

|     |                                       |       |                                     |
|-----|---------------------------------------|-------|-------------------------------------|
| $e$ | Fundamental charge                    | $m_e$ | Mass of an electron                 |
| $K$ | Coulomb constant = $1/4\pi\epsilon_0$ | $g$   | Magnitude of Free Fall Acceleration |

*Unless otherwise directed, drag should be neglected.*

*Any integrals in free-response problems must be evaluated. Questions about magnitudes will state so explicitly.*

- I. (20 points) A very long, uniformly charged insulating cylinder has radius  $R$  and linear charge density  $\lambda$ . Find the electric field magnitude at a distance  $r = R/2$  from the cylinder axis. Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

Use Gauss' Law,  $\epsilon_0 \Phi = q_{\text{in}}$ . Choose a Gaussian Surface with the symmetry of the charge distribution, and that passes through the point where the field is to be calculated. Letting "very long" be effectively infinite, that would be a finite cylinder of length  $L$  and radius  $R/2$ , concentric with the cylinder of charge. The flux is

$$\Phi = \oint \vec{E} \cdot d\vec{A} = EA_{\text{curved}} = E 2\pi \left(\frac{R}{2}\right) L = E \pi R L$$

as the flux through the end-caps of the cylindrical Gaussian Surface is zero. The charge inside is

$$q_{\text{in}} = \rho V = \rho \pi \left(\frac{R}{2}\right)^2 L = \rho \frac{\pi}{4} R^2 L$$

where  $\rho$  is the volume charge density and  $V$  is the volume of charge within the Gaussian Surface. We need, however, to relate  $\rho$  to the linear charge density  $\lambda$ . Consider a length  $L'$  of the charged cylinder, containing a charge  $Q$ . Its volume is  $V'$ .

$$Q = \rho V' = \lambda L' \quad \Rightarrow \quad \rho \pi R^2 L' = \lambda L' \quad \Rightarrow \quad \rho = \frac{\lambda}{\pi R^2}$$

Put these results together, and solve for  $E$ .

$$\epsilon_0 \Phi = q_{\text{in}} \quad \Rightarrow \quad \epsilon_0 E \pi R L = \rho \frac{\pi}{4} R^2 L = \left(\frac{\lambda}{\pi R^2}\right) \frac{\pi}{4} R^2 L \quad \Rightarrow \quad E = \frac{\lambda}{4\pi\epsilon_0 R}$$

1. (6 points) A very long conducting cylinder has radius  $R$  and linear charge density  $\lambda$ . How does the electric field magnitude at a distance  $r = R/2$  from the cylinder axis compare to the field you found for the insulating cylinder above?

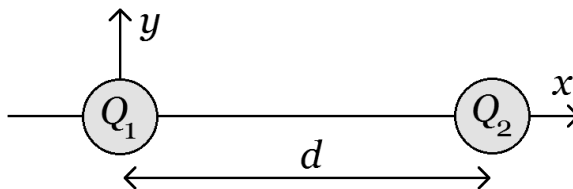
The electric field within a conductor at equilibrium is zero, so

**The field magnitude in the conducting cylinder is less, because it must be zero.**

2. (6 points) Particles with charges  $Q_1 = 3e$  and  $Q_2 = -9e$  are fixed in place with separation  $d$ , as shown. Letting the electric potential be zero at infinity, in what region or regions of the  $x$  axis will there be a finite  $x$  position at which the electric potential is also zero?

The electric potential with respect to zero at infinity is  $V = Kq/r$ . For a point to be at zero potential, it must be closer to the smaller-magnitude charge than it is to the greater-magnitude charge. That can only happen in regions

$$x < 0 \quad \text{and} \quad 0 < x < d$$



- II. (20 points) In the question above, find a finite position on the  $x$  axis at which the electric potential is zero. Express your answer in terms of parameters defined in the problem, and physical or mathematical constants.

