Solutions Printed Name	Nine-digit GT ID

signature

Spring 2019 **PHYS 2211 ABC** **Test 01**

Test Form: **1A**

- Put nothing other than your name and nine-digit GT ID in the blocks above. Print clearly so that OCR software can properly identify you. Sign your name on the line immediately below your printed name.
- Free-response problems are numbered I–III. Show all your work clearly, including all steps and logic. Write darkly. Blue or black ink is recommended. Do not make any erasures in your free-response work. Cross out anything you do not want evaluated. Box your answer.
- Multiple-choice questions are numbered 1–6. For each, select the answer most nearly correct, circle it on yourtest, and fill the bubble for your answer on this front page.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated.
- The standard formula sheet is on the back of this page, which may be removed from the quiz form if you wish, but it must be submitted.
- If the page for a free-response problem has insufficient space for your work, ask a proctor for an additional sheet. If you wish this work to be evaluated, put your name on the sheet and make a note on the problem page, so graders know where to find your work. Place any added pages at the **back** of your test, when submitting your exam.
- You may use a calculator that cannot store letters, but no other aids or electronic devices.
- Scores will be posted when your test has been graded. Test grades become final when the next is given.

Fill in bubbles for your Multiple Choice answers darkly and neatly.

1 ⓐ ● ⓒ ⓓ ⓔ

2 ⓐ ⓑ ⓒ ● ⓔ

3 ● ⓑ ⓒ ⓓ ⓔ

4 ⓐ ⓑ ⓒ ● ⓔ

5 ⓐ ⓑ ⓒ ⓓ ●

6 ● ⓑ ⓒ ⓓ ⓔ

The following problem will be hand-graded. Show all supporting work for this problem.

[1] *(20 points)* The Tortoise and the Hare are in a footrace covering a total straight-line distance D. The Tortoise walks at a constant speed v and the Hare runs at a constant speed $3v$. At the start of the race, the Hare shows off by intentionally running in the wrong direction. When he sees the Tortoise reach the halfway mark, he turns around and starts running toward the finish line. The Tortoise wins the race, but the Hare continues to the finish line anyway, at speed $3v$.

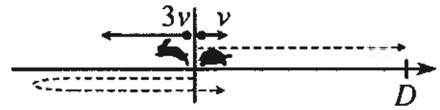
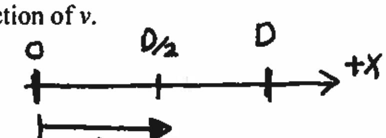


figure is not necessarily to scale

What is the Hare's average velocity during the time interval required for the hare to reach the finish line? Express your answer as a velocity vector with a magnitude that is some multiple or fraction of v.

① Find time for tortoise to travel $D/2$, at constant speed V

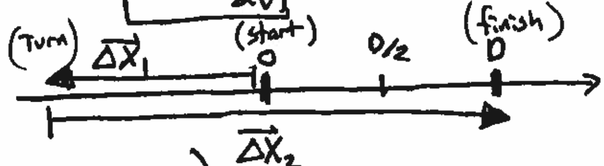


$\vec{\Delta X}_T = \langle +D/2 \rangle$ in some time Δt_1

$\vec{\Delta x} = \vec{V}\Delta t \rightarrow \langle +D/2 \rangle = \langle +V \rangle \Delta t_1$

$\rightarrow \boxed{\Delta t_1 = \frac{D}{2V}}$

② Find wrong-way displacement of hare, at constant speed $3V$



Hare:
$\vec{\Delta X}_1$ = wrong-way displacement
$\vec{\Delta X}_2$ = right-way displacement

$\vec{\Delta X}_1 = \vec{V}_1 \Delta t_1 \rightarrow \vec{\Delta X}_1 = \langle -3V \rangle (D/2V) \rightarrow \boxed{\vec{\Delta X}_1 = \langle -\tfrac{3}{2} D \rangle}$

③ To get to finish line, hare must displace $\vec{\Delta X}_2 = \langle +\tfrac{5}{2} D \rangle$ in some time Δt_2

$\vec{\Delta X}_2 = \langle +3V \rangle \Delta t_2 \rightarrow \langle +\tfrac{5}{2} D \rangle = \langle +3V \rangle \Delta t_2 \rightarrow \boxed{\Delta t_2 = \frac{5D}{6V}}$

