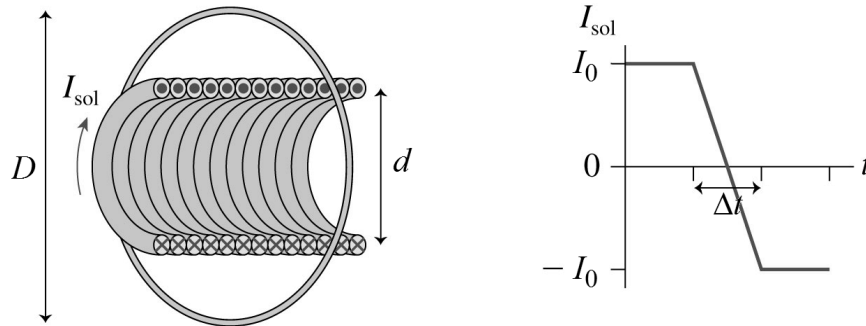

e	Fundamental charge	m_e	Mass of an electron
μ_0	Permeability constant	c	Speed of light
K	Coulomb constant $= 1/4\pi\epsilon_0$	g	Magnitude of Free Fall Acceleration

Unless otherwise directed, all wires and batteries are ideal.

Any integrals in free-response problems must be evaluated. Questions about magnitudes will state so explicitly.

- I. (20 points) A loop with diameter D and resistance R surrounds an ideal solenoid with diameter d . The solenoid has length L , has N turns, and carries the current shown in the graph, which changes from $+I_0$ to $-I_0$ in time Δt . A positive current is clockwise when seen from the left. Determine the magnitude of the current in the loop at the instant the current in the solenoid is zero. Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.



Use Faraday's Law to find the emf $\mathcal{E}$ induced in the loop. In terms of magnitudes,

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt} \left[\int B \cos \theta dA \right] = \frac{d}{dt} [BA] = A \frac{dB}{dt}$$

where the magnetic field due to the solenoid is parallel to the area vector of the loop, so $\cos \theta = 1$. Note that the area A is the area of magnetic flux through the loop, not the entire area of the loop. Magnetic flux is only within the solenoid, so the area A is the cross-sectional area of the solenoid, $A = \frac{\pi}{4}d^2$. The magnetic field in an ideal solenoid is $B = \mu_0 I_{\text{sol}} n$.

$$\mathcal{E} = A \frac{dB}{dt} = \frac{\pi}{4} d^2 \frac{d}{dt} [\mu_0 I_{\text{sol}} n] = \frac{\mu_0 \pi}{4} d^2 \frac{N}{L} \frac{dI_{\text{sol}}}{dt}$$

Since the current in the solenoid changes linearly over the time interval Δt ,

$$\mathcal{E} = \frac{\mu_0 \pi N d^2}{4L} \frac{\Delta I_{\text{sol}}}{\Delta t} = \frac{\mu_0 \pi N d^2}{4L} \frac{2I_0}{\Delta t} = \frac{\mu_0 \pi N d^2 I_0}{2L \Delta t}$$

From the definition of resistance, the current in the loop is

$$I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{\mu_0 \pi N d^2 I_0}{2LR \Delta t}$$

1. (6 points) In the problem above, what is the direction of the current, if any, in the loop during the time Δt ?

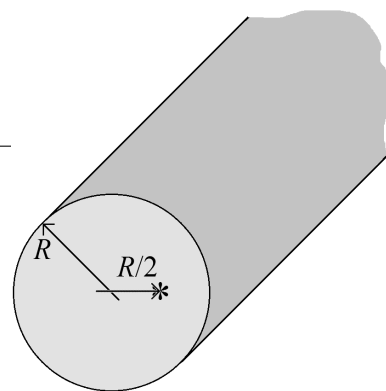
.

Using the Right-Hand-Rule, the magnetic field in the solenoid is initially into the page. As the current decreases, it becomes less and less into the page, and then as the current increases in the opposite direction the field becomes more and more out of the page. Therefore, the change in flux is out of the page throughout the time interval Δt .

Lenz' Law tells us that the current induced in the loop will flow so the the induced field it produces will oppose this change in flux. Therefore, the current must flow to produce field into the page. Using the Right-Hand-Rule, the current in the loop flows

Only clockwise.

- II. (20 points) A long, straight, cylindrical conductor of radius R carries a uniform current I out of the page. What is the magnitude of the magnetic field at the asterisk, a distance $R/2$ from the cylinder's axis? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.



