Physics 2211 B/C Test form 363 Name Solution Set

Spring 2016

Exam 3 Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

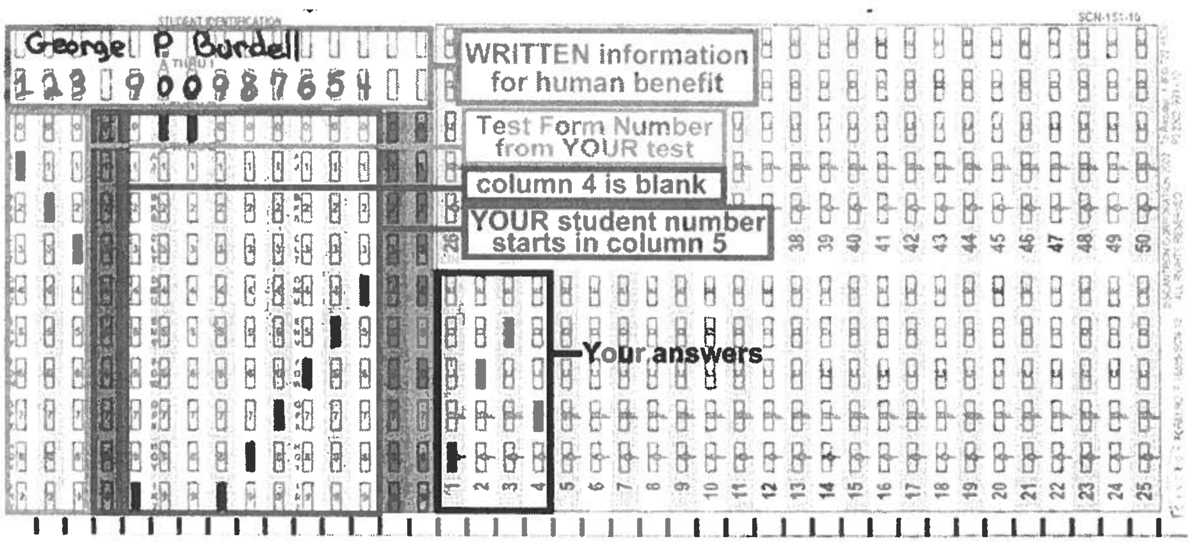
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

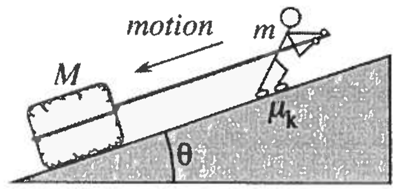
Your test form is: 363



Our next test will be on Monday, April 11!

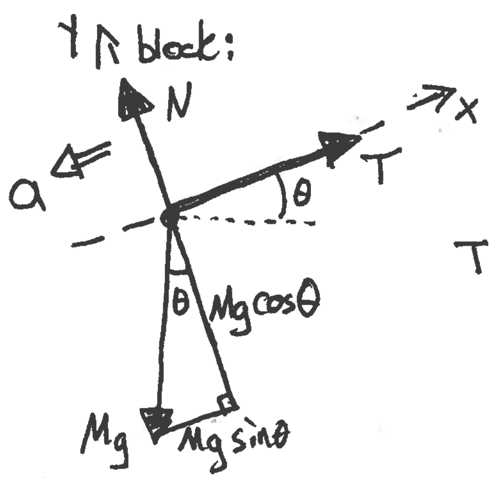
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

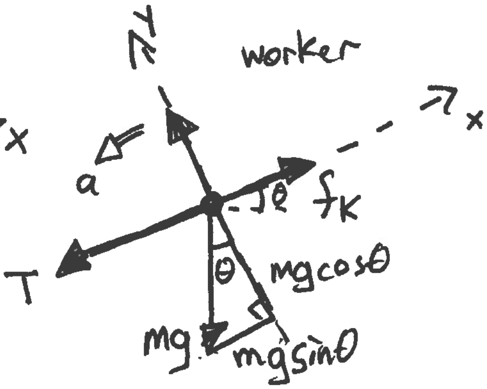
[1] *(20 points)* A worker of mass m is trying to use a rope to pull a large block of ice (having mass $M = 2m$) up a ramp that is inclined at an angle $\theta = 27°$ above the horizontal. Unfortunately, there is not enough traction, so the worker and block begin to slide down the ramp. The coefficient of kinetic friction *between the worker and the ramp* is $\mu_k = 0.42$, and friction between the block and the ramp is negligible.



