Physics 2211A/B Test form 746 Name Solutions

Fall 2017

Test 4 Recitation Section (see cover page):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

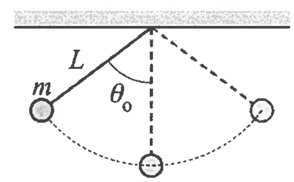
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

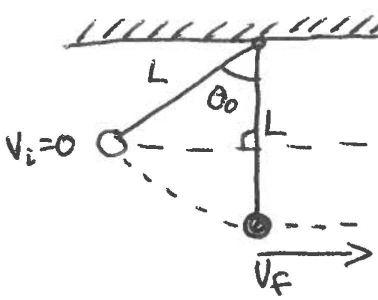
[I] *(20 points)* A simple pendulum consists of a ball of mass m attached to a string of length L. The mass is raised to an angle $\theta_o = 53.1°$ from the vertical, and released from rest. What should be the test strength (i.e. the maximum tension the cord can sustain) in order for the mass to swing through a full arc without breaking? Express your answer as a multiple of the ball's weight, mg.



Hint: use conservation of energy as part of your solution.

Tension will be greatest when speed of ball is greatest — at bottom of arc

Set ceiling to be $y=0$



Conservation of Energy

$K_i=0$

$U_i=-\frac{3}{5}mgL$

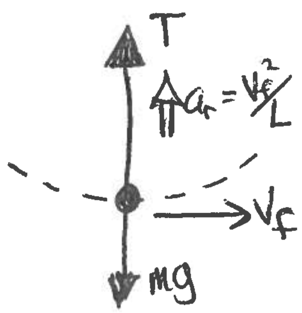
$K_f=\frac{1}{2}mv_f^2$

$U_f=-mgL$

$K_i+U_i=K_f+U_f$

$0+(-\frac{3}{5}mgL)=\frac{1}{2}mv_f^2+(-mgL) \rightarrow \frac{1}{2}mv_f^2=\frac{2}{5}mgL \rightarrow \boxed{v_f^2=\frac{4}{5}gL}$

Now analyze forces at lowest point: 2nd Law / circular motion



radial accel is $a_r=v_f^2/L=\frac{4}{5}g$

so $\Sigma\vec{F}_r=m\vec{a}_r \rightarrow \langle +T\rangle+\langle -mg\rangle=m\langle +\frac{4}{5}g\rangle$

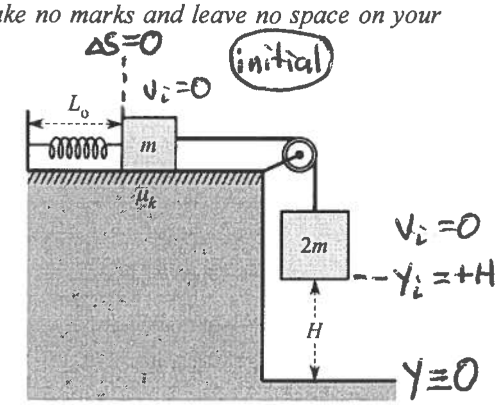
$T=mg+\frac{4}{5}mg$

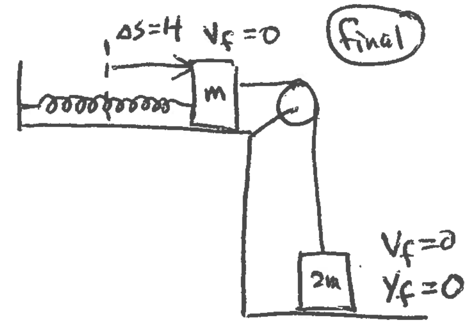
$\boxed{T=\frac{9}{5}mg \text{ or } 1.8mg}$

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] *(20 points)* In the figure at right, block m rests on a rough surface with coefficient of friction $\mu_k = 0.25$. Block m is attached to the wall by a spring that is initially at its natural length L_o. It is also attached, via an ideal cord, to a hanging block of mass $2m$ that is a height H above the ground. The blocks are released from rest and begin to move—picking up speed at first, but eventually slowing to a stop at the exact moment block $2m$ reaches the ground.

Use the Work-Energy Principle to determine the elastic constant of the spring. Express your answer in terms of g, m and H.