.

Use Ampere's Law. Choose an Amperian Loop of radius $R/2$, centered on the cylinder axis.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru}} \quad \Rightarrow \quad B 2\pi \left(\frac{R}{2} \right) = \mu_0 I \left(\frac{A_{\text{loop}}}{A_{\text{wire}}} \right)$$

where the current through the loop must be proportional to the area of the loop, as the current is distributed uniformly.

$$B \pi R = \mu_0 I \left(\frac{\pi (R/2)^2}{\pi R^2} \right) = \mu_0 I / 4 \quad \Rightarrow \quad B = \frac{\mu_0 I}{4\pi R}$$

2. (6 points) In the problem above, what is the direction of the magnetic field at the asterisk?

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As the current is out of the page, the Right-Hand-Rule tells us that the magnetic field at the asterisk is

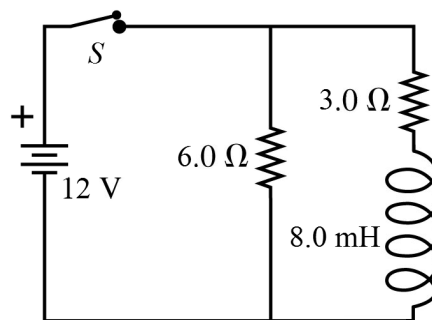
Up the page.

3. (6 points) The switch S has been open for a long time. Immediately upon closing it, what current is supplied by the battery?

.

After the switch has been open for a long time, no current will be flowing through the inductor. Inductors oppose changes in current, so immediately upon closing the switch, there will still be no current through the inductor. If no current flows through the inductor, no current can flow through the $3.0\ \Omega$ resistor. The battery supplies current only to the $6.0\ \Omega$ resistor. From the definition of resistance $\Delta V = IR$, that current is

$$I = \frac{\Delta V}{R} = \frac{12\text{ V}}{6.0\ \Omega} = 2.0\text{ A}$$

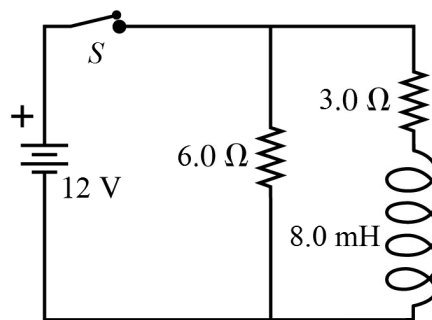


4. (6 points) The switch S has been closed for a long time. What current is supplied by the battery?

.

After the switch has been closed for a long time, the current through the inductor will not be changing. Therefore, there will be no induced emf in the inductor, so the $6.0\ \Omega$ and $3.0\ \Omega$ resistors have the same potential of 12 V across them. From the definition of resistance $\Delta V = IR$, the current through the $6.0\ \Omega$ resistor is $12\text{ V}/6.0\ \Omega = 2.0\text{ A}$ and the current through the $3.0\ \Omega$ resistor is $12\text{ V}/3.0\ \Omega = 4.0\text{ A}$. The battery must supply a total current, then, of

$$6.0\text{ A}$$



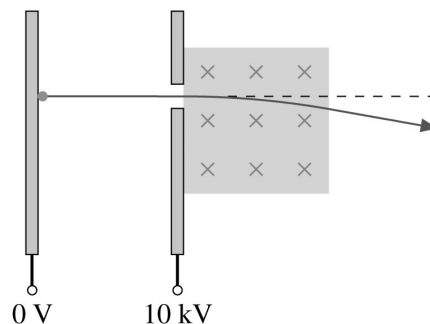
5. (6 points) Consider a particle that is released from rest at the 0 V plate of a capacitor. It accelerates toward the 10 kV plate, and passes through a small hole. Entering a uniform magnetic field directed into the page, its path is deflected down the page. Under what circumstances, if any, is this possible?

.....

Electric field points from high potential to low potential. If the particle accelerates toward the high-potential plate, the force $\vec{F}_E = q\vec{E}$ on it must be opposite the electric field, so the particle must be negatively charged.

If a negatively-charged particle enters the magnetic field as shown, the force $\vec{F}_B = q\vec{v} \times \vec{B}$ would be down the page. So,

This is possible if the particle has negative charge.