Construct free body diagrams for both the worker and the block, decompose any force vectors that are not along your cartesian axes, and then write out statements of the second law for each object. ***The quality of your diagrams will be graded!***

Use your expressions to determine the magnitude of acceleration, *a,* for the block and the worker, and then find the magnitude of the tension force in the rope. Express the acceleration as a multiple of *g*, and the tension as a multiple of the worker's actual weight, *mg*.





$\Sigma \vec{F}_y = m\vec{a}_y$

$\boxed{\langle +n\rangle + \langle -mg\cos\theta\rangle = 0}$

$\Sigma \vec{F}_x = m\vec{a}_x$

$\boxed{\langle +f_k\rangle + \langle -T\rangle + \langle -mg\sin\theta\rangle = m\langle -a\rangle}$

$\Sigma \vec{F}_y = m\vec{a}_y$

$\boxed{\langle +N\rangle + \langle -Mg\cos\theta\rangle = 0}$

$\Sigma \vec{F}_x = m\vec{a}_x$

$\boxed{\langle +T\rangle + \langle -Mg\sin\theta\rangle = M\langle -a\rangle}$

analyze worker

$n = mg\cos\theta \rightarrow f_k = \mu_k n = \mu_k mg\cos\theta$

then $\boxed{(\mu_k mg\cos\theta) - T - mg\sin\theta = -ma}$

analyze block $\boxed{T - Mg\sin\theta = M(-a)}$

Two equations in unknowns T, a

Adding both equations: $\mu_k mg\cos\theta - \cancel{T} - mg\sin\theta + \cancel{T} - Mg\sin\theta = -(M+m)a$

subbing $M = 2m$:

$\mu_k mg\cos\theta - mg\sin\theta - 2mg\sin\theta = -3ma$

$a = g\sin\theta - \frac{\mu_k}{3} g\cos\theta = g\left[\sin\theta - \frac{\mu_k}{3}\cos\theta\right]$

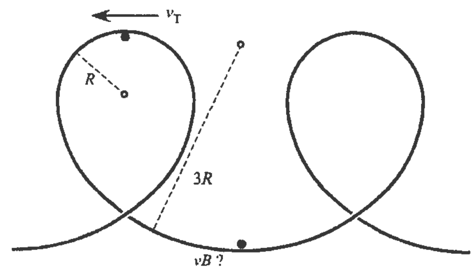
$\boxed{a = 0.30g}$

Then $T = M(g\sin\theta - a) = 2mg\left[\sin\theta - \sin\theta + \frac{\mu_k}{3}\cos\theta\right]$

$\boxed{T = \frac{2}{3}\mu_k mg\cos\theta = 0.25mg}$

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* Modern roller coasters have vertical loops with a small radius of curvature at the top (to keep the inverted passengers pressed into their seats), and a large radius of curvature at the bottom (to prevent the passengers from being pressed <u>too</u> firmly into their seats). Consider the double-loop at right, with a radius of curvature R at the top, and a radius of curvature $3R$ at the bottom.

