Physics 2211M Test form 512 Name Solution Set

Summer 2015

Test 1 Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

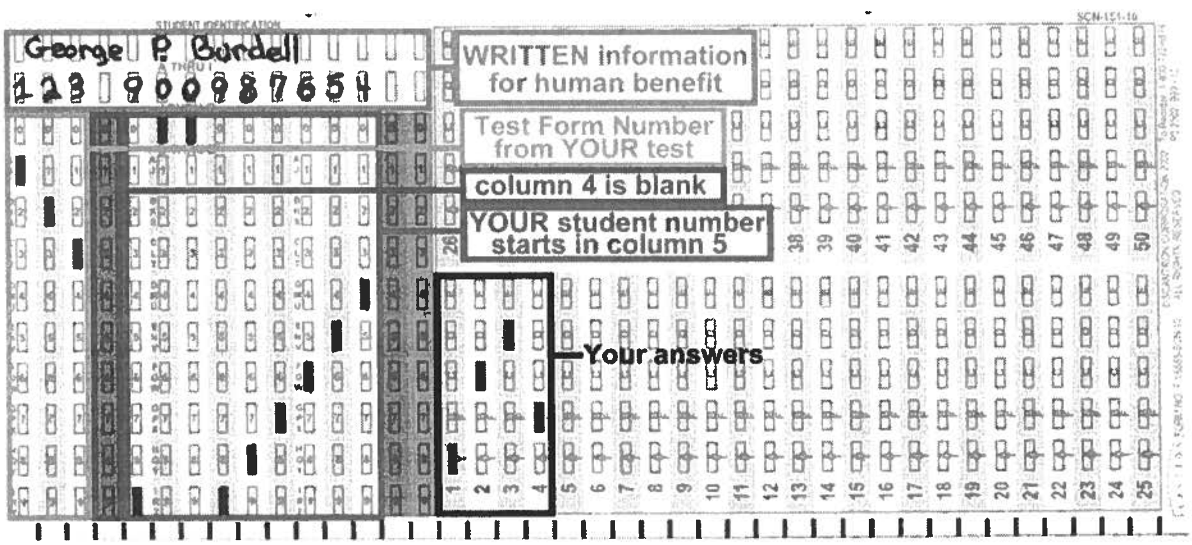
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

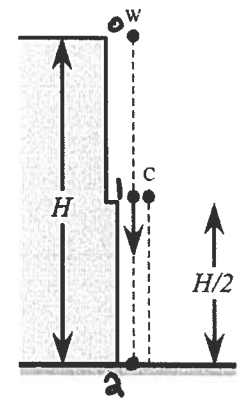
Your test form is: 512



Our next test will be on Monday, June 22!

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[I] *(20 points)* Wally drops a watermelon (from rest) off a building of height H. On the way down, it just misses a balcony at height $H/2$, falling to the ground below. Chloe is standing on the balcony. She drops a cantelope (from rest) at the exact moment the watermelon passes by on the way down. How long after the watermelon strikes the ground will the cantelope strike the ground? Express your answer in terms of the sybmols H (height of building) and g (magnitude of gravitational acceleration). *Do not use the value for g—just use the symbol "g" itself!*



① Break up fall of W into two parts

Top → balcony Δt_{01}
balcony → ground Δt_{12}

$$\underbrace{\Delta t_{02}}_{\text{total time in air for } W} = \Delta t_{01} + \Delta t_{12}$$

but: Δt_{02} = time to free-fall a distance H, from rest:

$$\vec{\Delta y} = \cancel{v_{0y}}\Delta t + \tfrac{1}{2}\langle -g\rangle \Delta t^2 \longrightarrow \langle -H\rangle = \tfrac{1}{2}\langle -g\rangle \Delta t_{02}^2 \rightarrow \boxed{\Delta t_{02} = \sqrt{\tfrac{2H}{g}}}$$

also: Δt_{01} = time to free-fall a distance $H/2$, from rest:

$$\langle -H/2\rangle = \tfrac{1}{2}\langle -g\rangle \Delta t_{01}^2 \rightarrow \boxed{\Delta t_{01} = \sqrt{\tfrac{H}{g}}}$$

