Solutions

Printed Name

Nine-digit GT ID

signature

Summer 2019 **PHYS 2211 M** **Test 02**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.

Test Form:

2A

- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

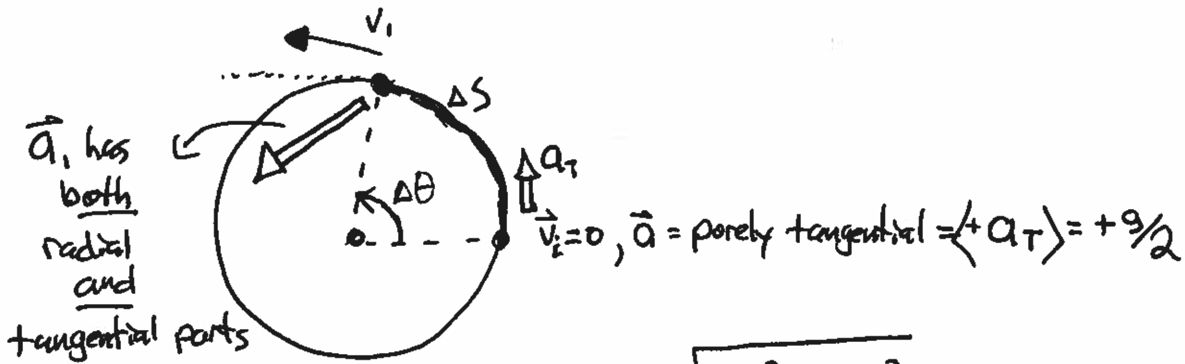
4 ⓐ ⓑ ⓒ ⓓ ⓔ

5 ⓐ ⓑ ⓒ ⓓ ⓔ

6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. *Show all supporting work for this problem.*

[1] *(20 points)* A performance sportcar is driving on a flat circular track of radius R. *(Professional driver on closed course; do not try this at home.)* Starting from rest, the car maintains a steady tangential acceleration of magnitude $a_t = 1/2\ g$. How far around the track will the car have travelled, at the moment the car (and driver) experience an overall acceleration of magnitude $2g$? Express your answer in both radians and revolutions.



- At position 1, $|\vec{a}| = 2g = \sqrt{a_T^2 + a_r^2}$

$$4g^2 = a_T^2 + a_r^2 \rightarrow a_r^2 = 4g^2 - \frac{g^2}{4} = \frac{15}{4}g^2$$

$$\text{or } \boxed{a_r = \frac{\sqrt{15}}{2} g}$$

- At this point, we also know that $a_r = \frac{v_1^2}{R}$

$$\text{so } \boxed{v_1^2 = a_r R = \frac{\sqrt{15}}{2} gR}$$

- Now, analyze tangential motion to determine where this occurs.

speed equation $v_f^2 = v_i^2 + 2\vec{a}_T \cdot \vec{\Delta s}$ where Δs = arc length $= R\Delta\theta$

$$v_1^2 = 0 + 2(g/2)R\Delta\theta$$

$$\frac{\sqrt{15}}{2} g\cancel{R} = g\cancel{R}\Delta\theta$$

$$\Delta\theta = \frac{\sqrt{15}}{2} \text{ radians} = \boxed{1.94 \text{ rad}}$$

since 1 rev $= 2\pi$ rad,

$$\Delta\theta = \frac{1.94}{2\pi} = \boxed{0.308 \text{ rev}}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*

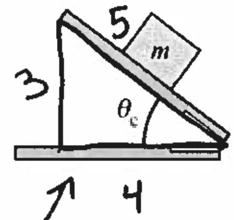
$\sin 36.9 = \frac{3}{5} = 0.6$

$\cos 36.9 = \frac{4}{5} = 0.8$

$\tan 36.9 = \frac{3}{4} = 0.75$

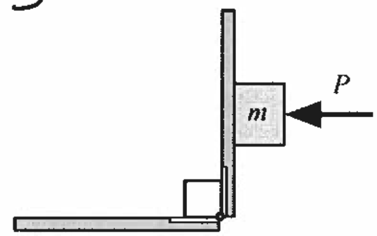
[II] *(20 points)* A block rests on a hinged board that can rotate through any angle, from horizontal ($\theta = 0°$) to vertical ($\theta = 90°$). You find that the block will remain in equilibrium on the board for any angle up to $\theta_c = 36.9°$ (top figure), but for any angle greater than this critical value, the block will begin to slip down the board. The board is then rotated to the vertical orientation and locked into place. What horizontal applied force P (bottom figure) would be required to hold the block against the board, without slipping?