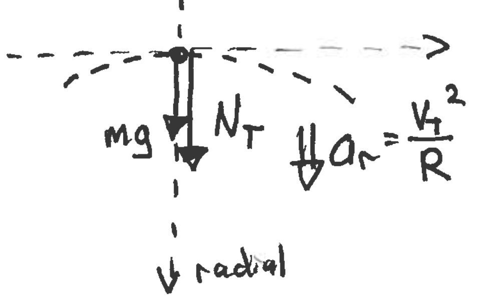


When a passenger passes through the top of the loop (position T) moving at a speed v_T, she feels that she is pressed into her seat with an apparent force equal to her true weight, mg. When she passes through the bottom of the loop (position B), she feels that she is pressed into her seat with an apparent force equal to **three times** her true weight.

Determine the speed v_B of the roller-coaster car as it passes through the bottom of the loop, expressed as a multiple of v_T.

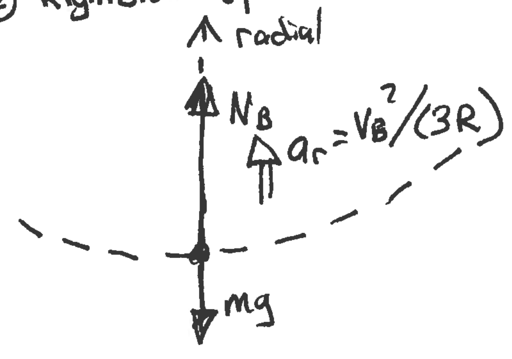
"apparent force" is actually the normal force by the seat on passenger

① inverted at top of loop:



$$\Sigma \vec{F}_r = m\vec{a}_r$$
$$\langle +mg \rangle + \langle +N_T \rangle = m\left\langle +\frac{V_T^2}{R} \right\rangle$$
→ here, $N_T = mg$
$$2\cancel{m}g = \cancel{m}V_T^2/R \qquad \boxed{V_T^2 = 2gR}$$

② Rightside-up at bottom of loop



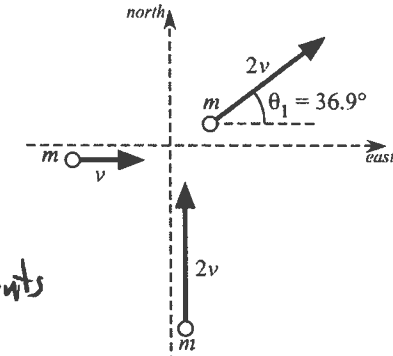
$$\Sigma \vec{F}_r = m\vec{a}_r$$
$$\langle -mg \rangle + \langle +N_B \rangle = m\langle + V_B^2/3R \rangle$$
→ here, $N_B = 3mg$
$$\text{so } -mg + 3mg = 2\cancel{m}g = \cancel{m}\, V_B^2/3R$$
$$\boxed{V_B^2 = 6gR}$$

③ Comparing: $\frac{V_T^2}{V_B^2} = \frac{2gR}{6gR} = \frac{1}{3}$ → $V_B^2 = 3V_T^2$

$$\boxed{V_B = \sqrt{3}\, V_T}$$

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* Two identical sportscars (having masses m) collide at an intersection. The first is initially travelling due east at speed v, and the second is initially travelling due north at speed $2v$. <u>Immediately</u> after the collision, the first car is observed to be skidding in a direction $\theta_1 = 36.9°$ north of east, with a speed $2v$.



Determine the velocity of the second car <u>immediately</u> after the collision. Express your answer first as a Cartesian component vector, then as a magnitude and direction relative to N/S/E/W axes.

Apply two <u>separate</u> conservation of momentum statements

Let $\vec{V}_{2f} = \langle V_x\rangle\hat{i} + \langle V_y\rangle\hat{j}$ → <u>signs</u> are unknown, as yet

$\vec{P}_x$ = constant: $\langle +mV\rangle + \langle 0\rangle = \langle +m2V\cos\theta_1\rangle + \langle mV_x\rangle$

$\langle V_x\rangle = V - 2V\cos\theta_1 = V - 2V\left(\frac{4}{5}\right)$

$= V\left(1 - \frac{8}{5}\right)$

$\boxed{\vec{V}_x = \langle -\frac{3}{5}V\rangle}$ → westward!