Hence: time for W to fall from balcony to ground is:

$$\Delta t_{12} = \Delta t_{02} - \Delta t_{01} = \sqrt{\tfrac{2H}{g}} - \sqrt{\tfrac{H}{g}} \Longrightarrow \boxed{\Delta t_{12} = (\sqrt{2}-1)\sqrt{\tfrac{H}{g}}} \approx 0.41\sqrt{\tfrac{H}{g}}$$

② for C, it's much simpler; find time to free-fall from rest, balcony to ground

Here, $\vec{\Delta y}_c = \langle -H/2\rangle$, so $\langle -H/2\rangle = \tfrac{1}{2}\langle -g\rangle \Delta t_c^2 \rightarrow \boxed{\Delta t_c = \sqrt{\tfrac{H}{g}}}$

③ Find the difference: since C lands last, we know $\Delta t_c > \Delta t_{12}$

so, time lag is $\Delta t_{lag} = \Delta t_c - \Delta t_{12}$ [to ensure a positive value]

$$= \sqrt{\tfrac{H}{g}} - \sqrt{\tfrac{H}{g}}\left[\sqrt{2}-1\right]$$

$$\boxed{\Delta t_{lag} = 2\sqrt{\tfrac{H}{g}} - \sqrt{2}\sqrt{\tfrac{H}{g}} = (2-\sqrt{2})\sqrt{\tfrac{H}{g}}}$$

$$\approx 0.586\sqrt{\tfrac{H}{g}}$$

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

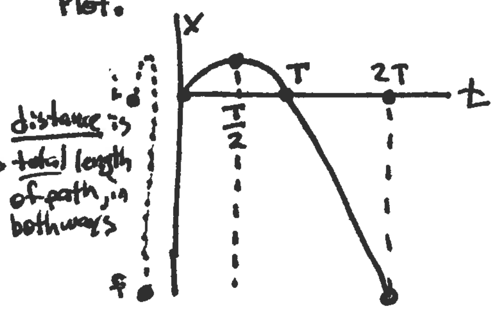
[II] *(20 points)* An object moves along the x-axis, with its position given by the time-dependent function:

$$\vec{x}(t) = \langle +At - \frac{A}{T}t^2 \rangle$$

Here, A is a positive constant having units of [Length/Time] and T is a positive constant having unts of [Time]. Determine the average velocity (a vector) ***and*** average speed (a scalar) for the object during the time interval $t = 0 \to t = 2T$.

Hint: You might find it helpful to draw a rough plot of $\vec{x}(t)$; it is a simple quadratic function of time. Quadratics (i.e. parabolas) have some obvious features, when you look at them...

Plot:



- $X(t) = 0$ for $t = 0$ and $t = T$
- quadratic term is negative, so parabola is concave down
- parabola is symmetric about apex, so the maximum must occur at $x = T/2$ (easily confirmed with some calculus)

Note: object travels in pos direction fo $t \in [0, T/2]$, then stops, turns around, and travels in neg direction for $t \in [T/2, 2T]$

[1] Average velocity does not care about turnaround — only net displacement matters

$$\vec{V}_{av} \equiv \frac{\vec{\Delta X}}{\Delta t} = \frac{\vec{X}_f - \vec{X}_i}{\Delta t} = \frac{\vec{X}(2T) - \vec{X}(0)}{2T} = \frac{\langle +A \cdot 2T - \frac{A}{T}(4T^2) \rangle - \langle 0 \rangle}{2T}$$

$$\vec{V}_{av} = \frac{\langle +2AT - 4AT \rangle}{2T} = \frac{\langle -2AT \rangle}{2T} \rightarrow \boxed{\vec{V}_{av} = \langle -A \rangle}$$

dimension check: A has units $\left[\frac{\text{length}}{\text{time}}\right]$ → right kind of units for a velocity

