

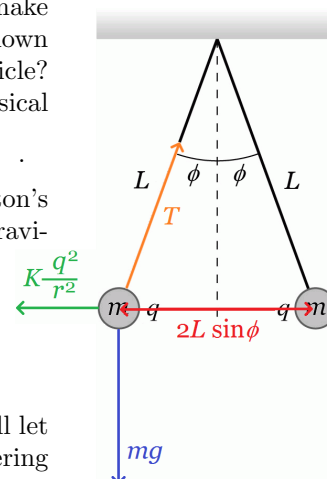
e Fundamental charge
 K Coulomb constant $= 1/4\pi\epsilon_0$

m_e Mass of an electron
 g Magnitude of Free Fall Acceleration

Unless otherwise directed, drag should be neglected.

Any integrals in free-response problems must be evaluated. Questions about magnitudes will state so explicitly.

- I. (20 points) Two point particles, each of mass m , on threads of length L that make angles ϕ with the vertical, repel each other after being *equally charged* as shown in the figure below. What is the magnitude of the charge q on each particle? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (On Earth.)



The situation is symmetric, so only one particle need be analyzed. Use Newton's Second Law. Sketch a Free Body Diagram. There is a tension force T , and gravitational force mg , and an electric force

$$F_E = K \frac{q_1 q_2}{r^2} = K \frac{q^2}{(2L \sin \phi)^2}$$

(from Coulomb's Law) acting on each particle. Choose a coordinate system. I'll let x be to the right, and y be upward, but I haven't marked them, to avoid cluttering the figure.

In the y direction, in terms of magnitudes, and showing signs explicitly,

$$\sum F_y = T_y - mg = ma_y = 0 \quad \Rightarrow \quad T \cos \phi = mg \quad \Rightarrow \quad T = \frac{mg}{\cos \phi}$$

In the x direction, again in terms of magnitudes, and showing signs explicitly,

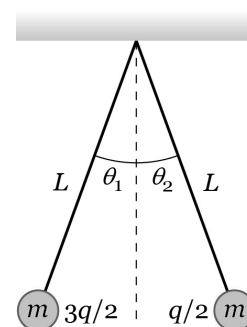
$$\sum F_x = F_E - T_x = ma_x = 0 \quad \Rightarrow \quad K \frac{q^2}{(2L \sin \phi)^2} = T \sin \phi$$

Substitute the expression found for T , and solve for q .

$$K \frac{q^2}{(2L \sin \phi)^2} = \left(\frac{mg}{\cos \phi} \right) \sin \phi \quad \Rightarrow \quad \frac{q^2}{(2L \sin \phi)^2} = \frac{mg}{K} \tan \phi \quad \Rightarrow \quad \frac{q}{2L \sin \phi} = \sqrt{\frac{mg}{K} \tan \phi}$$

$$q = 2L \sin \phi \sqrt{\frac{mg}{K} \tan \phi}$$

1. (6 points) In the problem above, charge $q/2$ is transferred from the right particle to the left particle, so the threads now make angles θ_1 and θ_2 with the vertical, which (despite the figure) may or may not be equal to each other. How do the angles θ_1 and θ_2 compare to each other, and to the original angle ϕ ?



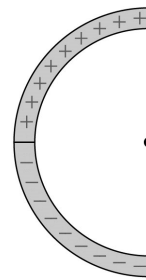
If the distance between the particles didn't change, the force between them would change from

$$K \frac{q^2}{(2L \sin \phi)^2} \quad \text{to} \quad K \frac{\left(\frac{3}{2}Q\right)\left(\frac{1}{2}Q\right)}{(2L \sin \phi)^2} = \frac{3}{4} K \frac{q^2}{(2L \sin \phi)^2}$$

which is a reduction. With less horizontal electric force, less horizontal tension force would be required. The angles (which must still be the same, as the forces are symmetric even though the charges aren't) become smaller than ϕ .

$$\theta_1 = \theta_2 < \phi$$

2. (6 points) A thin insulating rod forms a semicircle of radius R . Positive charge $+Q$ is uniformly distributed on the upper half of the rod, and negative charge $-Q$ is uniformly distributed on the bottom half, as shown. What is the direction, if any, of the electric field at the center of the semicircle?



A positive element of charge on the top half of the semicircle will produce an element of field down and to the right. A negative element of charge on the bottom half of the semicircle will produce an element of charge down and to the left. The left and right components will cancel, leaving the field

Down the page.

- II. (20 points) In the question above, what is the magnitude of the electric field at the center of the semicircle? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. *Note: If you **start** with a memorized expression for the field at the center of a circular arc, you will be penalized for showing insufficient work.*

Let the x direction be to the right, and the y direction be upward. From symmetry, the magnitude of the field must be twice the magnitude of the y component of the field due to one half of the semicircle. Consider an element of field dE_+ due to an element of charge dQ on the positive part of the semicircle. That element of charge is point-like, so

$$dE_+ = K \frac{dQ}{r^2} = K \frac{\lambda ds}{R^2} = \frac{K}{R^2} \lambda R d\theta = \frac{K}{R} \lambda d\theta$$

where the element of charge occupies an element of arc length ds , and the linear charge density λ is

$$\lambda = \frac{dQ}{ds} = \frac{dQ}{R d\theta} = \frac{Q}{S} = \frac{Q}{R\pi/2}$$