$\vec{P}_y$ = constant:

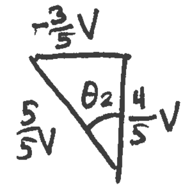
$\langle 0\rangle + \langle +\cancel{m}2V\rangle = \langle +\cancel{m}2V\sin\theta_1\rangle + \cancel{m}\langle V_y\rangle$

$\langle V_y\rangle = \langle 2V(1-\sin\theta_1)\rangle = \langle 2V(1-\frac{3}{5})\rangle$

$\vec{V}_y = \langle +\frac{4}{5}V\rangle$

so $\boxed{\vec{V}_{2f} = \langle -\frac{3}{5}V\rangle\hat{i} + \langle +\frac{4}{5}V\rangle\hat{j}}$

hey — another 345 triangle!



$\boxed{|\vec{V}_{2f}| = V}$

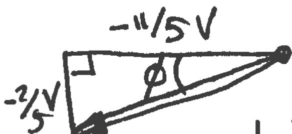
direction $\boxed{\theta_2 = 36.9° \text{ W of N}}$

[III+] *(4 points extra credit)* <u>Immediately</u> after the collision, what is the velocity of the second car, *relative to the first car*? Express your answer first as a Cartesian component vector,then as a magnitude and direction relative to N/S/E/W axes.

$\vec{V}_{2,Earth} = \vec{V}_{2,1} + \vec{V}_{1,Earth} \rightarrow \vec{V}_{2,1} = \vec{V}_{2E} - \vec{V}_{1E}$

so $\vec{V}_{21} = \left[\langle -\frac{3}{5}V\rangle\hat{i} + \langle +\frac{4}{5}V\rangle\hat{j}\right] - \left[\langle +\frac{4}{5}2V\rangle\hat{i} + \langle +\frac{3}{5}2V\rangle\hat{j}\right]$

$\boxed{\vec{V}_{2,1} = \langle -\frac{11}{5}V\rangle\hat{i} + \langle -\frac{2}{5}V\rangle\hat{j}}$



$|\vec{V}_{21}| = \sqrt{\frac{121}{25}V^2 + \frac{4}{25}V^2} = \sqrt{\frac{125}{25}V^2}$

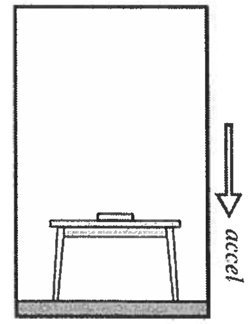
$\boxed{|\vec{V}_{21}| = \sqrt{5}\,V}$

$\tan\phi = \frac{|-\frac{2}{5}V|}{|-\frac{11}{5}V|} = \frac{2}{11}$

$\boxed{\phi = 10.3° \text{ South of west}}$

Question value 5 points

(1) A book rests atop a table, which rests on the floor of an elevator that is accelerating *downwards*. According to the 3rd Law, what force (if any) is paired with the $\boxed{\text{normal force}}$ exerted $\boxed{\text{by the table}}$ on the book?



(a) The gravitational force by the Earth on the book.
(b) The $\boxed{\text{normal force}}$ by the book $\boxed{\text{on the table.}}$
(c) Because the book and table are *accelerating*, they are not subject to the 3rd Law
(d) The gravitational force by the Earth on the table.
(e) The normal force by the elevator floor on the table.

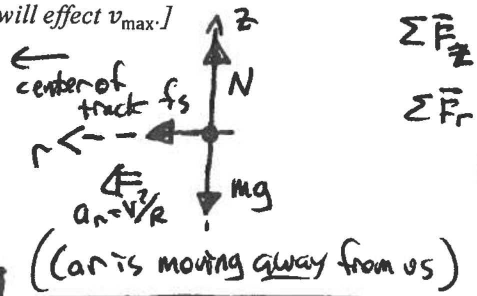
3rd Law: $\vec{F}_{A\,on\,B} \Longleftrightarrow \vec{F}_{B\,on\,A}$