[2] "Average speed" is NOT $\|\text{Average Velocity}\|$

Defined as: $\frac{\text{total distance}}{\text{total time}}$ where "total distance" = $|\text{initial pos displacement}| + |\text{final neg. disp}|$

$$\vec{\Delta X}_{pos} = \vec{X}(T/2) - \vec{X}(0) = \langle +A\tfrac{T}{2} - \tfrac{A}{T}(\tfrac{T^2}{4}) \rangle - \langle 0 \rangle = \langle +\tfrac{AT}{4} \rangle$$

$$\vec{\Delta X}_{neg} = \vec{X}(2T) - \vec{X}(T/2) = \langle -2AT \rangle - \langle +\tfrac{AT}{4} \rangle = \langle -\tfrac{9}{4}AT \rangle$$

so: total distance is $D = \left|+\frac{AT}{4}\right| + \left|-\frac{9}{4}AT\right| = \frac{10}{4}AT = \frac{5}{2}AT$

then $V_{av} = \frac{D}{\Delta t} = \frac{5/2\,AT}{2T} = \boxed{\frac{5}{4}A}$

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

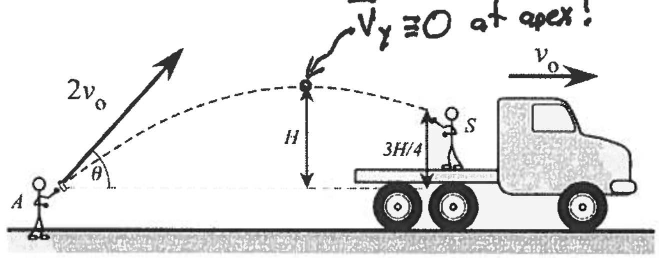
[III] *(20 points)* Aristotle is standing stationary on the ground, as Socrates passes by standing on the back of a flatbed truck that is moving at a constant speed v_0. Aristotle tosses a can of Red Bull™ to Socrates; it leaves his hands moving with a speed $2v_0$, traveling at an angle $\theta = 53.1°$ above the horizontal. The can rises to a maximum height H above its launch height before descending into Socrates' outstetched hand. If Socrates' height on the flatbed truck is $3H/4$, at what angle is the can moving relative to the vertical, according to Socrates, as he catches it?

Hint 1: 53.1° is one of the angles in the 'magic' 3–4–5 triangle!

Hint 2: There is only relative horizontal motion between Socrates and Aristotle; they both see the same vertical motion.

Steps:

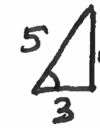
1. analyze motion of can from Aristotle's point of view
2. Correct for Socrates' relative horizontal motion
3. Compare $\vec{V}_{yf}$ and $\vec{V}_{fx}$ as seen by S to compute final projectile angle



1A Decompose:

$\vec{V}_{yi} = \langle +2V_0 \sin\theta \rangle = \langle +2V_0 \cdot \frac{4}{5} \rangle = \langle +\frac{8}{5}V_0 \rangle$

$\vec{V}_{xi} = \langle +2V_0 \cos\theta \rangle = \langle +2V_0 \cdot \frac{3}{5} \rangle = \langle +\frac{6}{5}V_0 \rangle$



$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

3-4-5 trigonometry

1B Analyze vertical motion to apex $\vec{\Delta y} = \langle +H \rangle$ as $\vec{V}_y: +\frac{8}{5}V_0 \to 0$

$$V_{yf}^2 = V_{yi}^2 + 2\langle -g \rangle \vec{\Delta y} \longrightarrow 0 = \left(\tfrac{8}{5}V_0\right)^2 + 2(-g)(+H) \longrightarrow H = \frac{32V_0^2}{25g}$$

