Solution Set

Printed Name

Nine-digit GT ID

signature

Summer 2019 **PHYS 2211 M** **Test 01**

Test Form:

1A

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

1 ⓐ ⓑ ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ⓓ ⓔ

3 ⓐ ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ⓓ ⓔ

5 ⓐ ⓑ ⓒ ⓓ ⓔ

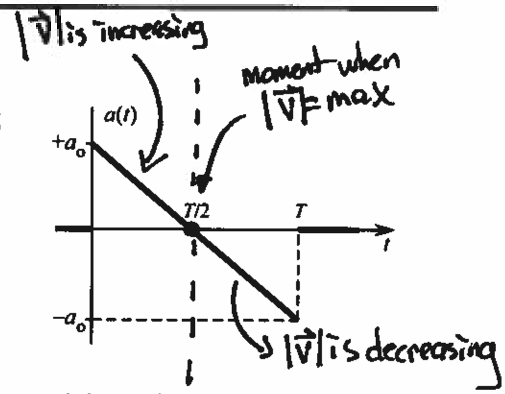
6 ⓐ ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. *Show all supporting work for this problem.*

[I] *(20 points)* A robotic toy car starts from rest. It moves in a straight line, experiencing an acceleration given by the formula:

$$a(t) = \begin{cases} a_0\left(1-\dfrac{2t}{T}\right) & 0 \le t \le T \\ 0 & t < 0 \text{ or } t > T \end{cases}$$

where a_0 and T are constants.



As you watch the car move, you note that it passes through the origin at the exact moment it is moving with maximum speed. Find an expression for the initial position of the car. Express your answer as a vector, in terms of the constants a_0 and T.

Note that speed (magnitude of velocity) is max when

$\frac{dV}{dt} = 0 \rightarrow$ ie when $a = 0$ so $\boxed{\text{max speed at } x = T/2}$

(obvious from graph)

To find position, first note that

$$\vec{V}(t) = \int \vec{a}(t)\,dt + \vec{C}_1$$
$$= \int a_0\left[1-\tfrac{2t}{T}\right]dt + \vec{C}_1$$
$$= a_0\left[t - \tfrac{t^2}{T}\right] + \vec{C}_1$$

require $\vec{V} = 0$ when $t = 0$: $\quad 0 = a_0\left[0 - \frac{0^2}{T}\right] + \vec{C}_1$

$\rightarrow \vec{C}_1 = 0$

so $\vec{V}(t) = a_0\left[t - \frac{t^2}{T}\right]$

then $\vec{X} = \int \vec{v}(t)\,dt + \vec{C}_2$

$$= \int a_0\left[t - \tfrac{t^2}{T}\right]dt + \vec{C}_2$$
$$= a_0\left[\tfrac{t^2}{2} - \tfrac{t^3}{3T}\right] + \vec{C}_2$$

we are given $\vec{X} = 0$ when $t = T/2$, so:

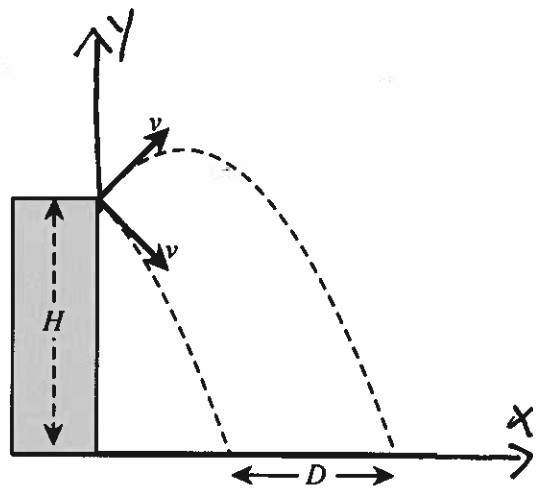
$$0 = a_0\left[\frac{T^2}{2\cdot 4} - \frac{T^3}{3T\cdot 8}\right] + \vec{C}_2 = a_0 T^2\left[\tfrac{3}{24} - \tfrac{2}{24}\right] + \vec{C}_2$$

$0 = \frac{a_0 T^2}{12} + \vec{C}_2 \rightarrow \vec{C}_2 = \left\langle -\frac{a_0 T^2}{12} \right\rangle$ (interpret value as a vector)

then $\vec{X}(t=0) = a_0\left[\frac{t^2}{2} - \frac{t^3}{3T}\right]_{t=0} + \vec{C}_2$

$$\boxed{\vec{X}(0) = \left\langle -\frac{a_0 T^2}{12} \right\rangle}$$

The following problem will be hand-graded. *Show all supporting work for this problem.*



[II] *(20 points)* Doug and Uma are standing on the roof of a building of height H. They each throw their cell phones off the roof, with initial speed v. Doug throws his phone down, at an angle 45° *below* the horizontal. Uma throws her phone up, at an angle 45° *above* the horizontal. How far away from each other do the two phones strike the ground?

