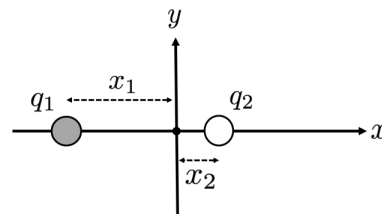


- I. (14 points) Two positively-charged particles are located on the x axis as shown. The first particle carries charge $q_1 = +5.0 \text{ nC}$ and is located at $x_1 = -7.0 \text{ cm}$ from the origin. The second particle carries charge $q_2 = +8.0 \text{ nC}$ and is located at $x_2 = +3.0 \text{ cm}$ from the origin. At what location is the total electric field zero (other than infinity)?



Since both particles carry charge of the same sign, the fields they create are in opposite directions between them. At $x > x_2$ or $x < x_1$, however, they create fields in the same direction. The two fields can only add to zero between the charges, where they have opposite directions. The two fields must have the same magnitude at that point. In terms of magnitudes, then

$$E_1 = E_2 \quad \Rightarrow \quad K \frac{q_1}{r_1^2} = K \frac{q_2}{r_2^2}$$

where r_1 and r_2 are the distances of the point in question from q_1 and q_2 , respectively. Let d represent the distance between the charges, 10.0 cm . Then $r_2 = d - r_1$, and

$$K \frac{q_1}{r_1^2} = K \frac{q_2}{(d - r_1)^2} \quad \Rightarrow \quad q_1 (d - r_1)^2 = q_2 r_1^2 \quad \Rightarrow \quad d^2 - 2dr_1 + r_1^2 = \frac{q_2}{q_1} r_1^2$$

which is quadratic in r_1 .

$$\left(\frac{q_2}{q_1} - 1 \right) r_1^2 + 2dr_1 - d^2 = 0$$

Using the quadratic formula,

$$r_1 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \text{where} \quad A = \frac{q_2}{q_1} - 1 = \frac{8.0 \text{ nC}}{5.0 \text{ nC}} - 1$$

$$B = 2d = 2(10.0 \text{ cm}) = 20.0 \text{ cm}$$

$$C = -d^2 = -(10.0 \text{ cm})^2 = -100 \text{ cm}^2$$

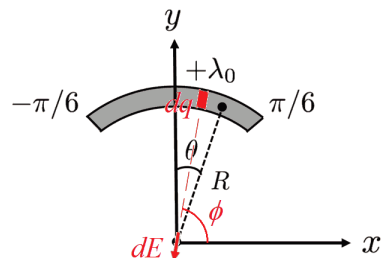
which yields

$$r_1 = 4.4 \text{ cm} \quad \text{or} \quad r_1 = -38 \text{ cm}$$

but since the point of zero field must lie between the two charged particles, it cannot be 38 cm from particle 1. The point between the particles that is 4.4 cm from particle 1 lies at

$$x = x_1 + r_1 = -7.0 \text{ cm} + 4.4 \text{ cm} = -2.6 \text{ cm}$$

- II. (18 points) A flexible rod is bent into an circular arc of radius R as shown. The arc subtends a total angle of $\pi/3$. It is charged and carries a uniform positive linear charge density $+\lambda_0$. Find an expression for the magnitude of the electric field E_0 at the origin, in terms of parameters defined in the problem and physical or mathematical constants.



An element of charge dq makes an element of field $d\vec{E}$. Since the element of charge is point-like, the magnitude of the element of field is

$$dE = K \frac{dq}{r^2}$$

As the rod is symmetrical about the y axis, the x components of the field cancel.

$$E_0 = \int dE_y = \int dE \sin \phi = \int K \frac{dq}{r^2} \sin \phi$$

Since the arc is circular, $r = R$, a constant. The element of charge can be related to ϕ through an element of arc length ds .

$$\lambda = \frac{dq}{ds} \quad \Rightarrow \quad dq = \lambda ds = \lambda_0 R d\phi$$

