
I. (16 points) The Earth has a radius of 6.38×10^6 m. The period of rotation of Earth is 24 hours (actually it's a bit less than 24 hours, but ignore that detail). What would the period of rotation have to be, in hours, so an object at the equator would have a centripetal acceleration magnitude equal to g ($9.8\,\text{m/s}^2$)?

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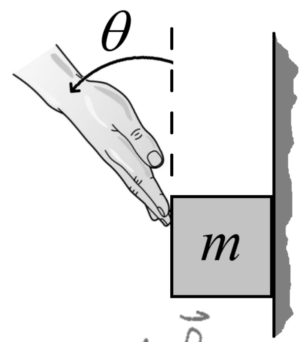
Centripetal acceleration is $a_c = v_t^2/r$ and tangential speed is $v_t = 2\pi r/T$, where T is the period.

$$a_c = g = \frac{v_t^2}{r} \quad \Rightarrow \quad v_t = \sqrt{gr} = \frac{2\pi r}{T} \quad \Rightarrow \quad T = \frac{2\pi r}{\sqrt{gr}} = 2\pi\sqrt{\frac{r}{g}}$$

$$= 2\pi\sqrt{\frac{6.38 \times 10^6\,\text{m}}{9.8\,\text{m/s}^2}} \times \frac{1\,\text{hr}}{3600\,\text{s}} = 1.4\,\text{hr}$$

II. (16 points) A block of mass m is being slid down along a wall by an applied pushing force of magnitude A that makes an angle θ with the vertical, as illustrated. The coefficient of static friction between the block and the wall is μ_s, while the coefficient of kinetic friction is μ_k. What is the acceleration magnitude the block in terms parameters defined in the problem and physical or mathematical constants? (*On Earth.*)



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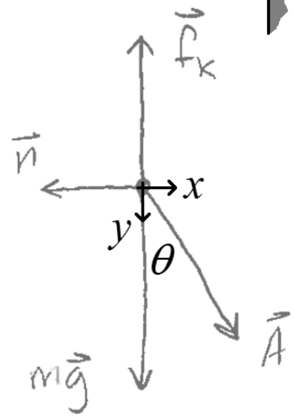
Use Newton's Second Law. Choose a coordinate system. I'll choose x horizontal to the right, and y vertical downward. Write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes.



$$\sum F_x = A_x - n = ma_x = 0 \qquad \Rightarrow \qquad n = A_x = A \sin\theta$$

$$\sum F_y = A_y + mg - f_k = ma_y \qquad \Rightarrow \qquad a_y = \frac{A_y + mg - \mu_k n}{m}$$

Substitute the expression for the normal force n from the x equation into the y equation.

$$a_y = \frac{A\cos\theta + mg - \mu_k A \sin\theta}{m} = g + \frac{A}{m}(\cos\theta - \mu_k \sin\theta)$$

1. (6 points) In the problem above, what is the direction of the block's acceleration?

The direction of the block's acceleration ...

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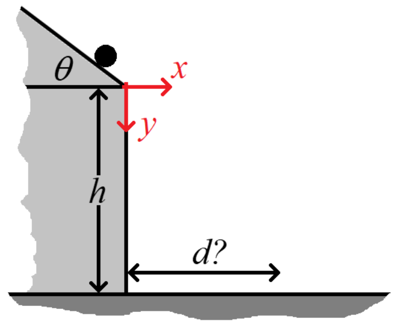
cannot be determined without numeric values for the parameters defined in the problem.

as the upward friction force may be more, less, or the same as the downward sum of the gravitational force and vertical component of the applied force, depending on the value of the coefficient of kinetic friction.

III. (16 points) A bowling ball rolls off a roof that makes an angle of 37° with the horizontal, as shown. At the instant it leaves the roof, it is a distance $h = 14\,\mathrm{m}$ above the level ground, and traveling at $4.5\,\mathrm{m/s}$. At what distance d from the building does the ball strike the ground? (*On Earth.*)



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This is a projectile motion problem (constant acceleration in two dimensions, with an acceleration of zero horizontally, and an acceleration of g vertically). Use the vertical information to find the time for the ball to reach the ground. Choose a coordinate system. I've chosen x to be horizontal to the right, and y to be vertical downward, with the origin where the ball leaves the roof.

$$y = y_0 + v_{0y}\,\Delta t + \tfrac{1}{2}a_y\,(\Delta t)^2 \qquad \text{where} \qquad a_y = g \quad \text{and} \quad v_{0y} = v_0 \sin\theta$$