④ Total time for Hare to reach finish line: $\Delta t_{TOT} = \Delta t_1 + \Delta t_2$

$$= \frac{D}{2V} + \frac{5D}{6V}$$

$$= \frac{3D}{6V} + \frac{5D}{6V}$$

$$\boxed{\Delta t_{TOT} = \frac{8D}{6V} = \frac{4D}{3V}}$$

⑤ Hare's average velocity during entire run:

$$\vec{V}_{av} = \frac{\vec{\Delta X}_{TOT}}{\Delta t_{TOT}} = \frac{\langle +D \rangle}{(4D/3V)}$$

$$= \left\langle + \frac{\cancel{D} \cdot 3V}{4\cancel{D}} \right\rangle$$

$$\boxed{\vec{V}_{av} = \langle +\tfrac{3}{4} V \rangle} \rightarrow \text{answer is a vector: sign and magnitude (direction)}$$

ie slower than tortoise!

The following problem will be hand-graded. Show all supporting work for this problem.

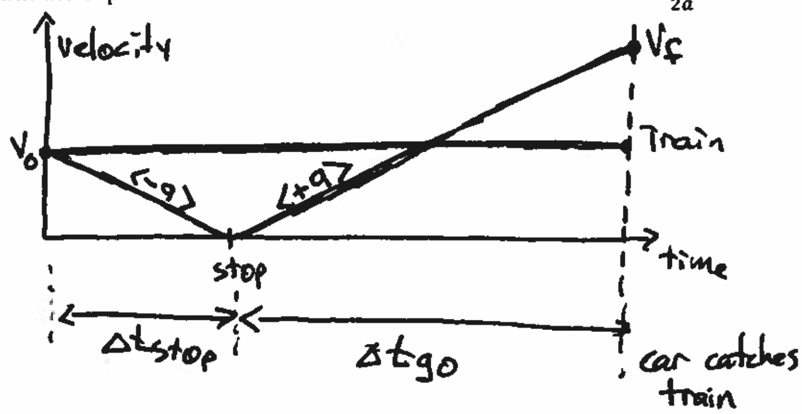
[II] *(20 points)* A car and a train move together along straight, parallel paths with the same initial speed v_o. At time $t = 0$ the car driver notices a red light ahead and slows down with constant acceleration of magnitude a. Just as the car comes to a full stop, the light immediately turns green, and the car then speeds up with a constant acceleration of the same magnitude a, continuing until it catches up to the train. During the same time interval, the train continues to travel at the constant speed v_o.

What total distance does the car travel before catching back up to the train? How fast is it travelling at that moment? Express both answers in terms of the parameters v_0 and a.

Hint: the expression $au^2 + bu + c = 0$ has solutions $u = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

- Car and Train must experience the same overall displacements

$$\vec{\Delta X_c} = \vec{\Delta X_T}$$



① Car requires two intervals

stopping: $\vec{V_i} = \langle +V_0 \rangle$, $\vec{V_f} = \langle 0 \rangle$

$$\vec{\Delta V} = \langle -V_0 \rangle = \langle -a \rangle \Delta t_s$$

$\boxed{\Delta t_s = V_0/a}$ → use this to find displacement while stopping:

$$\vec{\Delta X_{c,s}} = \vec{V_i}\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2 \rightarrow \langle +V_0 \rangle \left(\frac{V_0}{a}\right) + \frac{1}{2}\langle -a \rangle \left(\frac{V_0}{a}\right)^2$$

$$\vec{\Delta X_{c,s}} = +\left\langle \frac{V_0^2}{a} - \frac{1}{2}\frac{V_0^2}{a} \right\rangle = \boxed{\left\langle + \frac{V_0^2}{2a} \right\rangle}$$

going after stop: $(\vec{V_i} \text{ now } = 0)$

$$\vec{\Delta X_{c,g}} = \vec{V_i}\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2 \rightarrow \boxed{\vec{\Delta X_{c,g}} = \tfrac{1}{2}\langle +a \rangle \Delta t_2^2}$$

