
e	Fundamental charge	m_e	Mass of an electron
K	Coulomb constant = $1/4\pi\epsilon_0$	g	Magnitude of Free Fall Acceleration

Unless otherwise directed, all wires and batteries are ideal.

Any integrals in free-response problems must be evaluated. Questions about magnitudes will state so explicitly.

- I. (20 points) A battery with an emf $\mathcal{E}$ is connected to the two capacitors as shown. Afterward, the charge on capacitor C_2 is Q . What is the capacitance of C_2 in terms of any or all of C_1 , Q , and $\mathcal{E}$, and physical or mathematical constants?
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Using Kirchhoff's Loop Law, moving clockwise around the loop from the bottom left corner,

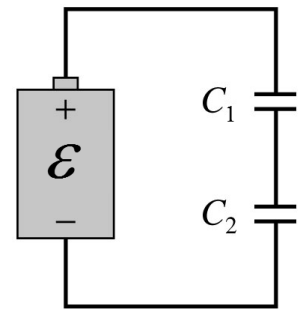
$$+\mathcal{E} - \Delta V_1 - \Delta V_2 = 0$$

so from the definition of capacitance $Q = C \Delta V$,

$$+\mathcal{E} - \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = 0 \quad \Rightarrow \quad +\mathcal{E} - \frac{Q}{C_1} - \frac{Q}{C_2} = 0$$

where $Q_1 = Q_2 = Q$ as the capacitors are in series. Solve for C_2 .

$$\frac{Q}{C_1} + \frac{Q}{C_2} = \mathcal{E} \quad \Rightarrow \quad \frac{1}{C_1} + \frac{1}{C_2} = \frac{\mathcal{E}}{Q} \quad \Rightarrow \quad C_2 = \left(\frac{\mathcal{E}}{Q} - \frac{1}{C_1} \right)^{-1}$$



1. (6 points) Consider the ratio of the energies stored in the capacitors above, U_1/U_2 . If the emf of the battery was doubled, how would the new ratio of energies, U'_1/U'_2 , compare to the original ratio?
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Capacitors in series must have the same charge, and $U = Q^2/2C$, so since the capacitances don't change

$$U'_1/U'_2 = U_1/U_2$$

- II. (20 points) A capacitor with initial potential difference ΔV_0 is discharged through a resistor when the switch is closed at time $t = 0$. At time $t = T$, the potential difference across the capacitor is $\Delta V = \Delta V_0/10$. What was the potential difference across the capacitor at time $t = T/2$?

Announced during quiz: express your answer in terms of ΔV_0 .

The potential across the capacitor will decay as it discharges through the resistor.

$$\Delta V_C = \Delta V_{C0} e^{-t/\tau}$$

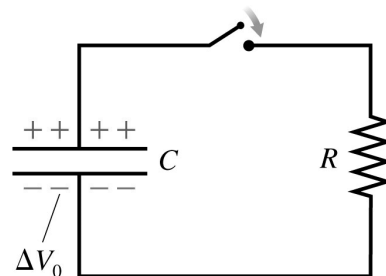
Use the information at time $t = T$ to find the time constant τ .

$$\frac{\Delta V_0}{10} = \Delta V_0 e^{-T/\tau} \quad \Rightarrow \quad \frac{1}{10} = e^{-T/\tau}$$

$$\ln 10 = T/\tau \quad \Rightarrow \quad \tau = T/\ln 10$$

Now the potential at time $t = T/2$ can be determined.

$$\Delta V_C = \Delta V_{C0} e^{-t/\tau} = \Delta V_0 e^{-\frac{T/2}{T/\ln 10}} = \Delta V_0 e^{-\frac{\ln 10}{2}} = \Delta V_0 e^{-\ln \sqrt{10}} = \frac{\Delta V_0}{\sqrt{10}}$$



2. (6 points) As the magnitude of the potential difference across the capacitor decreases with time in the problem above, how, if at all, does the magnitude of the potential difference across the resistor change?

Apply Kirchhoff's Loop Law to the circuit, moving clockwise around the loop from the bottom left corner.

$$+\Delta V_C - \Delta V_R = 0 \quad \Rightarrow \quad +\Delta V_R = \Delta V_C$$

The potential across the resistor must be the same as the potential across the capacitor at all times. As the magnitude of the potential difference across the capacitor decreases with time,

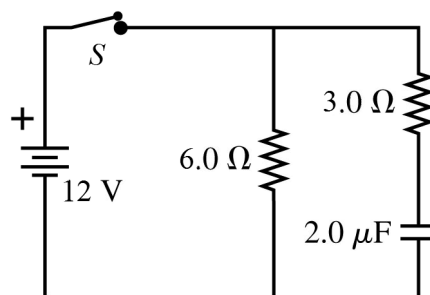
The magnitude of the potential difference across the resistor decreases.