Express your answer in terms of H, v, and/or g.

① For both phones, $\vec{V}_x = \langle +V_0 \cos 45^\circ \rangle = \langle \frac{+V_0}{\sqrt{2}} \rangle$] horizontal velocity components

Meanwhile, vertical velocity components are:

up: $\vec{V}_{yi} = \langle + V_0 \sin 45^\circ \rangle = \langle + \frac{V_0}{\sqrt{2}} \rangle$

down: $\vec{V}_{yi} = \langle - V_0 \sin 45^\circ \rangle = \langle - \frac{V_0}{\sqrt{2}} \rangle$

② Find time of flight based on vertical displacement: $\vec{\Delta y} = \langle -H \rangle$ for both phones

thrown down: $\vec{\Delta y} = \vec{V}_{iy} \Delta t + \frac{1}{2} \langle g \rangle \Delta t^2$

(✱) $\langle -H \rangle = \langle -\frac{V_0}{\sqrt{2}} \rangle \Delta t + \frac{1}{2} \langle -g \rangle \Delta t^2$

→ quadratic equation $\frac{1}{2} g \Delta t^2 + \frac{V_0}{\sqrt{2}} \Delta t - H = 0$

solution $\Delta t = \dfrac{-\frac{V_0}{\sqrt{2}} \pm \sqrt{\frac{V_0^2}{2} + 2gH}}{g}$ (choose positive root)

so $\boxed{\Delta t_{down} = \dfrac{-\frac{V_0}{\sqrt{2}} + \sqrt{\frac{V_0^2}{2} + 2gH}}{g}}$

also, from (✱) we see that Δt_{up} is found by replacing $\frac{-V_0}{\sqrt{2}} \to \frac{+V_0}{\sqrt{2}}$ (i.e. changing sign of $\vec{V}_{yi}$)

so $\boxed{\Delta t_{up} = \dfrac{\frac{+V_0}{\sqrt{2}} + \sqrt{\frac{V_0^2}{2} + 2gH}}{g}}$

Finally, distance between landing points is $\vec{\Delta X}_{up} - \vec{\Delta X}_{down} = \vec{V}_x \Delta t_{up} - \vec{V}_x \Delta t_{down}$

→ separation is $V_x (\Delta t_{up} - \Delta t_{down})$

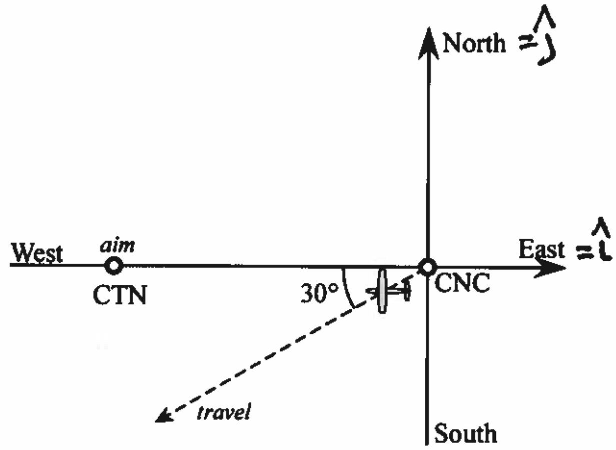
$= \frac{V_0}{\sqrt{2}} \left(\frac{\frac{V_0}{\sqrt{2}} + \cdots}{g} - \frac{-\frac{V_0}{\sqrt{2}} - \cdots}{g} \right) = \frac{V_0}{\sqrt{2}} \left(\frac{2V_0}{\sqrt{2} g} \right) = \boxed{\frac{V_0^2}{g}}$

The following problem will be hand-graded. *Show all supporting work for this problem.*

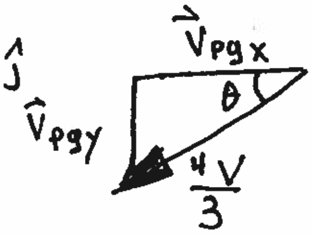
[III] *(20 points)* You are flying a plane west from Charlotte, NC to Chattanooga, TN. You aim the nose of your plane due west, and set the autopilot to maintain a constant airspeed v. After 15 minutes of travel, your GPS receiver informs you that due to the wind, you are actually traveling relative to the ground at a speed $4/3\,v$, in a direction 30° south of west (See figure). What direction should you actually point the plane, in order to fly due west along the ground?