Since the charges have opposite sign, they'll contribute opposite potential to any point. For the potential at a point to be zero, each charge must contribute potential of the same magnitude. Letting  $d_1$  be the distance from charge 1, and  $d_2$  be distance from charge 2,

$$\frac{K|Q_1|}{d_1} = \frac{K|Q_2|}{d_2} \quad \Rightarrow \quad \frac{K3e}{d_1} = \frac{K9e}{d_2} \quad \Rightarrow \quad \frac{3}{d_1} = \frac{9}{d_2} \quad \Rightarrow \quad d_2 = 3d_1$$

There are two positions on the  $x$  axis at which the potential will be zero, although only one need be found. Consider first the region  $0 < x < d$ . Here

$$d_1 + d_2 = d \quad \Rightarrow \quad d_2 = d - d_1 \quad \text{so} \quad d_2 = 3d_1 \quad \Rightarrow \quad d - d_1 = 3d_1 \quad \Rightarrow \quad d_1 = \frac{d}{4} = x$$

Consider next the region  $x < 0$ . Here

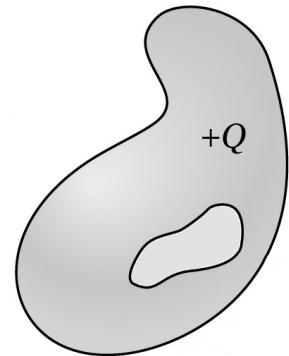
$$d_2 = d_1 + d = d_2 \quad \text{so} \quad d_2 = 3d_1 \quad \Rightarrow \quad 3d_1 = d_1 + d \quad \Rightarrow \quad d_1 = \frac{d}{2} \quad \Rightarrow \quad x = -\frac{d}{2}$$

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3. (6 points) A conducting object has a void within it. If the object carries a net positive charge  $+Q$ , what is the charge on the surface of the void?

. . . . .

Use Gauss' Law. Sketch a Gaussian Surface in the bulk of the conducting object. The electric field in a conductor at equilibrium is zero, so the flux is zero, so the net charge within the Gaussian Surface is zero. With no other charges to consider, the charge on the surface of the void is

**Zero.**

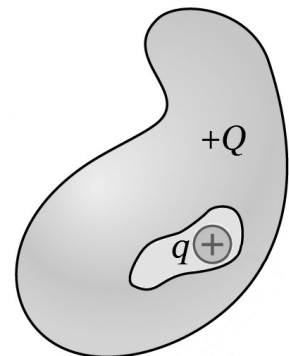


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4. (6 points) A conducting object has a void within it. A positive point charge  $+q$  is placed within the void. If the object carries a net positive charge  $+Q$ , what is the charge on the outer surface of the object, and on the surface of the void?

. . . . .

Use Gauss' Law. Sketch a Gaussian Surface in the bulk of the conducting object. The electric field in a conductor at equilibrium is zero, so the flux is zero, so the net charge within the Gaussian Surface is zero. With charge  $+q$  within the void, there must be an opposite charge of equal magnitude on the surface of the void. For the charge on the conducting object to be conserved, there must be

**$Q + q$  on outer surface,  $-q$  on surface of void.**

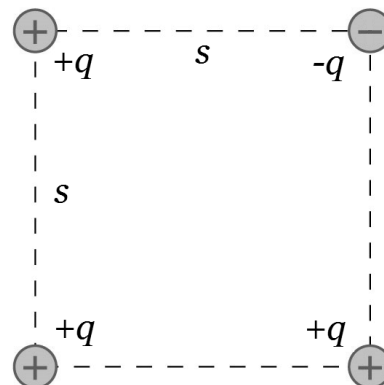


5. (6 points) Three positively-charged particles and one negatively-charged particle, each with charge magnitude  $q$ , are placed at the vertexes of a square with sides of length  $s$ . What is the potential energy of this system of charges, with respect to zero at infinite separation?

The potential energy of a system of charges is the sum of the potential energies of each pair of charges in the system. There are six pairs in this system of four charges — let them be “TL” (top left), “TR” (top right), “BL” (bottom left), and “BR” (bottom right).

The potential energy of the TL-BL pair is equal and opposite that of the TR-BR pair. The potential energy of the TL-TR pair is equal and opposite that of the BL-BR pair. The potential energy of the TL-BR pair is equal and opposite that of the TR-BL pair. Therefore, the energy of the system is

Zero.



6. (6 points) In the problem above, an external force is used to remove the negatively-charged particle from the system, by taking it infinitely far away. What is the sign, if any, of the work done by this external force? How, if at all, does the potential energy of the system change?