1C Analyze motion to catch, after net $\vec{\Delta y} = \langle +\frac{3}{4}H \rangle$

$$V_{yf}^2 = V_{yi}^2 + 2\langle -g \rangle \vec{\Delta y} \longrightarrow V_{yf}^2 = \left(\tfrac{8}{5}V_0\right)^2 + 2(-g)\left(\tfrac{3}{4} \cdot \tfrac{32V_0^2}{25g}\right) = \left(\tfrac{16V_0^2}{25}\right) = \left(\tfrac{4V_0}{5}\right)^2$$

since can is on the way down, $\boxed{\vec{V}_{yf} = \langle -\frac{4}{5}V_0 \rangle \text{ as seen by both A and S}}$

---

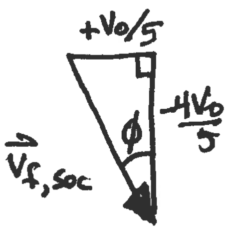
2 Correct for relative horizontal motion: $\vec{V}_x(\text{can to } S) = \vec{V}_x(\text{can to } A) + \vec{V}_x(A \text{ to } S)$

(note well: to S, A seems to be moving left at speed V_0) $= \langle +\frac{6}{5}V_0 \rangle + \langle -V_0 \rangle$

$\boxed{\text{so, to } S,\ \vec{V}_{fx} = \langle +\frac{1}{5}V_0 \rangle}$ logic check: S is moving away, so can should seem to move slower

---

3



As seen by S, can's angle relative to vertical satisfies relation

$$\tan\phi = \frac{V_0/5}{4V_0/5} = \frac{1}{4} \Rightarrow \boxed{\phi = 14.0° \text{ from vertical}}$$

(travelling downward)

Question value 4 points

(1) Fill in the blanks: When the acceleration of a moving object is ___________, it's velocity is ___________.

(a) opposite to the velocity ; increasing in magnitude
(b) negative ; opposite to its speed
(c) perpendicular to the velocity ; changing direction *and* increasing in magnitude
(d) negative ; decreasing in magnitude
(e) perpendicular to the velocity ; changing direction but not changing in magnitude

$a_{\parallel}$ effects speed only
$a_{\perp}$ effects direction only

Question value 4 points

(2) Fill in the blanks: two observers who are in motion relative to one another, with constant velocities, will always measure the same ___________ for a particular moving object, but not the same ___________ for that object.

(a) velocity ; speed
(b) acceleration ; velocity
(c) average velocity ; instantaneous velocity
(d) velocity ; acceleration
(e) velocity ; position

if relative velocity $= \vec{V}_{AB} =$ constant, then

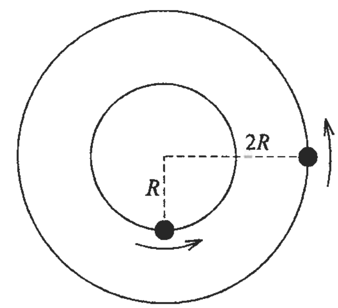
$\vec{V}_{o\,to\,B} = \vec{V}_{o\,to\,A} + \vec{V}_{AB}$: different velocities

then, since $\vec{a} = d\vec{V}/dt$,

$\vec{a}_{o\,to\,B} = \vec{a}_{o\,to\,A} + \underbrace{\frac{d\vec{V}_{AB}}{dt}}_{zero}$: same accelerations

Question value 4 points

(3) Two objects move in circular paths at constant speed, with the same period T. Object A moves in a circle of radius R, and object B moves in a circle of radius $2R$. Compare the speeds and acceleration magnitudes of the two objects.



(a) $v_B = 2v_A$, and $|a_B| = \frac{1}{2}\,|a_A|$
(b) $v_B = v_A$, while $|a_B| = |a_A| = 0$
(c) $v_B = 2v_A$, and $|a_B| = 2\,|a_A|$
(d) $v_B = \frac{1}{2}v_A$, and $|a_B| = |a_A| \neq 0$
(e) $v_B = v_A$, and $|a_B| = \frac{1}{2}|a_A|$

$V_A = \frac{2\pi R}{T}$

$V_B = \frac{2\pi(2R)}{T} = 2\left(\frac{2\pi R}{T}\right) = 2V_A$

$a_A = \frac{V_A^2}{R} \quad a_B = \frac{V_B^2}{2R} = \frac{(2V_A)^2}{2R} = \frac{4V_A^2}{2R} = 2\frac{V_A^2}{R} = 2a_A$

Question value 4 points

(4) For the two objects in the preceding question, how do the *angular* speeds (ω) and *angular* accelerations (α) compare with one another?