Express your answer as a numerical angle to three-digit precision, measured relative to a cardinal direction (i.e. relative to North, South, East, or West).

Hint: start by figuring out the wind velocity.



① Aim due west: $\vec{V}_{plane\ to\ air} = \langle -V\rangle \hat{i}$



but $\vec{V}_{plane\ to\ ground} = \langle -\frac{4}{3}V\cos 30^\circ\rangle\hat{i} + \langle -\frac{4}{3}V\sin 30^\circ\rangle\hat{j}$

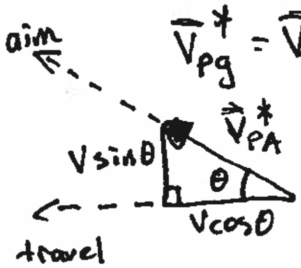
So, we can infer speed of wind, from

$$\vec{V}_{pg} = \vec{V}_{pa} + \vec{V}_{ag} \rightarrow \vec{V}_{ag} = \vec{V}_{pg} - \vec{V}_{pa}$$

→ wind velocity is $\vec{V}_{ag} = \langle -\frac{4}{3}V\cdot\frac{\sqrt{3}}{2}\rangle\hat{i} + \langle -\frac{4}{3}V\cdot\frac{1}{2}\rangle\hat{j} - \left[\langle -V\rangle\hat{i}\right]$

$$= \underbrace{\langle V(1-\tfrac{2}{\sqrt{3}})\rangle\hat{i}}_{\text{westward}} + \underbrace{\langle -\tfrac{2}{3}V\rangle\hat{j}}_{\text{southward}}$$

② To fly due west, you want plane to aim somewhat northward, to cancel southward wind component, so that new $\vec{V}_{pgy} = 0$



$\vec{V}_{pg}^* = \vec{V}_{PA}^* + \vec{V}_{Ag}$ → only y-component is important here:

$$\vec{V}_{pgy}^* = 0 = \vec{V}_{PAy}^* + \vec{V}_{Agy}$$

$$0 = \langle +V\sin\theta\rangle + \langle -\tfrac{2}{3}V\rangle$$

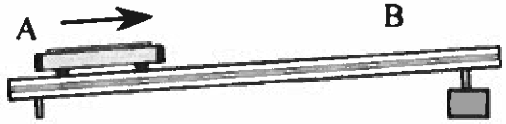
$$\Longrightarrow \sin\theta = \frac{2}{3}$$

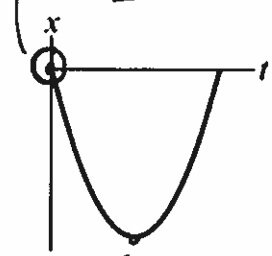
$$\boxed{\theta = \sin^{-1}\left(\tfrac{2}{3}\right) = 41.8^\circ}$$

Starts at A, at $x=0$

$\boxed{A = \text{origin}}$

The next two questions involve the following situation:
A cart is given an initial shove up an inclined ramp. The cart starts at A, coasts up the track and stops at B, and then returns back down the track to A. An observer collects position data and constructs the x-vs-t graph shown at right.





Question value 4 points

(1) What coordinate system was the observer using to collect data?

(a) The coordinate system that was used cannot be inferred from the graph.

(b) ~~A system with the origin at B~~ and the positive direction pointing *towards* A.

$\boxed{\text{(c)}}$ A system with the origin at A and the positive direction pointing *away* from B.

(d) ~~A system with the origin at B~~ and the positive direction pointing *away* from A.

(e) A system with the origin at A and the positive direction pointing *towards* B.

at B:
$\vec{x}$ takes maximum <u>negative</u> value

→ From A to B = negative

→ $\boxed{\text{Positive direction is away from B}}$

or: downslope = positive
Upslope = negative

Question value 4 points

(2) At what point (if any) during the motion does the acceleration of the cart have a *negative* value?

(q) The acceleration is negative *only* as it moves from A to B.

(b) The acceleration is negative while it is moving from A to B, and from B to A, but *not* at the moment that it is stopped at B.

(c) At all points during the cart's motion the acceleration is negative.

(d) At no point during the cart's motion is the acceleration negative.

(e) The acceleration is negative *only* as it moves from B to A.