• each exerts same kind of force [Normal force, here] on each other

Question value 5 points

(2) Three identical cars drive around three different circular tracks. (All three tracks are *level* and *unbanked*.) Car *A* drives on a dry track (coefficient of static friction = μ) of radius *R*. Car *B* drives around a dry track (coefficient of static friction = μ) of radius *R*/2. Car *C* drives around an wet track (coefficient of static friction = μ/3) of radius 3*R*. Rank, from greatest to least, the *top* speeds that each car can attain, without skidding. *[Hint: solve once using generic R and μ—then consider how different R and μ values will effect* v_{max}*.]*

$\boxed{\text{(a) } v_A = v_C > v_B}$
(b) $v_C > v_A > v_B$
(c) $v_B > v_A = v_C$
(d) $v_A = v_B > v_C$
(e) $v_B > v_A > v_C$



$\Sigma \vec{F}_z = 0 = \langle +N \rangle + \langle -mg \rangle$

$\Sigma \vec{F}_r = m\vec{a}_r \rightarrow \langle +f_s \rangle = m\langle +v^2/R \rangle$

max speed → max friction: $f_s = \mu_s N = \mu_s mg$

so $\mu_s \cancel{m}g = \cancel{m}v^2/R$

$\boxed{V_{max} = \sqrt{\mu_s g R}}$

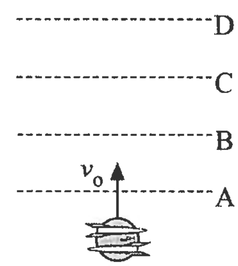
$V_A = \sqrt{\mu_s g R}$

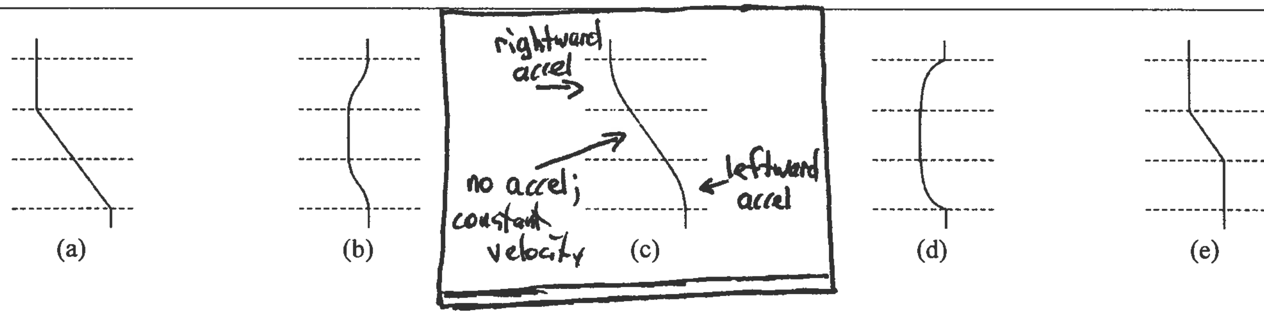
$V_B = \sqrt{\mu_s g R/2} < V_A \quad V_C = \sqrt{\frac{\mu}{3} g 3R} = V_A$

Question value 5 points

(3) A hockey puck on frictionless ice has two tiny rocket motors glued to its top, pointing in opposite directions. The puck is given a push in a direction perpendicular to the rockets' orientations. As the puck crosses line A, the leftward-firing rocket is ignited, burning out as the puck crosses line B. As the puck crosses line C, the rightward-firing rocket ignites, burning out as the puck crosses line D. The two rockets are identical. Which of the trajectories below best characterizes the motion of the puck?

top-down view of puck on ice

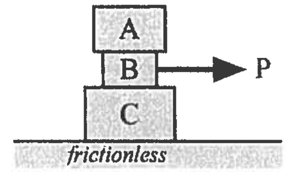




(a) (b) (c) (d) (e)

Note that since rockets are identical, leftward velocity added by #1 will be exactly cancelled by the rightward velocity added by #2

The next two questions involve the following situation:
Blocks A, B and C are stacked as shown at right, on a *frictionless* surface. Their relative masses are $m_A = 2M$, $m_B = M$, and $m_C = 3M$. When block B is pulled to the right by a force P, all three blocks move together, without slipping *relative to each other*. (That is, they do slip collectively along the surface.)



friction is the only force acting on C in horizontal direction

→ since C = half of total mass, horizontal force on C must be half of total force on system

Question value 5 points
(4) What is the magnitude of the friction force by block B on block C?