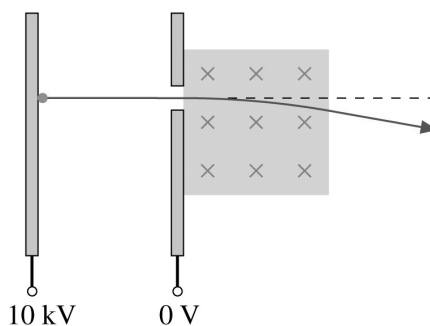
6. (6 points) Consider a particle that is released from rest at the 10 kV plate of a capacitor. It accelerates toward the 0 V plate, and passes through a small hole. Entering a uniform magnetic field directed into the page, its path is deflected down the page. Under what circumstances, if any, is this possible?

.....

Electric field points from high potential to low potential. If the particle accelerates toward the low-potential plate, the force $\vec{F}_E = q\vec{E}$ on it must be in the direction of the electric field, so the particle must be positively charged.

If a positively-charged particle enters the magnetic field as shown, the force $\vec{F}_B = q\vec{v} \times \vec{B}$ would be up the page. So,

This is not possible.



-
7. (6 points) Current is flowing through the inductor from b to a and increasing in magnitude. What is the polarity of the inductor? Is it supplying or storing energy?

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The inductor's emf opposes the change in current. Since the current is flowing leftward and increasing, current to the right would oppose the change. Using the "imagine a battery" idea, a battery with its positive end on the right would produce current toward the right. The current leftward is flowing into the positive end of that imagined battery, which would tend to re-charge it.

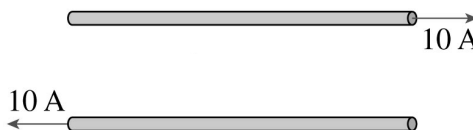


Point b is at higher potential than point a . The inductor is storing energy.

-
8. (6 points) When the current in each of the long straight wires shown is 10 A, each wire experiences a force magnitude F_0 . Describe the forces on the wires if the current in the bottom wire is increased to 20 A.

.

Using the Right-Hand-Rule, the magnetic field at the top wire due to the bottom wire is into the page. Using the Right-Hand-Rule again, the magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ is up the page. From Newton's Third Law, the magnetic force on the bottom wire must be down the page. Ampere's Law tells us that doubling the current in the bottom wire will double the field at the top wire, and so the force on the top wire will be doubled.



There is a force $2F_0$ upward on the top wire, and a force $2F_0$ downward on the bottom wire.

9. (6 points) A positively-charged particle is moving up the page with velocity $\vec{v}$. Rank the magnetic field magnitudes at points i , ii , and iii , from greatest to least.

Use the Biot-Savart Law.

$$\vec{B} = \frac{\mu_0 q}{4\pi} \frac{\vec{v} \times \hat{r}}{r^2} \Rightarrow B = \frac{\mu_0 q}{4\pi} \frac{v \sin \theta}{r^2}$$

So at point iii , the field magnitude is

$$B_{iii} = \frac{\mu_0 q}{4\pi} \frac{v \sin(90^\circ)}{r_{iii}^2} = \frac{\mu_0 q}{4\pi} \frac{v}{r_{iii}^2}$$

At point i the field is

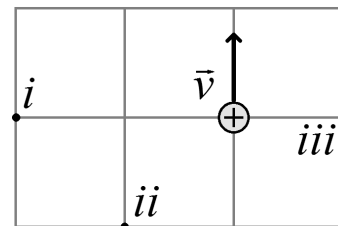
$$B_i = \frac{\mu_0 q}{4\pi} \frac{v \sin(90^\circ)}{r_i^2} = \frac{\mu_0 q}{4\pi} \frac{v}{(2r_{iii})^2} = B_{iii}/4$$

And at point ii the field is

$$B_{ii} = \frac{\mu_0 q}{4\pi} \frac{v \sin(135^\circ)}{r_{ii}^2} = \frac{\mu_0 q}{4\pi} \frac{v\sqrt{2}/2}{(r_{iii}\sqrt{2})^2} = B_{iii}\sqrt{2}/4$$

So

$$iii > ii > i$$



10. (6 points) The loop lies in the plane of the page, and current flows clockwise around it. A uniform magnetic field points up the page. If there is a net torque on the loop, which side of the loop would tend to come out of the page toward you?

The loop's magnetic moment $\vec{\mu} = NI\vec{A}$ is into the page. The torque will tend to align the magnetic moment with the external field. The side that would tend to come out of the page is

The top side.