From prior question, we have established "negative" means "upslope"

so: <u>When</u> is acceleration directed upslope?

→ <u>NOT</u> going from A to B: velocity upslope, slowing down = downslope accel

→ <u>NOT</u> at B: velocity changing from upslope to downslope = downslope accel

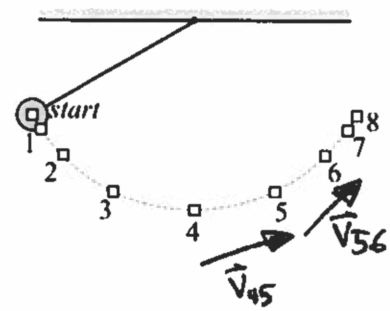
→ <u>NOT</u> going from B to A: velocity downslope, speeding up = downslope accel

$\vec{a}$ = downslope at all times $\Rightarrow$ $\vec{a}$ is positively directed/positively-valued at all times

$\boxed{\vec{a} \text{ is } \underline{\text{never}}\text{: negatively directed/negatively valued}}$

Question value 8 points

(3) The figure at right displays a motion diagram for a pendulum that is released at time zero. Which of the arrows below *best* characterizes the direction of the acceleration vector for the pendulum bob during frame #5?



$\boxed{\text{(a) ↖}}$ best

(b) ↓ way off

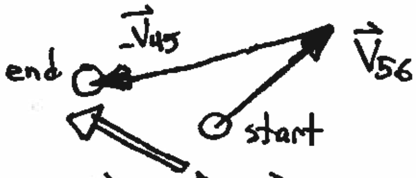
(c) ↗ way off

(d) ← not bad

(e) ↘ way off

direction of $\vec{a}_5$ can be found by comparing $\vec{V}_{45}$ and $\vec{V}_{56}$, using graphical construction

$$\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t} \rightarrow \vec{a} \sim \vec{V}_{56} - \vec{V}_{45} = \vec{V}_{56} + (-\vec{V}_{45})$$



$$\vec{a} \sim \vec{V}_{56} - \vec{V}_{45}$$

Question value 8 points

(4) You are the passenger in a car that is traveling 25° north of east at speed v. The driver suddenly steps on the accelerator and veers left. As this is happening, the car experiences an acceleration of magnitude a, directed 60° north of east. At what rate is the speed of the car increasing, as this occurs?