→ 3-4-5 triangle



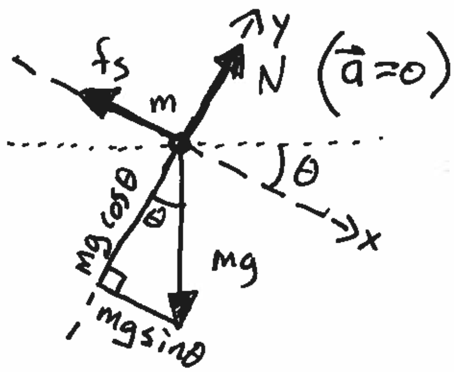
(*i*) Draw a free body diagram for the top figure, and apply the 2nd Law to determine a numerical value for the coefficient of friction.

(*ii*) Draw a free body diagram for the bottom figure, and apply the 2nd Law to to determine P, expressed as a multiple of the gravitational force mg.



The quality and clarity of your diagrams will be graded!

(i) Note block is about to slip, so we know that static friction ⟶ max value



$\Sigma \vec{F}_x = 0 \rightarrow \langle +N \rangle + \langle -mg\cos\theta \rangle = 0$

$N = mg\cos\theta$

so $f_s = \underline{max} = \mu_s N = \mu_s mg\cos\theta$

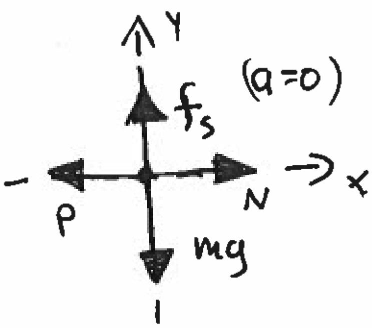
$\Sigma \vec{F}_x = 0 \rightarrow \langle +mg\sin\theta \rangle + \langle -f_s \rangle = 0$

$mg\sin\theta = f_s$

$mg\sin\theta = \mu_s mg\cos\theta$

$$\boxed{\mu_s = \tan\theta = \frac{3}{4} = 0.75}$$

(ii) Now, we don't know block is about to slip



$\Sigma \vec{F}_x = 0 \rightarrow \langle +N \rangle + \langle -P \rangle = 0 \rightarrow \boxed{N = P}$

$\Sigma \vec{F}_y = 0 \rightarrow \langle +f_s \rangle + \langle -mg \rangle = 0 \rightarrow \boxed{f_s = mg}$

If block is not slipping, then $f_s \le f_{s,max}$

$f_s \le \mu_s N$

$mg \le \mu_s P$

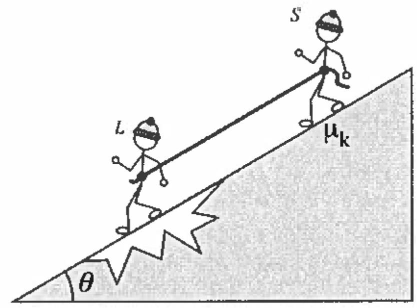
so $P \ge \frac{mg}{\mu_s} = \frac{mg}{3/4}$ $\quad \boxed{P \ge \frac{4}{3} mg}$

i.e. any force larger than this value

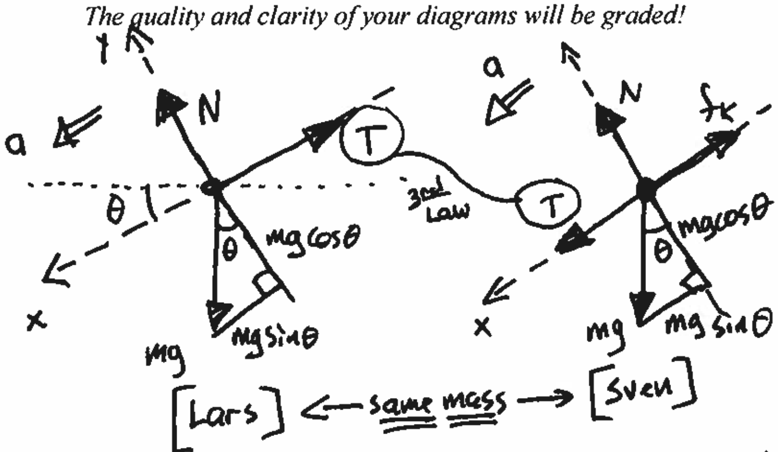
The following problem will be hand-graded. *Show all supporting work for this problem.*

[III] *(20 points)* Identical twins Lars and Sven are mountain-climbers, tethered together as they descend a steep, snowy 30° slope. Suddenly, Lars hits an icy patch with effectively zero friction, causing both climbers to begin slipping downslope. The coefficient of kinetic friction between Sven and the snow is $\mu_k = 0.25$.

Draw free body diagrams for both skiers, and use the 2nd and 3rd Laws to determine the tention in the tether. Express your answer as a multiple of the climbers' (identical) true weights, *mg*.