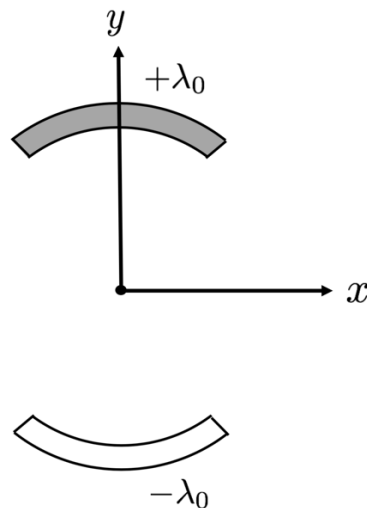
So

$$\begin{aligned} E_0 &= \int_{\pi/3}^{2\pi/3} K \frac{\lambda_0 R d\phi}{R^2} \sin \phi = \frac{K\lambda_0}{R} \int_{\pi/3}^{2\pi/3} \sin \phi d\phi = \frac{-K\lambda_0}{R} \cos \phi \Big|_{\pi/3}^{2\pi/3} \\ &= \frac{-K\lambda_0}{R} \left[\cos \left(\frac{2\pi}{3} \right) - \cos \left(\frac{\pi}{3} \right) \right] = \frac{-K\lambda_0}{R} \left[-\frac{1}{2} - \frac{1}{2} \right] = \frac{K\lambda_0}{R} \end{aligned}$$

1. (5 points) An arc similar to the one in the problem above, but carrying a uniform linear charge density of opposite sign $-\lambda_0$, is added as shown. Both arcs share the same center. What is the magnitude of the new electric field $|\vec{E}_1|$ at the origin, compared to $|\vec{E}_0|$ found in the problem above?

The original positive arc on the $+y$ axis makes field in the $-y$ direction at the origin (a positive probe charge at the origin would be repelled). The new negative arc on the $-y$ axis also makes field in the $-y$ direction at the origin (a positive probe charge at the origin would be attracted.) Since these fields are in the same direction, their magnitudes add.

$$|\vec{E}_1| = 2 |\vec{E}_0|$$



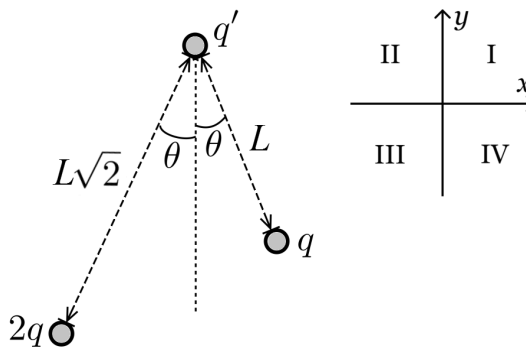
2. (5 points) What is the direction of the electric field at the location of the charge q' , due to the other two charges?
Note: All three charges are positive.

The electric force magnitude between two point charges (Coulomb's Law) is

$$F = K \frac{q_1 q_2}{r^2}$$

The particle on the left has twice the charge as the particle on the right, but it is also $\sqrt{2}$ times farther away from the particle with charge q' . Thus, the particle on the left and the particle on the right each exert the same force magnitude on q' . Since the angles θ are the same, the x components of these forces cancel, and the net force on q' must be entirely in the $+y$ direction. That is,

Directly along an axis.



- III. (18 points) In the question above, $q = 2.0 \text{ nC}$, $q' = 5.0 \text{ nC}$, $L = 3.0 \text{ mm}$, and $\theta = 41^\circ$. Calculate the magnitude of the net electrostatic force on charge q' .