② train has constant velocity during these two intervals: $\vec{\Delta X_T} = \langle +V_0 \rangle (\Delta t_1 + \Delta t_2)$

$$= \langle +V_0 \rangle \left(\frac{V_0}{a} + \Delta t_2\right)$$

③ Set displacements to be equal:

$$\vec{\Delta X_{c,TOT}} = \vec{\Delta X_T} \rightarrow \frac{V_0^2}{2a} + \tfrac{1}{2}a\Delta t_2^2 = \frac{V_0^2}{a} + V_0 \Delta t_2$$

$$\tfrac{1}{2}a\Delta t_2^2 - V_0\Delta t_2 - \frac{V_0^2}{2a} = 0 \qquad \Delta t_2 = \frac{+V_0 \pm \sqrt{V_0^2 - 4(\tfrac{1}{2}a)\left(\frac{-V_0^2}{2a}\right)}}{2(\tfrac{1}{2}a)}$$

$$\Rightarrow \boxed{\Delta t_2 = \frac{V_0(1+\sqrt{2})}{a}} \rightarrow \text{then } \vec{\Delta X_{c,TOT}} = \vec{\Delta X_T} = V_0\left(\frac{V_0}{a} + \frac{V_0}{a}(1+\sqrt{2})\right)$$

$$\boxed{|\vec{\Delta X_c}| = \frac{V_0^2}{a}(2+\sqrt{2})}$$ total distance travelled by car

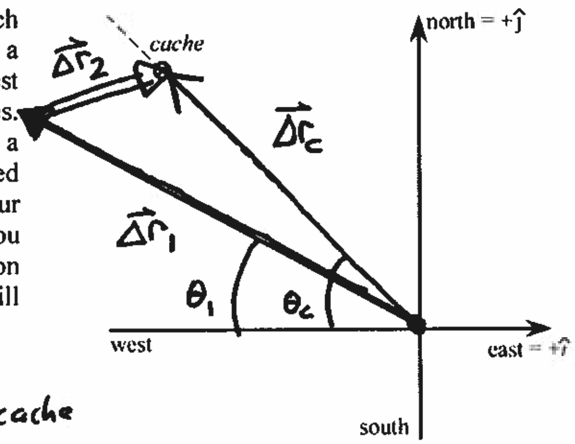
Final speed is $\vec{V_f} = \cancel{\vec{V_0}}^{\,0} + \vec{a}\Delta t_2$

$$V_f = (+a)\left(\frac{V_0}{a}\right)(1+\sqrt{2})$$

$$\boxed{V_f = V_0(1+\sqrt{2})}$$ speed of car as it passes train

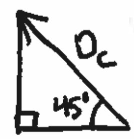
The following problem will be hand-graded. *Show all supporting work for this problem.*

[III] *(20 points)* You are a cast member on a reality TV show in which contestants must navigate in the wilderness using only a map, a compass and a stopwatch. Your task is to travel 5.0 miles northwest (i.e. exactly 45° north of west) to retrieve a cache of water and supplies. However, after traveling for 1.4 hrs at a steady speed of 5.0 mph, in a direction that you believe to be northwest, you still have not reached the cache. Examining the compass closely, you realize that one of your rivals has sabotaged it using a tiny magnet. Removing the magnet, you realize that you have really been travelling 10° off course, in a direction 35° north of west. What displacement (magnitude and direction) will take you directly to the cache? [Note: two-digit precision]



- let $\vec{\Delta r_c}$ be correct displacement straight to cache

$$\vec{\Delta r_c} = D_c \text{ at angle } \theta_c = 45^\circ = \langle -D_c \cos 45^\circ \rangle \hat{i} + \langle + D_c \sin 45^\circ \rangle \hat{j}$$



- let $\vec{\Delta r_1}$ = initial incorrect displacement

distance $D_1 = V_1 \Delta t = 7.0 \text{ mi}$

so $\vec{\Delta r_1} = \langle -D_1 \cos\theta_1 \rangle \hat{i} + \langle +D_1 \sin\theta_1 \rangle \hat{j}$

- let $\vec{\Delta r_2}$ = displacement that gets you to cache. From figure above:

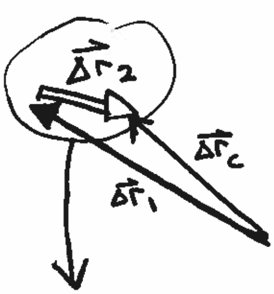
$$\vec{\Delta r_c} = \vec{\Delta r_1} + \vec{\Delta r_2}$$

$$\boxed{\vec{\Delta r_2} = \vec{\Delta r_c} - \vec{\Delta r_1}}$$

$$= \left[\langle -D_c \cos 45^\circ \rangle \hat{i} + \langle +D_c \sin 45^\circ \rangle \hat{j} \right] - \left[\langle -D_1 \cos\theta_1 \rangle \hat{i} + \langle +D_1 \sin\theta_1 \rangle \hat{j} \right]$$

$$= \left[\langle -3.53 \text{ mi} \rangle \hat{i} + \langle +3.53 \text{ mi} \rangle \hat{j} \right] - \left[\langle -5.73 \text{ mi} \rangle \hat{i} + \langle +4.02 \text{ mi} \rangle \hat{j} \right]$$