(a) $f_{BC} = \frac{1}{3}P$
(b) $f_{BC} = \frac{2}{3}P$
(c) The force magnitudes cannot be determined without knowing the coefficientsof friction μ_{BC}.
(d) $f_{BC} = \frac{5}{6}P$
(e) $f_{BC} = \frac{1}{2}P$

hand-wavy $\boxed{f_{BC} = \frac{1}{2}P}$

see?

formal math

system: 6M ● → P, → a

$\Sigma \vec{F}_x = m\vec{a}_x$

$\langle +P \rangle = 6M\langle +a \rangle$

$a = \frac{P}{6M}$

just C: 3M ● → f_{BC}, → a

$\Sigma \vec{F}_x = m\vec{a}_x$

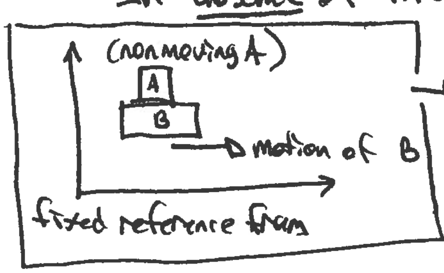
$\langle +f_{BC} \rangle = 3M\langle +a \rangle = 3M\langle +\frac{P}{6M} \rangle$

$\boxed{\langle +f_{BC} \rangle = \langle +\frac{P}{2} \rangle}$

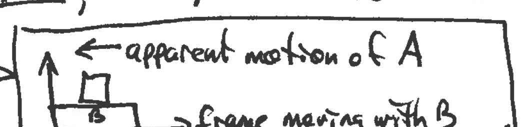
Question value 5 points
(5) What are the directions of the two friction forces on block B?

(a) Friction by A on B is to ~~the~~ right and friction by C on B is to the left.
(b) The force directions cannot be compared without knowing the coefficients of friction, μ_{AB} and μ_{BC}.
(c) **Friction by A on B is to the left and friction by C on B is to the left.**
(d) Friction by A on B is to ~~the~~ right and friction by C on B is to ~~the~~ right.
(e) Friction by A on B is to the left and friction by C on B is to ~~the~~ right.

In absence of friction, B would be pulled left, and A, C would remain stationary



→ from Block B's POV →



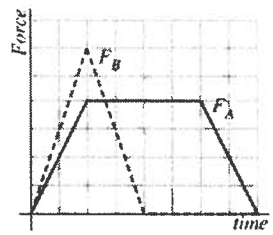
⇒ A seems to slip left along B ; friction by B on A is $\boxed{\text{to the right}}$

Identical arguments apply to the B/C friction force.

Therefore, friction by A on B and friction by C on B are directed $\boxed{\text{to the left}}$

Question value 5 points

(6) Two identical blocks are at rest on a frictionless surface. Block A is subjected to a force along the positive x-direction, of time-dependent magnitude F_A graphed at right, while block B is subjected to a force along the x-direction of time-dependent magnitude F_B. Compare the magnitude of impulse delivered to block A to the magnitude of impulse delivered to block B.