3. (6 points) The switch S has been open for a long time. Immediately upon closing it, what current is supplied by the battery?

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Immediately upon closing the switch, there is no charge on the capacitor, and thus no potential difference across it. The $6.0\,\Omega$ and $3.0\,\Omega$ resistors have the same potential of $12\,\text{V}$ across them. From the definition of resistance $\Delta V = IR$, the current through the $6.0\,\Omega$ resistor is $12\,\text{V}/6.0\,\Omega = 2.0\,\text{A}$ and the current through the $3.0\,\Omega$ resistor is $12\,\text{V}/3.0\,\Omega = 4.0\,\text{A}$. The battery must supply a total current, then, of

6.0 A

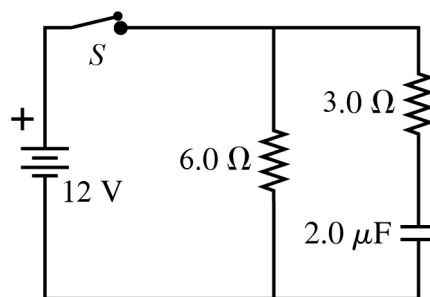


4. (6 points) The switch S has been closed for a long time. What current is supplied by the battery?

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After the switch has been closed for a long time, the capacitor will be fully charged. No current can flow through the $3.0\,\Omega$ resistor, or the charge on the capacitor would still be changing. The battery supplies current only to the $6.0\,\Omega$ resistor. From the definition of resistance $\Delta V = IR$, that current is

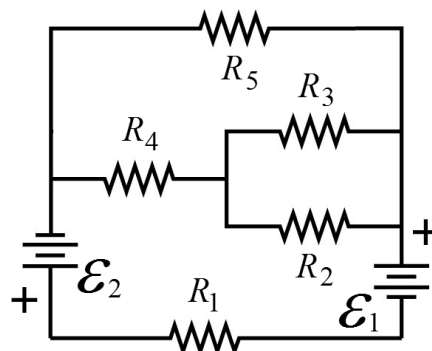
$$I = \frac{\Delta V}{R} = \frac{12\,\text{V}}{6.0\,\Omega} = \mathbf{2.0\,\text{A}}$$



5. (6 points) Let the currents I_1 – I_5 through the resistors R_1 – R_5 , respectively, be positive when flowing to the right in the circuit shown. Which is a valid expression of Kirchhoff's Loop Law?

Kirchhoff's Loop Law states that the sum of potential changes around a closed loop must be zero. With the positive direction of current flow being defined toward the right, potential decreases when traversing a resistor from left to right, and increases when traversing a resistor from right to left. The only option provided that satisfies these conditions is

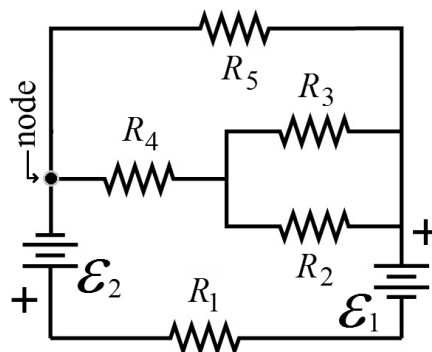
$$+\mathcal{E}_1 + I_2 R_2 - I_3 R_3 + I_5 R_5 + \mathcal{E}_2 - I_1 R_1 = 0$$



6. (6 points) Let the currents I_1 – I_5 through the resistors R_1 – R_5 , respectively, be positive when flowing to the right in the circuit shown. Which is a valid expression of Kirchhoff's Node or Junction Law at the indicated node, just to the left of R_4 ?

Kirchhoff's Node or Junction Law states that all current flowing into a junction must also flow out. One can express this mathematically by setting the sum of currents into a node equal to the sum out of the node, or by giving incoming and outgoing currents opposite signs and setting the sum of all currents at the node equal to zero. In the circuit shown, and with positive current flow being defined to the right, currents I_1 , I_4 , and I_5 flow out of the indicated node.

$$-I_1 - I_4 - I_5 = 0$$

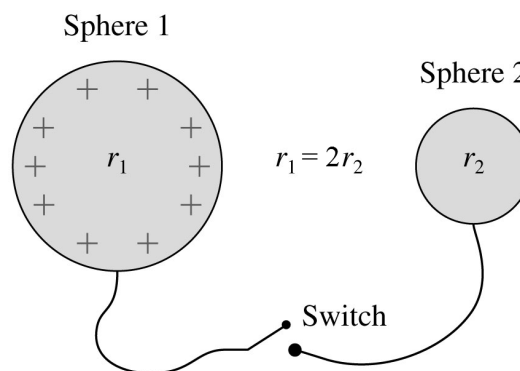


7. (6 points) The two conducting metal spheres pictured below have unequal radii, with $r_1 = 2r_2$, and are connected by a very thin wire. A charge Q_0 is initially placed on sphere 1. The switch in the wire is then closed, allowing charge to move back and forth freely. What will be the charge ratio Q_2/Q_1 when equilibrium is reached? Assume that the two spheres are separated by a very large distance.