Let us first use Coulomb's Law to find the magnitude of the force on q' from the charged particle on the right.

$$F_{qq'} = K \frac{qq'}{L^2} = (8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(2.0 \times 10^{-9} \text{ C})(5.0 \times 10^{-9} \text{ C})}{(3.0 \times 10^{-3} \text{ m})^2} = 9.99 \times 10^{-3} \text{ N}$$

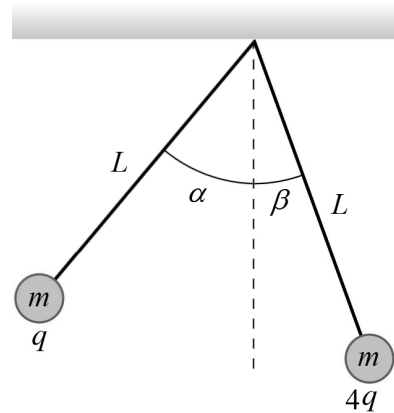
The y component of this is

$$F_{qq'_y} = F_{qq'} \cos \theta = (9.99 \times 10^{-3} \text{ N}) \cos (41^\circ) = 7.54 \times 10^{-3} \text{ N}$$

and the total force from both particles is twice that from one, so

$$F_{\text{total}} = 2F_{qq'_y} = 2(7.54 \times 10^{-3} \text{ N}) = \textcolor{red}{1.5 \times 10^{-2} \text{ N}}$$

3. (5 points) Two small balls, each with mass m , hang from massless cords of length L . One ball has charge q and hangs at an angle α from the vertical. The other ball has charge $4q$ and hangs at angle β from the vertical. In the figure, $\alpha = 2\beta$. Is that the actual relationship between α and β ? If not, what is the relationship? (*On Earth. Do NOT neglect gravity!*)

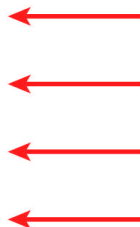
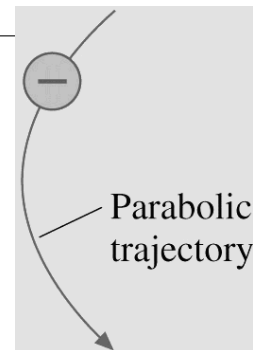


The balls have the same mass, and therefore there is the same gravitational force on each. Remembering Newton's Third Law, the electrostatic force must be the same on each ball. Since the cords are the same length, the forces on each ball are symmetric, and the geometry must be, too.

No, $\alpha = \beta$.

4. (5 points) A negatively-charged particle follows a parabolic trajectory. Which electric field is responsible?

Remembering the analogy with projectile motion, a parabolic trajectory is the result of moving under the influence of a constant force. In this case, that force must be to the right. A constant force is a consequence of a uniform field. Since the particle has negative charge, the force on it is opposite the electric field.

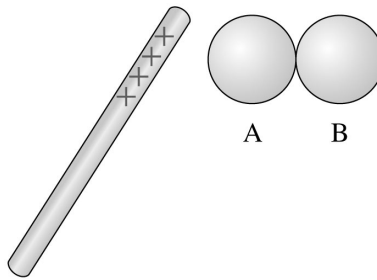


-
5. (5 points) How could two identical conducting spheres be given equal non-zero charges?

Start with the spheres touching. Bring a charged object nearby, as shown. Then ...

.....

As soon as the charged object is brought nearby, the combined two-sphere object is polarized, sphere A negative and sphere B positive, with equal magnitudes. Grounding sphere B briefly will remove its charge. When the charged object is removed, the charge on sphere A will distribute itself equally between the spheres. So,



briefly connect sphere B to ground, remove the charged object, and separate the spheres.

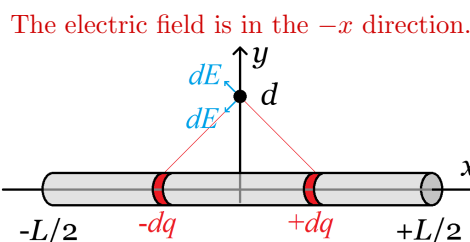
-
6. (5 points) A thin insulating rod of length L lies on the x axis, with its center at the origin, as shown. The rod has a non-uniform linear charge density λ that depends on position x according to

$$\lambda = \lambda_0 \sin\left(\pi \frac{x}{L}\right)$$

where λ_0 is a positive constant. What is the direction of the electric field, if any, at a distance d from the rod on the $+y$ axis?