(a) $J_B = \frac{3}{2}J_A$

(b) $J_B = J_A$

(c) $J_B = 2J_A$

(d) $\boxed{J_B = \frac{1}{2}J_A}$

(e) $J_B = 3J_A$

Impulse = Area under F-vs-t graph

For B (triangle) $J_B = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(4\text{ time units})\cdot(6\text{ force units})$

$J_B = 12$ "force·time" units $= \boxed{12\text{ impulse units}}$

for A (trapezoid) $J_A = (\text{height})\cdot\left(\frac{\text{Top width} + \text{Bottom width}}{2}\right)$

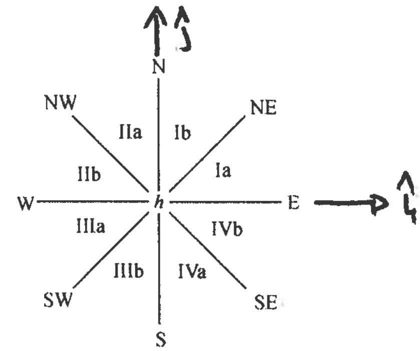
$= (4\text{ force units})\cdot\left(\frac{8\text{ time units} + 4\text{ time units}}{2}\right)$

$J_A = \boxed{24\text{ impulse units}}$

$$\boxed{J_B = \frac{1}{2}J_A}$$

Question value 5 points

(7) In the figure at right, the eight cardinal compass directions split the map into octants (each subtending a 45° arc): Ia/Ib, IIa/IIb, IIIa/IIIb, and IVa/IVb. Consider a car of mass m is initially travelling due east with speed v. It speeds up while turning right, ending up moving due south with a speed $2v$. Consider the vector impulse $\vec{J}$ delivered to the car during this process. The direction of $\vec{J}$ lies in which octant?



(a) $\boxed{\vec{J}\text{ lies in octant IIIb.}}$

(b) Impulse cannot be determined because the elapsed time was not specified.

(c) $\vec{J}$ lies in octant IIIa.

(d) $\vec{J}$ lies in octant IVa.

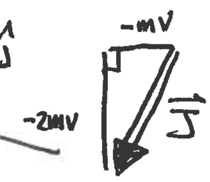
(e) $\vec{J}$ lies in octant IIa.

$\vec{J} = \Delta\vec{p} = \vec{P}_f - \vec{P}_i$ → if $\hat{i}$ = east, $\hat{j}$ = north, we have:

$\vec{P}_i = +mv\hat{i}$ $\quad \vec{P}_f = m(2v)(-\hat{j}) = -2mv\hat{j}$

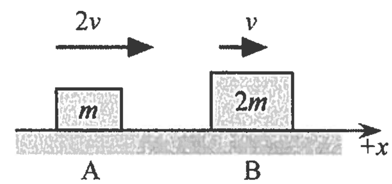
then $\Delta\vec{p} = (-2mv\hat{j}) - (+mv\hat{i}) = -mv\hat{i} - 2mv\hat{j}$

→ $\boxed{\vec{J}\text{ lies in octant IIIb}}$



Question value 5 points

(8) In the figure at right, block A overtakes block B. The resulting collision is *perfectly inelastic*. What will be the final velocity of block A, after the collision?



(a) $\vec{v}_{Af} = \langle +2v \rangle$

(b) $\vec{v}_{Af} = \langle +\frac{2}{3}v \rangle$

(c) $\vec{v}_{Af} = \langle +v \rangle$

(d) $\vec{v}_{Af} = \langle +\frac{3}{2}v \rangle$

(e) $\boxed{\vec{v}_{Af} = \langle +\frac{4}{3}v \rangle}$

perfectly inelastic:

they stick together: $\vec{V}_{Bf} = \vec{V}_{Af} = \langle +\underbrace{V}_{\text{final speed}} \rangle$

So: Conservation of momentum

$\vec{P}_i = \vec{P}_f \rightarrow m\langle +2V \rangle + 2m\langle +V \rangle = (m+2m)\langle +V \rangle$

$4\cancel{m}v = 3\cancel{m}V \rightarrow \boxed{V = \frac{4}{3}v}$

so $\boxed{\vec{V}_{Af} = \langle +\frac{4}{3}V \rangle}$