At equilibrium, the potential must be the same everywhere in and on a conductor. Since the spheres are separated by a very large distance, the potential *at their surface* can be calculated as if it were due to a point charge located at the sphere's center.

$$V_1 = V_2 \quad \Rightarrow \quad \frac{KQ_1}{r_1} = \frac{KQ_2}{r_2} \quad \Rightarrow \quad \frac{Q_1}{2r_2} = \frac{Q_2}{r_2}$$

$$\frac{Q_2}{Q_1} = \frac{1}{2}$$



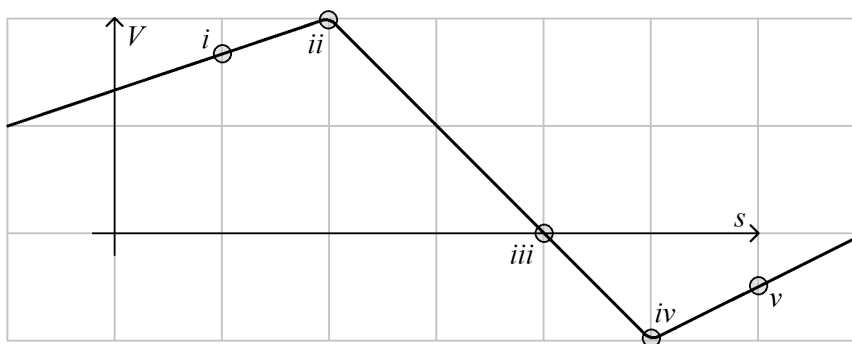
8. (6 points) The electric potential V is shown as a function of position s in a region of space. At which of the indicated locations is the magnitude of the electric field greatest?

As potential and field are related by

$$E_s = -\frac{\delta V}{\delta s}$$

the field magnitude is greatest where the slope of the V vs. s graph has greatest magnitude, which is

location *iii*



9. (6 points) Electrons move from left to right through the frustum of wire. Compare the electric field magnitudes and electron drift speeds at the right and left ends of the wire.

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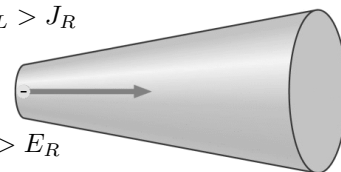
As charge is conserved, the current at the left and right ends must be the same, $I_L = I_R$.

To compare the fields, look at current density.

$$I_L = I_R \quad \Rightarrow \quad J_L A_L = J_R A_R \quad \text{but} \quad A_L < A_R \quad \text{so} \quad J_L > J_R$$

Current density can be related to field,

$$J = \sigma E \quad \text{so} \quad J_L > J_R \quad \Rightarrow \quad \sigma E_L > \sigma E_R \quad \Rightarrow \quad E_L > E_R$$



as the conductivity of a material is a constant.

Next, recall the relationship between current and drift speed, and that the charge carrier density of a material is a constant.

$$I = nAve \quad \text{so} \quad I_L = I_R \quad \Rightarrow \quad nA_L v_L e = nA_R v_R e \quad \Rightarrow \quad A_L v_L = A_R v_R$$

but

$$A_L < A_R \quad \text{so} \quad v_L > v_R$$

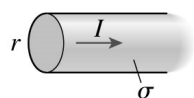
Field magnitude is greater on the left. Drift speed is greater on the left.

10. (6 points) Rank the current density magnitudes in these wires, from greatest to least.

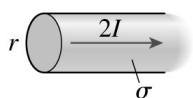
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The current density magnitude is

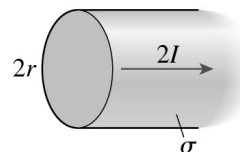
$$J = \frac{I}{A}$$



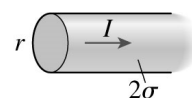
(i)



(ii)



(iii)



(iv)

Since the currents and areas are given for each situation, the conductivity σ is not needed. Comparing all the other situations to situation i ,

$$J_{ii} = 2J_i \quad J_{iii} = \frac{1}{2}J_i \quad J_{iv} = J_i \quad \text{so} \quad ii > i = iv > iii$$