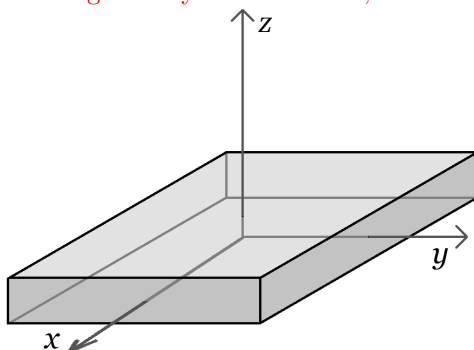
The internal electric forces from the positively-charged particles all attract the negatively-charged particle. An external force, then, would have to pull the negatively-charged particle out of the system, exerting its force in the same direction as the particle's displacement. This work done by the external force is the same as the potential energy change of the system.

The force does positive work. The potential energy of the system increases.

7. (6 points) The field *outside* a uniformly charged infinite solid slab is uniform, like that of an infinite uniform sheet of charge. What is the electric field magnitude like *inside* the slab?

. . . . .  
If the field were not zero on the center plane, it would have to point in some direction, and the symmetry of the field would not match the symmetry of the charge distribution. However, a Gaussian Surface entirely within the slab will contain charge, so the field within the slab cannot be zero everywhere. Only one of the choices offered satisfies these conditions.

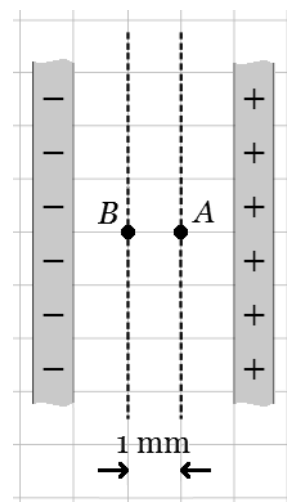
It is zero at the center, increasing linearly to the surface, where it matches the field outside.



8. (6 points) A charged particle ( $q = +2\text{ C}$ ), is released from rest at point  $A$  inside an ideal parallel-plate capacitor. At point  $B$ , the kinetic energy of the particle is  $4\text{ J}$ . If the electric potential of the positively charged plate is  $+8\text{ V}$ , what is the electric potential of the negatively charged plate?

. . . . .  
With no external forces doing work on the particle-capacitor system, and no non-conservative forces acting within it, energy is conserved. If the particle gains  $4\text{ J}$  of kinetic energy, then the system must lose  $4\text{ J}$  of potential energy. Since potential is the potential energy per charge, the potential at point  $B$  must be  $4\text{ J}/2\text{ C} = 2\text{ V}$  lower than at point  $A$ . Since the distance from  $A$  to  $B$  is one-third the distance between the plates, and the potential in a parallel-plate capacitor varies linearly with distance, the negative plate must be at a potential  $6\text{ V}$  lower than the positive plate.

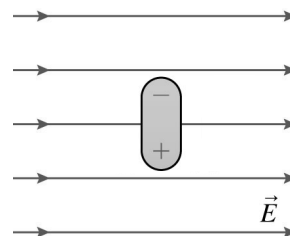
+2 V



9. (6 points) An electric dipole is held at rest in a uniform electric field, as shown. Which motion, if any, would require an external agent to do positive work on the system consisting of the dipole and the electric field?

The dipole will spontaneously rotate counter-clockwise, which must reduce the potential energy of the dipole-field system. Doing positive work on the system would increase its potential energy, which would rotate the dipole the opposite way.

Rotating the dipole clockwise would require positive external work.



10. (6 points) A positively-charged particle will be placed on the axis of a cylinder, either centered, or nearer the left end, as shown. In which configuration is the net flux through the cylinder greatest? In which is the net flux through the left end greatest?

As they cylinder contains the same net charge in each configuration, Gauss' Law tells us that the net flux through the cylinder is the same in each configuration. Gauss' Law, however, does not apply to open surfaces, such as one end of the cylinder. When the charge is moved from the center toward the left end, every field line that went through the left end will still do so, and additional field lines that previously went through the curved side of the cylinder will also now go through the left end, increasing the flux through that surface.

The net flux through the cylinder is the same in each configuration.  
The net flux through the left end is greatest when the particle is nearer the left end.