The quality and clarity of your diagrams will be graded!



Sven:

$\Sigma F_y = \langle +N \rangle + \langle -mg\cos\theta \rangle = 0$

$\boxed{N = mg\cos\theta}$

(also true for Lars!)

so $f_K = \mu_K N = \mu_K mg\cos\theta$

→ analyze downslope accel of both climbers: [using downslope = positive]

$$L: \quad \langle +mg\sin\theta \rangle + \langle -T \rangle = m\langle +a \rangle$$
$$S: \quad \langle +mg\sin\theta \rangle + \langle +T \rangle + \langle -f_K \rangle = m\langle +a \rangle$$

} same mass for both climbers

Since masses and accelerations are identical, we have

$$\cancel{mg\sin\theta} - T = \cancel{mg\sin\theta} + T - f_K$$
$$f_K = 2T$$

$$T = \frac{f_K}{2} = \frac{1}{2}\mu_K mg\cos\theta$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) mg\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{T = \frac{\sqrt{3}}{16} mg \approx 0.108 mg}$$

The next two questions involve the following situation:
A person having true weight $W_0 = 200$ lb steps into an express elevator and stands on a scale. Starting from rest, the elevator accelerates to its crusing speed v_c in a time T. While this is happening, the scale reads an apparent weight of 220 lb for the passenger. After the acceleration ends, the elevator coasts at constant speed.

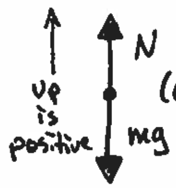
Question value 4 points

(1) What will be the passenger's apparent weight while coasting?

• "apparent weight" found from $\boxed{\text{normal force}}$

(a) 220 lb
(b) Exactly zero.
(c) 240 lb
(d) 180 lb
(e) 200 lb

coasting implies $v = \text{constant}$ and $\vec{a} = 0$



$$\langle +N \rangle + \langle -mg \rangle = 0$$

$$\boxed{N = mg = 200\,\text{lb}}$$

Question value 4 points

(2) As the elevator nears its destination, it begins to deccelerate, slowing from speed v_c to a stop in time $T/2$. What will the scale read during the decceleration?

(a) 210 lb
(b) 180 lb
(c) 190 lb
(d) 240 lb
(e) 160 lb

• speeding up: $mg = 200$ lb, but $N = 220$ lb

→ net upward force → net upward acceleration

[$\vec{a}$ = up and speed increasing implies $\vec{v}$ = up: ascending]

$\Uparrow a_1$ $\quad \langle +N \rangle + \langle -mg \rangle = m \langle +a_1 \rangle$

$220\,\text{lb} - 200\,\text{lb} = ma_1 \quad \boxed{ma_1 = 20\,\text{lb}}$

• Now, consider stopping: $\vec{v} = \langle +v_c \rangle \longrightarrow \vec{v}_f = 0$: $\vec{a}$ = downward

also: stopping in half the time implies $|\vec{a}|$ is twice as large as before

[$|\vec{a}| = \left|\frac{\Delta v}{\Delta t}\right|$ with same $|\Delta \vec{v}|$, but Δt goes from T to $T/2$]

$$\langle +N \rangle + \langle -mg \rangle = m \langle -a_2 \rangle$$

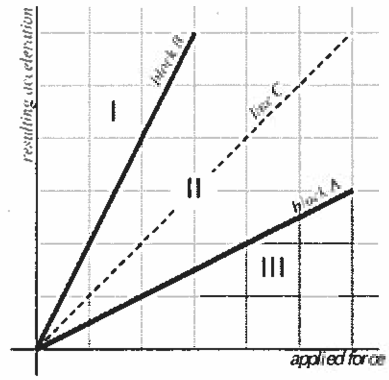
$$N - mg = -ma_2 = -2ma_1$$

$$N = mg - 2ma_1 = 200\,\text{lb} - 2(20\,\text{lb})$$

$$\boxed{N = 160\,\text{lb}}$$

Question value 8 points

(3) Two blocks A and B are used in an experiment. Each block is subjected to forces of different magnitude and the resulting accelerations are measured and plotted in the graph at right. If blocks A and B are glued together and the same experiment is done on the glued pair, where would the graph of a–vs–F lie?



(a) Somewhere above line B, in region I.

(b) Somewhere in region II, but not *necessarily* along line C.

(c) It *must* be *exactly* along line C.

(d) Somewhere below line A, in region III.

(e) There is insufficient information to determine where the graph would be.