.....

The sine function is odd, so for every element of charge dq at position $+x$, there is an element of charge with the same magnitude but opposite sign at position $-x$. Since λ_0 is positive, the charge on the $+x$ axis is positive, and the charge on the $-x$ axis is negative. The elements of field produced by these elements of charge are shown. Note that the y components cancel.



-
7. (5 points) In situation (i), an electric dipole is released near a fixed particle with positive charge $+q$. In situation (ii), an electric dipole is released near a fixed particle with negative charge $-q$. The two situations are separate (that is, the particles and dipoles do not interact with those in the other situation). After each dipole is released and is free to rotate and/or translate, how does each move?
-

In situation (i), the negative end of the dipole is attracted to the charged particle and the positive end is repelled. A net torque is the result, aligning the dipole with the field. In situation (ii), the positive end of the dipole is attracted to the charged particle and the negative end is repelled. Once again, net torque is the result, aligning the dipole with the field. Then in both situations, the field due to the charged particle is non-uniform, with the attracted end of the dipole now in greater field strength. The net force on each dipole is attractive.

Both dipoles rotate and move toward the fixed charge.



-
8. (5 points) A H^+ ion is hovering stationary in space when an electric field is switched for 0.50 seconds. After the field is switched off, the H^+ ion's speed is measured. The procedure is then repeated with a Ca^{2+} ion, which is exposed to the same electric field for the same length of time. What is the relation between the H^+ and Ca^{2+} ions' speeds? The ions are far enough apart that you can ignore any interaction between them. $m_H = 1m_p$ and $m_{Ca} = 40m_p$.
-

Remember Newton's Second Law, and the relationship between electric force and field.

$$F = ma = qE \quad \text{so} \quad a = \frac{qE}{m}$$

For constant acceleration,

$$v_f = v_i + a \Delta t$$

In this case, the initial velocity is zero.

$$\begin{aligned} \Delta t = \frac{v_f}{a} = \frac{v_H}{a_H} = \frac{v_{Ca}}{a_{Ca}} &\Rightarrow \frac{v_H}{q_H E / m_H} = \frac{v_{Ca}}{q_{Ca} E / m_{Ca}} \Rightarrow \frac{v_H m_H}{q_H} = \frac{v_{Ca} m_{Ca}}{q_{Ca}} \\ &\Rightarrow \frac{v_H (m_p)}{e} = \frac{v_{Ca} (40m_p)}{2e} \Rightarrow v_H = 20v_{Ca} \end{aligned}$$

-
9. (5 points) A very small copper sphere and a very small iron sphere are initially neutral. 10 million electrons are transferred from the copper sphere to the iron sphere. If the two spheres are $2.0\,\mu\text{m}$ apart, what is the electrostatic force between them?

.

Find the charge on each sphere,

$$q = (10 \times 10^6 \text{ electrons}) (1.602 \times 10^{-19} \text{ C/electron}) = 1.602 \times 10^{-12} \text{ C}$$

and then use Coulomb's Law

$$F = K \frac{q_1 q_2}{r^2} = (8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \left(\frac{(1.602 \times 10^{-12} \text{ C}) (1.602 \times 10^{-12} \text{ C})}{(2.0 \times 10^{-6} \text{ m})^2} \right) = 5.8 \times 10^{-3} \text{ N}$$

-
10. (5 points) An uncharged ball of aluminum foil is hanging from a string. Which of the situations below will produce a force of repulsion on the foil ball?

.

A neutral object, like the aluminum foil ball, will become polarized in an external electric field. If the field is uniform, there will be no net force on the object. If the field is non-uniform, there will be a net force toward greater field strength. Since field strength cannot increase with distance from a charged object,

There is no way to repel a neutral ball using a charged object.