The y component of this element is $dE_{+y} = dE_+ \sin \theta$ where θ is measured, as usual, from the $+x$ axis. The field, then, is

$$\begin{aligned} E_{+y} &= \int dE_{+y} = \int dE_+ \sin \theta = \int_{\pi/2}^{\pi} \left(\frac{K}{R} \lambda d\theta \right) \sin \theta = \frac{K\lambda}{R} \int_{\pi/2}^{\pi} \sin \theta d\theta = \frac{-K\lambda}{R} \cos \theta \Big|_{\pi/2}^{\pi} \\ &= \frac{-K\lambda}{R} \left[\cos \left(\frac{\pi}{2} \right) - \cos(\pi) \right] = \frac{-K\lambda}{R} [0 - 1] = \frac{K\lambda}{R} = \frac{K}{R} \left(\frac{Q}{R\pi/2} \right) = \frac{2KQ}{\pi R^2} \end{aligned}$$

Remembering that the magnitude of the field must be twice the magnitude of the y component of the field due to one half of the semicircle

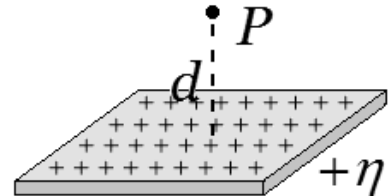
$$E = \frac{4KQ}{\pi R^2}$$

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3. (6 points) A thin infinite insulating sheet has positive uniform area charge density $+\eta$. The point P is a distance d from the sheet. Compare the sheet's electric field at point P when a positive charge $+q$ is placed there, to the electric field at that point when a negative charge $-2q$ is placed there.

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The field due to a source charge distribution (in this case, the sheet) is independent of what probe charge is used to measure it.

The field at P with $-2q$ has the same magnitude, and the same direction, as the field at P with $+q$.



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4. (6 points) A thin insulating spherical shell has radius R and uniformly distributed charge Q . Let F_{in} be the force magnitude exerted by the shell on a particle with charge q placed at a distance $R/2$ from the center. Let F_{out} be the force magnitude exerted by the shell on a particle with charge q placed at a distance $2R$ from the center. What is the ratio of F_{in} to F_{out} ?

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From Newton's Shell Theorems, we know that the net electric field is zero inside a uniform thin spherical shell of charge. Since F_{in} is zero, then regardless of F_{out} ,

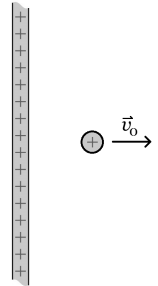
$$F_{\text{in}}/F_{\text{out}} = 0$$

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5. (6 points) A positively charged particle is held near a uniform infinite line of positive charge. The particle is then given an initial velocity $\vec{v}_0$ away from the line. How does the magnitude of the particle's velocity change, if at all, as it moves away from the line?

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The electric field due to an infinite line of positive charge points away from the line. Therefore, the force on a positively-charged particle is away from the line. Therefore, the acceleration of a positively-charged particle is away from the line. Since the particle in this case is given an initial velocity away from the line, the velocity and the acceleration are always in the same direction.

The magnitude of the velocity only increases.

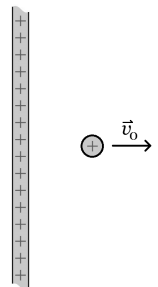


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6. (6 points) A positively charged particle is held near a uniform infinite line of positive charge. The particle is then given an initial velocity $\vec{v}_0$ away from the line. How does the magnitude of the particle's acceleration change, if at all, as it moves away from the line?

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The electric field due to an infinite line of positive charge points away from the line, but its magnitude decreases with distance (the field lines get farther apart). Therefore, the force on a positively-charged particle is away from the line, but has a magnitude that decreases with distance. Therefore,

The magnitude of the acceleration only decreases.

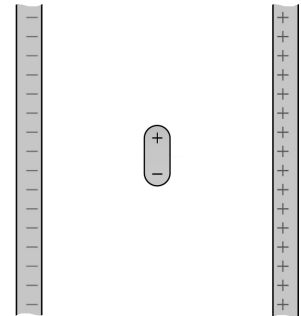


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7. (6 points) An electric dipole is centered in an ideal parallel-plate capacitor. How, if at all, does it move shortly after being released in the orientation shown?

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The electric field inside an ideal parallel-plate capacitor is uniform. There is no net force on a dipole in a uniform field, so the dipole will not accelerate away from its release point. However, the torque on a dipole in a field tends to align its dipole moment with the field.

It just rotates counter-clockwise.

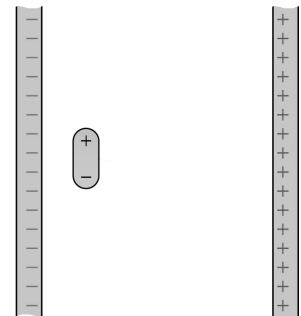


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8. (6 points) An electric dipole is in an ideal parallel-plate capacitor, much nearer the negative plate. How, if at all, does it move shortly after being released in the orientation shown?

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The electric field inside an ideal parallel-plate capacitor is **uniform**. There is no net force on a dipole in a uniform field, so the dipole will not accelerate away from its release point. However, the torque on a dipole in a field tends to align its dipole moment with the field.

It just rotates counter-clockwise.

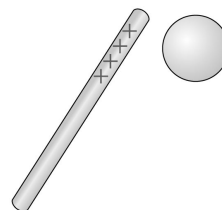


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9. (6 points) The conducting sphere is initially neutral. A positively charged rod is brought nearby, but not touching, as shown. What is the resulting charge on the sphere? What force does the rod exert on the sphere?

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No charge can be transferred to or from the sphere, so its net charge cannot change. The nearby rod will, however, polarize the sphere, making it an induced dipole in a non-uniform field.

The sphere is neutral. The rod exerts an attractive force on it.



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10. (6 points) The conducting sphere is grounded. A positively charged rod is brought nearby, but not touching, as shown. What is the resulting charge on the sphere? What force does the rod exert on the sphere?

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Negative charge will be attracted by the rod, from the ground into the sphere, resulting in opposite net charge on the rod and the sphere.

The sphere is negative. The rod exerts an attractive force on it.