$\vec{F} = m\vec{a}$

$\Rightarrow |\vec{a}| = \frac{|\vec{F}|}{m}$

so: slope of a-vs-F gives $\frac{1}{m}$

larger mass means flatter slope

note $M_{A+B} = M_A + M_B$ and is clearly $> M_A$

$\Rightarrow$ Slope for A+B is flatter than slope for A alone

$\Rightarrow$ plot must be in Region III

Question value 8 points

(4) A locomotive engine (mass M) pushes a group of railroad cars (total mass $9M$). The engine generates a propulsive force P, while the cars generate a total rolling friction force f. The train starts from rest, and gradually picks up speed. What is the magnitude of the direct contact force C, with which the locomotive pushes forward on the cars?

(a) $C = P - 0.9f$

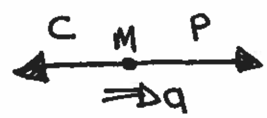
(b) $C = 0.9P + 0.1f$

(c) $C = 0.9P - f$

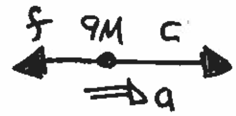
(d) $C = 0.1P + 0.9f$

(e) $C = 0.9P$

locomotive:



Freight cars:



Contact force acts:
- backward on engine
- forward on cars

2nd Law: $\langle +P \rangle + \langle -C \rangle = M\langle +a \rangle$

$\langle +C \rangle + \langle -f \rangle = 9M\langle +a \rangle$

eliminate unknown acceleration a, by plugging first equation into second:

$+C - f = 9(+P - C)$

$10C = 9P + f$

$$\boxed{C = \frac{9}{10}P + \frac{1}{10}f}$$

Question value 8 points

(5) When a cannonball is *dropped* from the top of a very high cliff, drag forces will cause it to accelerate until it reaches a final terminal speed v_T as it falls. Suppose instead that the cannonball is *fired* straight downwards with an initial speed that is *twice* the terminal speed of the dropped ball (i.e. $v_i = 2\,v_T$). What will be the initial acceleration of the cannonball at the moment it leaves the barrel of the cannon?

(a) $\boxed{3g\text{, upward}}$

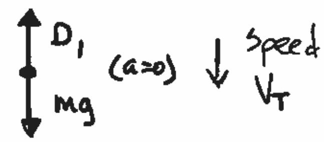
(b) $3g$, downward

(c) g, upward

(d) g, downward

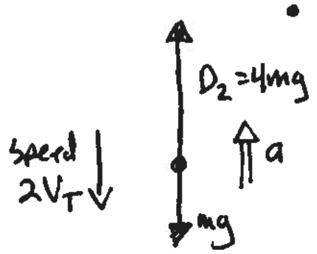
(e) $2g$, upward

• Terminal speed occurs when drag force equals grav force, and acceleration drops to zero:



• Consider ball launched at speed $2V_T$

since $D \sim v^2$, $v \to 2V_T$ causes $D_2 = 4D_1 = 4mg$



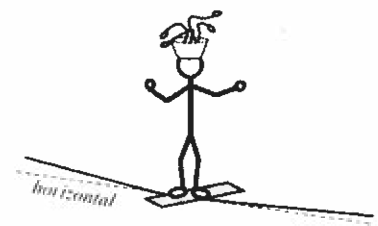
$\langle +D_2 \rangle + \langle -mg \rangle = m \langle +a \rangle$ → $D_2 > mg$ so we know $\Sigma \vec{F}$ is up → $\boxed{\vec{a} \text{ is upward}}$

$4mg - mg = ma$

$\boxed{a = 3g}$

Question value 8 points

(6) You are a performer in Cirque du Soleil, standing on a board that lies across a high wire, while simultaneously balancing a basket filled with poisonous snakes on your head. Your weight is W and the weight of the snake-filled basket is w. According to the third law, what force is paired with the upward normal force by the board on your feet?



(a) A downward gravitational force on your whole body equal to your weight W.

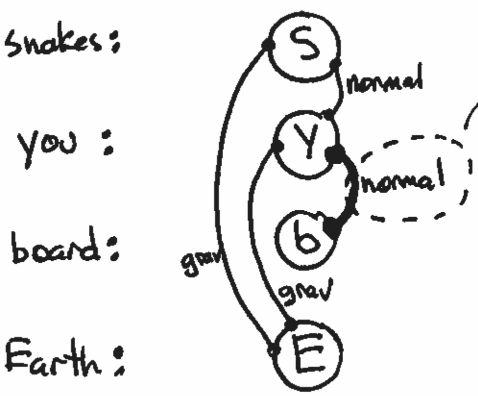
(b) A downward gravitational force on your whole body equal to the sum of the weights, $W + w$.

(c) A downward normal force by your feet on the board.

(d) A downward normal force by the basket o' snakes on your head.

(e) A downward gravitational force on your whole body, equal only to the added basket weight, w.

Don't let the snakes distract you!



This is the interaction that you are asked to consider!

Third Law says:

(Normal force, feet on board) = − (normal force, board on feet)