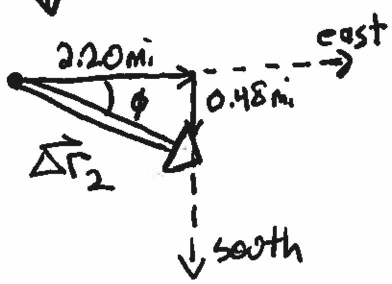
$$= \langle +2.20 \text{ mi} \rangle \hat{i} + \langle -0.48 \text{ mi} \rangle \hat{j}$$



Note that figure above was a "guess", and is not fully accurate
→ $\vec{\Delta r_2}$ is "east and south", actually – like figure at left

$$|\vec{\Delta r_2}| = \sqrt{(2.20 \text{ mi})^2 + (-0.48 \text{ mi})^2} = 2.25 \text{ mi}$$

rounds to $\boxed{2.3 \text{ mi}}$



direction angle is

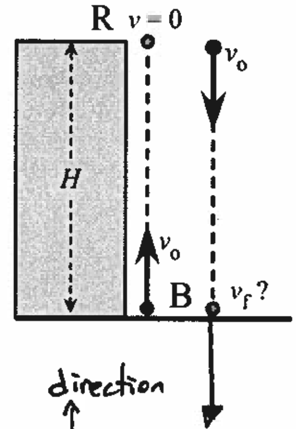
$$\phi = \tan^{-1}\left(\frac{0.48 \text{ mi}}{2.20 \text{ mi}}\right) = 12.3^\circ$$

rounds to $\boxed{12^\circ \text{ south of East}}$

relevant axes must be included!

Question value 8 points

(01) Rasputin stands on the roof of a building of height H, while Bernoulli stands at the base of the building. Bernoulli throws an avocado straight upward with an initial speed v_o that is just fast enough for the avocado to reach the roof, where Rasputin catches it as it comes (momentarily) to rest. If Rasputin throws the avocado back down at Bernoulli with the same speed v_o that it was thrown upward, with what speed will it hit the ground?



~~(a) $v_f = 2v_o$~~

(b) $\boxed{v_f = \sqrt{2}\, v_o}$

(c) $v_f = 2\, v_o$

(d) $v_f = 4\, v_o$

(e) $v_f = v_o$

① Throw upward: $\vec{v}_i = \langle +v_0 \rangle,\ \vec{v}_f = 0$

Speed equation gives:

$$v_f^2 = v_i^2 + 2\vec{a}\Delta\vec{y}$$

$$0 = v_0^2 + 2\langle -g\rangle\langle +H\rangle \rightarrow \boxed{v_0^2 = 2gH}$$

direction ↑

② Throw downward from roof: $\vec{v}_i = \langle -v_0 \rangle$, $\vec{v}_f = \langle -v_f \rangle$

(vector) ↳magnitude

$$v_f^2 = v_i^2 + 2\vec{a}\Delta\vec{y}$$

$$v_f^2 = v_0^2 + 2\langle -g\rangle\langle -H\rangle = v_0^2 + 2gH$$

but we just saw that $2gH = v_0^2$

$$\text{so } v_f^2 = v_0^2 + (v_0^2) = 2v_0^2 \rightarrow \boxed{v_f = \sqrt{2}\, v_0}$$

Question value 8 points

(02) You are in a boat that is travelling at speed v, in a direction 25° west of north. To your south, the shoreline runs along a line that is oriented 10° south of east. At what rate is your boat moving out to sea, *straight away from shore*?