(a) $\omega_B = 2\omega_A$, and $\alpha_B = 2\alpha_A$
(b) $\omega_B = \omega_A$, while $\alpha_B = \alpha_A = 0$
(c) $\omega_B = \frac{1}{2}\omega_A$, and $\alpha_B = \alpha_A \neq 0$
(d) $\omega_B = 2\omega_A$, while $\alpha_B = \alpha_A = 0$
(e) $\omega_B = \omega_A$, and $\alpha_B = \frac{1}{2}\alpha_A$

$\omega_A = \frac{2\pi \text{ rad}}{T} = \omega_B$

Both complete one revolution in the same total time
$\Rightarrow$ it should be obvious that they have the same angular speed

It is specified that both travel with constant speed, so $\omega_A = \omega_B =$ constant

Then $\alpha = \frac{d\omega}{dt}$ is necessarily zero for both

Question value 8 points

(5) A car completes two laps around a circular track of circumference C. The first lap is completed at a constant speed v, and the second lap is completed at a speed $2v$. What is the average speed of the car, for both laps? (Hint: how much *time* does each lap require?)

(a) 5/4 v
$\boxed{\text{(b) } 4/3\ v}$
<v?? nonsense! ~~(c) 2/3 v~~
(d) 3/2 v
(e) 5/3 v

1^{st} lap at speed $V = \frac{\text{distance}}{\text{time}} = \frac{C}{\Delta t_1} \rightarrow \Delta t_1 = \frac{C}{V}$

2^{nd} lap at speed $2V = \frac{\text{distance}}{\text{time}} = \frac{C}{\Delta t_2} \rightarrow \Delta t_2 = \frac{C}{2V}$

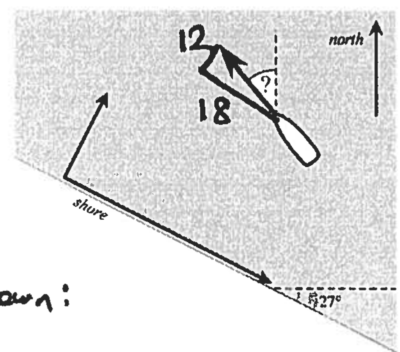
Hence, total time for both laps is $\Delta t_{TOT} = \frac{C}{V} + \frac{C}{2V} = \frac{3C}{2V}$

Then average speed for both laps combined is found from

$$V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{2C}{3C/2V} = \frac{2C}{3C} \cdot 2V = \boxed{\frac{4V}{3}}$$

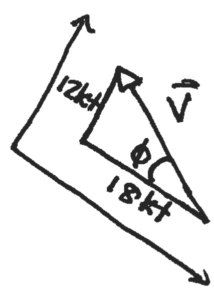
Question value 8 points

(6) You are a surveyor standing on a beach that lies along a line 27° south of east. As you look out to sea, you see a boat moving in a *roughly* north-westwardly direction; measurements with your surveying equipment tell you that the boat is moving parallel to the shore (i.e. to your left) with a speed of 18 knots, while at the same time it is moving perpendicular to shore (i.e straight out to sea) at 12 knots. In what *precise* direction is the boat actually moving, relative to due north on a map?



