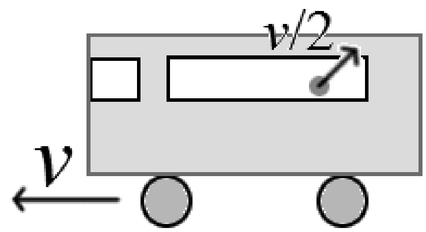
If the fridge "won't budge" then it has a constant velocity (zero). The acceleration of the fridge is zero, so the net force on the fridge must be zero. The horizontal force of static friction must exactly balance the horizontal component of the pulling force.

$$f_s = P\cos\theta = (1000\,\mathrm{N})\cos(15°) = 970\,\mathrm{N}$$

6. (8 points) A bus is moving toward the left with a constant speed v, and a boy throws a ball with speed $v/2$ and angle 45° above the horizontal as shown. Both speed and angle are measured relative to the bus. What is the trajectory of the ball seen by an observer standing on the ground near the bus? (*On Earth. Remember that drag should be ignored.*)



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Since the bus isn't moving up or down, the upward component of the ball's initial velocity is the same in both the boy's and the outside observer's frame.

$$v_{\mathrm{up}} = \left(\frac{v}{2}\right)\sin\theta = \left(\frac{v}{2}\right)\frac{\sqrt{2}}{2} = \frac{v\sqrt{2}}{4}$$

Since the bus is moving to the left, but the boy gives the ball's initial velocity a rightward component from his perspective, the outside observer sees a leftward component of

$$v_{\mathrm{left}} = v - \left(\frac{v}{2}\right)\cos\theta = v - \left(\frac{v}{2}\right)\frac{\sqrt{2}}{2} = v - \frac{v\sqrt{2}}{4} = v\left(\frac{4-\sqrt{2}}{4}\right) > 0$$

So the outside observer sees a ball launched up and to the left, which will follow a parabolic arc as expected for a projectile.

7. (8 points) While in an elevator accelerating upward, you decide to measure out 10 grams of sand. Should you use a pan balance, as on the left, in which the sand is compared to a 10 g cylinder? Or should you use a spring scale, as on the right, in which the upward force exerted by a spring is indicated by a needle pointing to 10 g? Or does it matter? (*On Earth.*)

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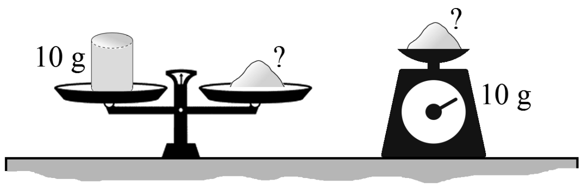
The pan balance will be level when the normal forces on the two pans are the same. When the masses on the two pans are the same, the normal forces will be the same, regardless of the acceleration, so the pan balance will always measure the correct amount of sand.

A spring scale depends on the normal forces, too, but it is marked in grams. When the markings were made, there was an assumption that the scale would be used on the surface of the Earth, not accelerating. If sand



is accelerating upward, the net force on it must be upward, so the normal force from the spring scale must be greater than the force of gravity. That it, it will take less than 10 g of sand for the scale to exert the normal force required to make the needle point to "10 g".

Use the pan balance. The spring scale will really have less than 10 g of sand.