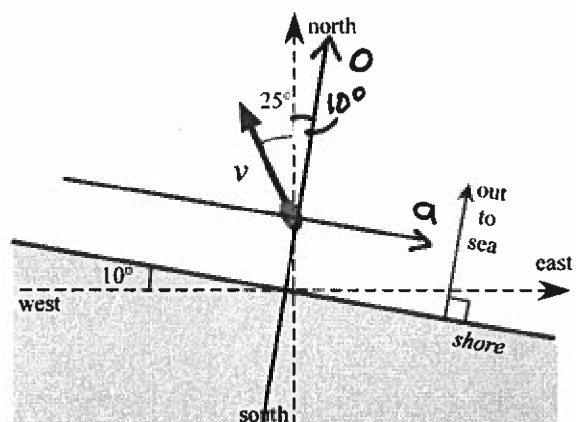
(a) $v \cos 80°$

(b) $v \cos 65°$

(c) $v \cos 55°$

(d) $\boxed{v \cos 35°}$

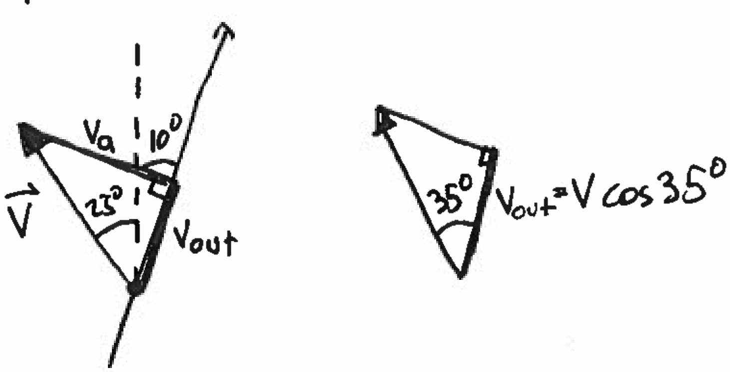
(e) $v \cos 25°$



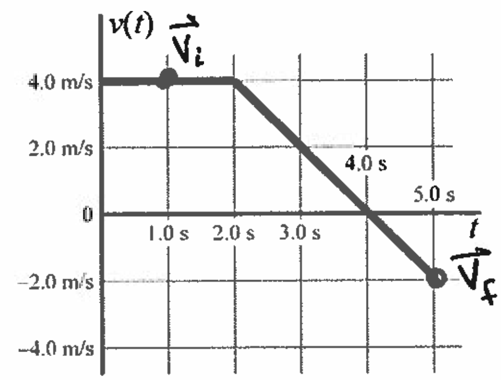
Establish coord system with axes:

- out to sea (o-axis)
- along shore (a-axis)

⇒ Goal is to find "o-component" of $\vec{v}$



The next two questions involve the following situation:
A particle moves along the x-axis. At time $t = 0$, it is located at $\vec{x}_o = \langle -3.0\ \text{m}\rangle$. Its velocity-versus-time graph is plotted at right.



Question value 4 points

(03) What is the average acceleration of the particle, between times $t = 1.0$ sec and $t = 5.0$ sec?

$\boxed{\text{(a)}\quad \vec{a} = \langle -1.5\ \text{m}/s^2\rangle}$

(b) $\vec{a} = \langle -2.5\ \text{m}/s^2\rangle$

(c) $\vec{a} = \langle +5.5\ \text{m}/s^2\rangle$

(d) $\vec{a} = \langle -2.0\ \text{m}/s^2\rangle$

(e) $\vec{a} = \langle +2.5\ \text{m}/s^2\rangle$

Definition:
$$\vec{a}_{av} = \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t}$$

From figure:
$$\vec{V}_f = \langle -2.0\,m/s\rangle$$
$$\vec{V}_i = \langle +4.0\,m/s\rangle$$
so $\Delta\vec{V} = \langle -2\,m/s\rangle - \langle +4\,m/s\rangle$
$$\Delta\vec{V} = \langle -6.0\,m/s\rangle$$

$$\vec{a}_{av} = \frac{\langle -6.0\,m/s\rangle}{4.0\,sec}$$

$$\boxed{\vec{a}_{av} = \langle -1.5\,m/s^2\rangle}$$

Question value 4 points

(04) Where will the particle be at time $t = 5.0$ sec?