Then

$$\tfrac{1}{2}g\,(\Delta t)^2 + v_0 \sin\theta\,\Delta t + (y_0 - y) = 0 \qquad \Rightarrow \qquad \Delta t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = \tfrac{1}{2}g = \tfrac{1}{2}\left(9.8\,\mathrm{m/s^2}\right) = 4.9\,\mathrm{m/s^2} \qquad B = v_0 \sin\theta = (4.5\,\mathrm{m/s})\sin 37^\circ = 2.7\,\mathrm{m/s}$$

$$C = y_0 - y = 0\,\mathrm{m} - 14\,\mathrm{m} = -14\,\mathrm{m}$$

so

$$\Delta t = \frac{-2.7\,\mathrm{m/s} \pm \sqrt{(2.7\,\mathrm{m/s})^2 - 4\,(4.9\,\mathrm{m/s^2})\,(-14\,\mathrm{m})}}{2\,(4.9\,\mathrm{m/s^2})} = 1.4\,\mathrm{s} \quad \text{or} \quad -2.0\,\mathrm{s}$$

Since the ball lands *after* it leaves the roof, choose the positive time. Then

$$x = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2}a_x\,(\Delta t)^2 = 0 + v_0 \cos\theta\,\Delta t + 0 = (14\,\mathrm{m/s})\cos 37^\circ\,(1.4\,\mathrm{s}) = 5.2\,\mathrm{m}$$

2. (6 points) In the problem above, in what direction is ball travelling as it strikes the ground?

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In the absence of gravity, the ball would travel in a straight line, in a direction 37° below the horizontal. Since there *is* gravity on Earth, the ball will fall below that straight line, and will be travelling in a direction greater than 37° below the horizontal when it strikes the ground. The horizontal component of its velocity, however, is constant and not zero, so it will not be travelling straight down.

The ball is travelling in a direction between 37° and 90° below the horizontal.

3. (8 points) A spaceship is subject to a force $3F_0$ in the positive direction of the x-axis, and to a second force $4F_0$, in the positive direction of the y-axis. This results in an acceleration of magnitude a. Then, the spaceship releases a probe with a mass 1/10 of that of the spaceship. This probe is subject to a force F_0 along the negative direction of the x-axis. What is the acceleration of the probe?

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The force magnitude on the spaceship is $\sqrt{(3F_0)^2 + (4F_0)^2} = 5F_0$. The probe is subject to one-fifth of that force, and has only one-tenth the spaceship's mass. Remembering Newton's Second Law,

$$\sum \vec{F} = m\vec{a}$$

the probe's acceleration must be twice that of the spaceship, and must be in the direction of the applied force, or

$2a$ along the negative direction of the x-axis.

4. (8 points) A blimp pilot wishes to fly directly East, but the wind is blowing from the North (toward the South). Therefore, the pilot points the blimp's nose at an angle θ north of East, enabling the blimp to move directly East over the ground.

After arriving at the destination, the pilot wishes to return directly West to the starting point. The wind is still blowing from the North at the same speed, so the pilot points the blimps nose at an angle ϕ north of West, enabling the blimp to move directly West over the ground.

If the blimp has the same speed through the air for both trips, how are the angles θ and ϕ related?

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In each case, the North component of the blimp's velocity through the air must be exactly opposite the air's velocity over the ground. Since the blimp's speed through the air is the same in each case, and the North component of its velocity is the same in each case, the angles must be the same.

$$\phi = \theta$$

5. (8 points) A car starts at rest on a circular track, then speeds up with constant tangential acceleration. Describe the direction of the car's *total* acceleration.

   The car's total acceleration ...

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   is initially straight ahead of the car, then becomes
   closer and closer to toward the center of the track as time goes on.

   Starting at rest, with a tangential speed of zero, initially the car has no centripetal acceleration. Its total acceleration is its tangential acceleration at that instant.

   As the car gains speed, its centripetal acceleration v_t^2/r increases, giving its total acceleration a greater and greater radial component.

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6. (8 points) A small skydiver of mass m and a large skydiver of mass $M > m$ are each falling at their own terminal speeds. There is a drag force d on the small skydiver and a drag force D on the large skydiver. Compare the *difference* between the drag force and the gravitational force on each skydiver. (*On Earth, do NOT neglect drag!*)

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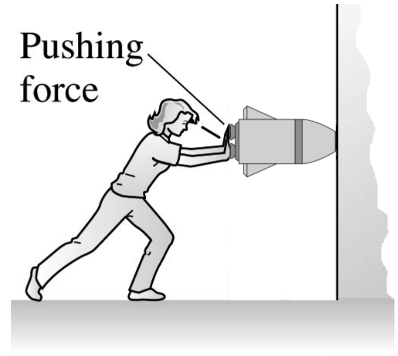
   At terminal speed, the upward drag force on each skydiver must balance the downward gravitational force on that same skydiver. The difference for each skydiver must be zero.

$$d - mg = D - Mg = 0$$

7. (8 points) Aaliyah pushes her model rocket horizontally against a wall. Static friction holds the rocket in place, so it doesn't move. If Aaliyah **doubles** the force with which she pushes her rocket, how is the static friction force affected? (*On Earth.*)



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The rocket does not accelerate vertically. Therefore, the upward static friction force must balance the downward gravitational force. The downward gravitational force does not change.

The static friction force remains the same.