(a) 7° west of north
(b) 53° west of north
$\boxed{\text{(c) 29° west of north}}$
(d) 71° west of north
(e) 34° west of north

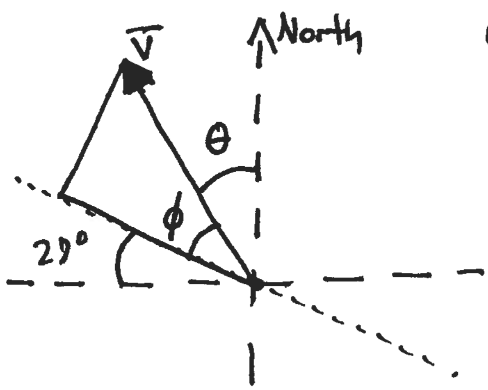
Relative to beach coords, shown:



$\tan\phi = \frac{opp}{adj} = \frac{12\,kt}{18\,kt} = \frac{2}{3}$

$\phi = 33.69°$

Now convert to NS coord axes



$\theta + \phi + 27° = 90°$

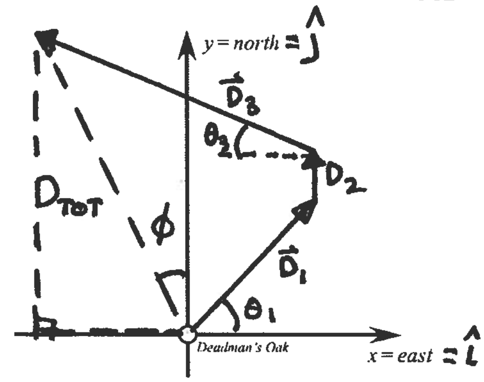
$\theta = 90° - 27° - \phi = 29.31°$

$\boxed{\text{rounds to } 29°}$

512

Question value 4 points

(7) Captain Jack Sparrow's treasure map reads: "From Deadman's Oak, walk 311 paces nor'east, then 74 paces north, then 512 paces west-nor'west, to find the buried treasure." How far, as the crow flies, is the buried treasure from the Deadman's Oak? (For ye' scurvy land-lubbers out there, nor'east means 45° north of east, and west-nor'west means 22.5° north of west.)



(a) 673 paces
(b) 551 paces
(c) 897 paces
(d) 603 paces
(e) 498 paces

$D_{TOT} = \left| \vec{D}_1 + \vec{D}_2 + \vec{D}_3 \right|$

Decompose each displacement: $\vec{D}_1 = \langle +D_1 \cos 45^\circ \rangle \hat{i} + \langle +D_1 \sin 45^\circ \rangle \hat{j} = \langle +220, +220 \rangle$ paces

$\vec{D}_2 = \langle 0 \rangle \hat{i} + \langle D_2 \rangle \hat{j} = \langle 0, +74 \rangle$ paces

$\vec{D}_3 = \langle -D_3 \cos 22.5^\circ \rangle \hat{i} + \langle +D_3 \sin 22.5^\circ \rangle \hat{j} = \langle -473, +196 \rangle$ paces

$\vec{D}_{TOT} = \langle -253, +490 \rangle$ paces

$\hat{i}$-component $\quad$ $\hat{j}$-component

Hence, $\left| \vec{D}_{TOT} \right| = \sqrt{(253)^2 + (490)^2}$ paces

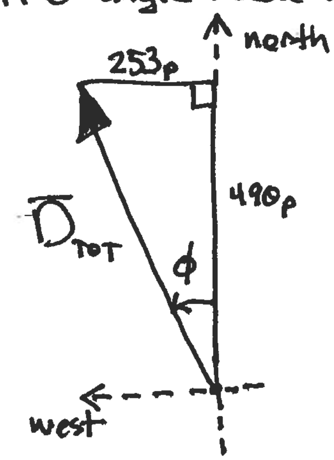
$\boxed{= 551 \text{ paces}}$

Question value 4 points

(8) In the preceding question, what direction should you walk, relative to due north, to go straight from Deadman's Oak to the buried treasure?

(a) 24.8° ~~east of north~~
(b) 19.7° west of north
(c) 27.3° ~~east of north~~
(d) 17.9° ~~east of north~~
(e) 27.3° west of north

given component form of $\vec{D}_{TOT}$ above, find angle made with +y axis:



$\tan\phi = \dfrac{|-253 \text{ paces}|}{|+490 \text{ paces}|}$

$\boxed{\phi = 27.3^\circ \text{ west of north}}$