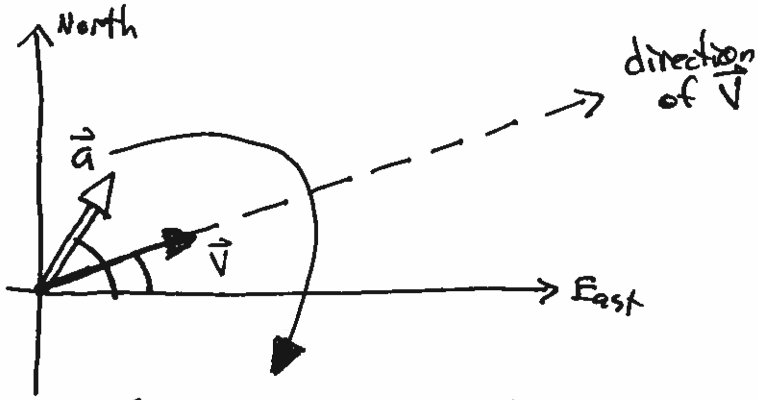
~~(a) $v \cos 60°$~~ wrong units

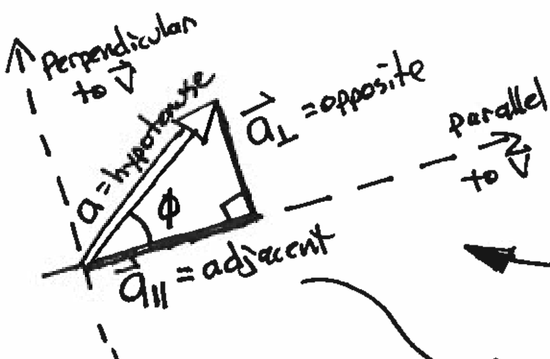
$\boxed{\text{(b) } a \cos 35°}$

(c) $a \cos 60°$

(d) $a \cos 85°$

~~(e) $v \cos 25°$~~ wrong units





recall $\vec{a}_{\perp \text{ to } \vec{v}}$ causes turning

$\vec{a}_{\parallel \text{ to } \vec{v}}$ causes speed change

we see that $|\vec{a}_\parallel| = |\vec{a}| \cos 35°$

so rate of speed change $= \boxed{a \cos 35°}$

angle between $\vec{a}$ and $\vec{v}$ is $\phi = 60° - 25° = \boxed{35°}$

Question value 8 points

(5) A car and a train move together along straight, parallel paths with the same initial speed. The car driver notices a red light ahead of him, and slows down with constant acceleration of magnitude a_{stop}. He comes to a stop right as he reaches the light, after an elapsed time $\Delta t_{stop} = T$. At that moment, the light turns green, and he begins to accelerate. What acceleration magnitude would allow him to pass the train after a further elapsed time $\Delta t_{go} = T$ after the light turns green? (*Hint: plot v-vs-t for both vehicles.*)

(a) $a_{go} = 3/2\, a_{stop}$

$\boxed{\text{(b) } a_{go} = 3\, a_{stop}}$

(c) $a_{go} = 2\, a_{stop}$

not nearly enough!

~~(d) $a_{go} = a_{stop}$~~

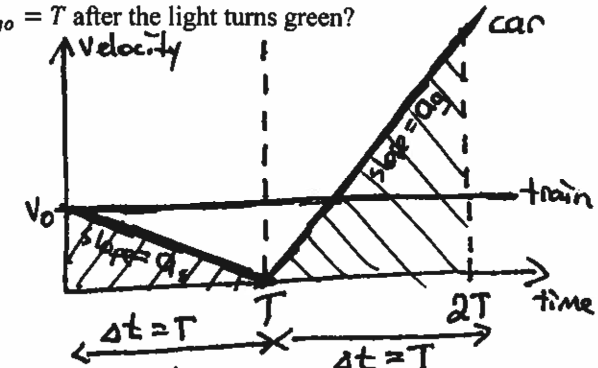
~~(e) $a_{go} = 2/3\, a_{stop}$~~

let V_0 = train speed = initial car speed

① stopping car: $\vec{\Delta v} = \vec{a}\Delta t$

$\langle -V_0 \rangle = \langle -a_{stop} \rangle \Delta t$

$\boxed{V_0 = a_s T}$



② Train displacement: $\vec{\Delta X_T} = \vec{V}\Delta t = V_0 \cdot 2T = \boxed{2a_s T^2}$

③ Total car displacement must match this, to catch train but displacement = areas = triangles, in graph

$$\vec{\Delta X_c} = \vec{\Delta X_{stop}} + \vec{\Delta X_{go}} = \tfrac{1}{2}(a_s T)T + \tfrac{1}{2}(a_g T)T = \tfrac{1}{2}a_s T^2 + \tfrac{1}{2}a_g T^2$$

so $\vec{\Delta X_c} = \vec{\Delta X_T} \rightarrow \tfrac{1}{2}a_s T^2 + \tfrac{1}{2}a_g T^2 = 2a_s T^2 \rightarrow \tfrac{1}{2}a_g T^2 = \tfrac{3}{2}a_s T^2$

$$\boxed{a_g = 3a_s}$$

Question value 8 points

(a) Clara walks east to the bank at a constant speed v. When she gets there, she realizes she left her ATM card at home, so she runs back home at a constant speed $2v$. What is her average speed during the round trip? → NOT "average velocity"!

(a) 3/2 v

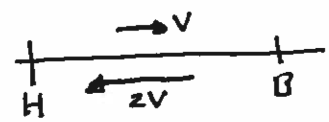
~~(b) zero~~

~~(c) 2/3 v~~ less than V??

(d) 5/3 v

$\boxed{\text{(e) } 4/3\, v}$

Let D = distance from home to bank



- time to reach bank: $\vec{V} = \frac{\vec{\Delta x}}{\Delta t} \rightarrow \Delta t = \frac{|\vec{\Delta x}|}{|\vec{V}|} = \frac{D}{V}$
- time to return: $\Delta t = \frac{|\vec{\Delta x}|}{|\vec{V}|} = \frac{D}{2V}$

total time for round trip is $\Delta t_{TOT} = \frac{D}{V} + \frac{D}{2V}$

$$\boxed{\Delta t_{TOT} = \frac{3D}{2V}}$$

Now, avg speed = $\frac{\text{total distance}}{\text{total time}}$

- total distance = round trip = 2D
- total time is per above

$$V_{av} = \frac{2D}{\frac{3D}{2V}} = 2D \cdot \frac{2V}{3D} = \frac{4V\cancel{D}}{3\cancel{D}} = \boxed{\frac{4}{3}V}$$