(a) $\vec{x}_f = \langle +12\ \text{m}\rangle$

(b) $\vec{x}_f = \langle +5.0\ \text{m}\rangle$

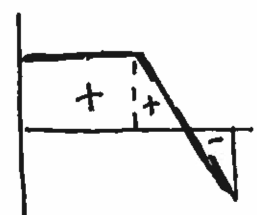
(c) $\vec{x}_f = \langle +7.0\ \text{m}\rangle$

$\boxed{\text{(d)}\quad \vec{x}_f = \langle +8.0\ \text{m}\rangle}$

(e) $\vec{x}_f = \langle +11\ \text{m}\rangle$

$$\vec{X}_f = \vec{X}_i + \Delta\vec{X}$$ where:

$$\vec{X}_i = \langle -3.0\,m\rangle$$
$\Delta\vec{X}$ = displacement
= area under curve



area = pos rectangle = $(+2s)(+4\,m/s)$
+ pos triangle + $\frac{1}{2}(2s)(4\,m/s)$
+ neg triangle + $(-)\frac{1}{2}(1s)(2\,m/s)$

net area $= \langle +11\ m\rangle$

$$\vec{X}_f = \langle -3.0\,m\rangle + \langle +11\,m\rangle$$

$$\boxed{\vec{X}_f = \langle +8.0\,m\rangle}$$

Question value 8 points

(05) A sportscar moves in a straight line, with its velocity given by the expresssion $v(t) = At - Bt^2$, where the quantities A and B are both positively-valued constants. It starts at the origin at time $t = 0$. What is the maximum speed of the car, during the time it is travelling in the positive direction?

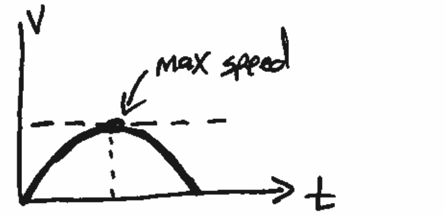
(a) $v_{\max} = \frac{A^2}{B}$

(b) $v_{\max} = \frac{B}{2A}$

(c) $v_{\max} = \frac{A^2}{2B}$

(d) $v_{\max} = \frac{B}{4A}$

(e) $\boxed{v_{\max} = \frac{A^2}{4B}}$



$V(t) = At - Bt^2$

max speed when $a = \frac{dV}{dt} \to 0$

$a = \frac{dV}{dt} = A - 2Bt$

$a = 0$ when $V = \max$, at time t_m

$0 = A - 2Bt_m \rightarrow \boxed{t_m = \frac{A}{2B}}$ plug back in to expression for $V(t)$

$V_{max} = V(t_m)$
$= A\left(\frac{A}{2B}\right) - B\left(\frac{A}{2B}\right)^2 = \frac{A^2}{2B} - \frac{A^2}{4B}$

$\boxed{V_{max} = \frac{A^2}{4B}}$

Question value 8 points

(06) A circus acrobat steps off a platform and drops straight down onto a trampoline, and then bounces straight back up to her original position. At the moment when she is at her lowest point of her "bounce" (i.e. with the maximum sag in the trampoline), the acrobat's acceleration is necessarily...

(a) $\boxed{\text{...upward.}}$

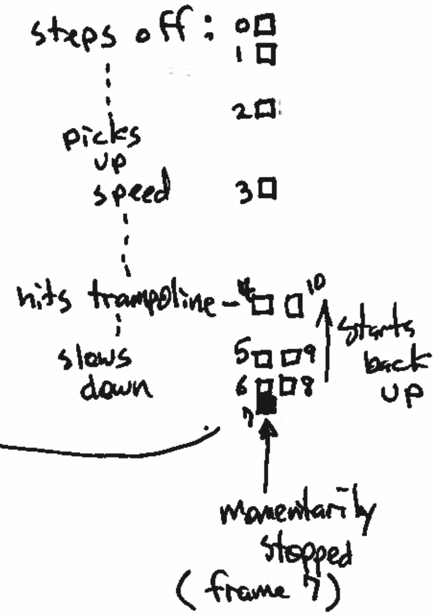
(b) ...zero.

(c) ...negative.

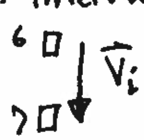
(d) ...positive.

(e) ...downward.

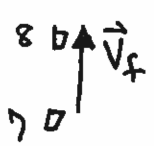
Construct a motion diagram



• last interval before stopping



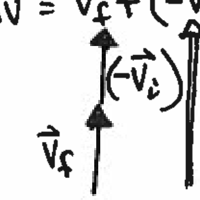
• First interval after stopping



$\vec{a} = \Delta\vec{V}/\Delta t$

→ construct $\Delta\vec{V}$ to get $\vec{a}$

$\Delta\vec{V} = \vec{V}_f + (-\vec{V}_i)$



$\Delta\vec{V} =$ up, so $\boxed{\vec{a} = \text{up}}$