Both blocks begin and end at rest

$\Rightarrow \Delta K_{system} = 0$

Treating system as "blocks + cord",
we have spring, friction, gravity doing work

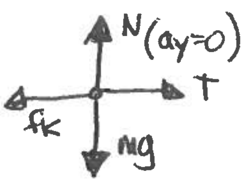
① Spring: $\Delta S_i = 0$, $\Delta S_f = +H$

$$W_s = -\tfrac{1}{2}k(\Delta S_f)^2 + \tfrac{1}{2}k(\Delta S_i)^2 = \boxed{-\tfrac{1}{2}kH^2}$$

② gravity (block 2m) $Y_i = +H$, $Y_f = 0$, $\vec{\Delta y} = \langle -H \rangle$

$$W_g = \vec{F_g} \cdot \vec{\Delta y} = \langle -2mg \rangle \langle -H \rangle = \boxed{+2mgH}$$

③ friction (block m)



$f_k = \mu_k N = \mu_k mg = \tfrac{1}{4}mg$ ($\mu_k = 0.25$)

$\rightarrow W_f = -f_k \Delta x = \boxed{-\tfrac{1}{4}mgH}$ (or $\Delta E_{TH} = +\tfrac{1}{4}mgH$)

Now apply work-energy principle: $\Delta E_{sys} = W_{ext}$

$\Delta K = W_s + W_g + W_f$ (or $\Delta K + \Delta E_{Th} = W_s + W_g$)

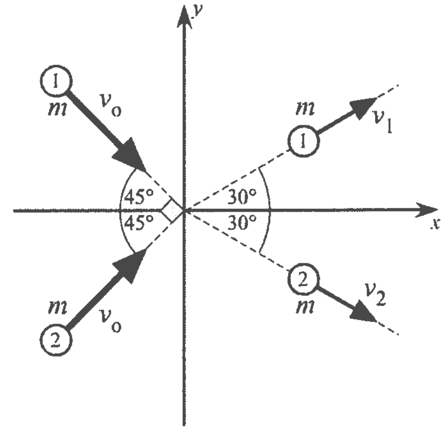
$$\rightarrow 0 = -\tfrac{1}{2}kH^2 + 2mgH - \tfrac{1}{4}mgH$$

$$\tfrac{1}{2}kH^2 = \tfrac{8}{4}mgH - \tfrac{1}{4}mgH = \tfrac{7}{4}mgH$$

$$\boxed{k = \frac{7mg}{2H}}$$

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[III] *(20 points)* Two identical pucks are sliding along a surface. At the moment they collide, they are moving at right angles to one another as shown, having the same speed v_0. After the collision, they are seen to be travelling with velocities directed at angles of 30.0° above and below the x-axis.

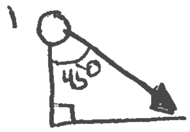


Determine the amount of kinetic energy **lost** in this collision (where $K_{lost} = |\Delta K| = |K_f - K_i|$). Express your answer as a fraction of the initial kinetic energy of the system—for example: "$K_{lost} = (3/7)K_i$".

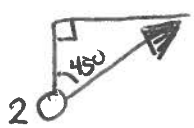
Hint: start by thinking about the y-direction...

$\vec{P}_i = \vec{P}_f$ in a collision

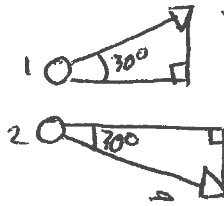
$\to \vec{P}_{ix} = \vec{P}_{fx}$ and $\boxed{\vec{P}_{iy} = \vec{P}_{fy}}$ do this first



$\vec{V}_{1i} = +\frac{1}{\sqrt{2}}V_0\hat{i} - \frac{1}{\sqrt{2}}V_0\hat{j}$



$\vec{V}_{2i} = +\frac{1}{\sqrt{2}}V_0\hat{i} + \frac{1}{\sqrt{2}}V_0\hat{j}$



$\vec{V}_{1f} = +\frac{\sqrt{3}}{2}V_1\hat{i} + \frac{1}{2}V_1\hat{j}$

$\vec{V}_{2f} = \frac{\sqrt{3}}{2}V_2\hat{i} - \frac{1}{2}V_2\hat{j}$

$\vec{P}_y$ is conserved: $\vec{P}_{iy} = \vec{P}_{fy}$

$$\underbrace{\left(-\frac{mV_0}{\sqrt{2}}\right) + \left(+\frac{mV_0}{\sqrt{2}}\right)}_{0} = \left(+\frac{mV_1}{2}\right) + \left(-\frac{mV_2}{2}\right)$$

$$0 = \frac{m}{2}(V_1 - V_2) \to \boxed{V_1 = V_2}$$

same final speed — so call both of these speeds "V_f"

$\vec{P}_x$ is conserved:

$$\left(+\frac{mV_0}{\sqrt{2}}\right) + \left(+\frac{mV_0}{\sqrt{2}}\right) = \left(+\frac{\sqrt{3}}{2}mV_f\right) + \left(\frac{\sqrt{3}}{2}mV_f\right)$$

$$\sqrt{2}\,\cancel{m}V_0 = \sqrt{3}\,\cancel{m}V_f \qquad \boxed{V_f = \sqrt{\tfrac{2}{3}}\,V_0}$$

Now – compare initial / final KE: $K_i = 2\left(\frac{1}{2}mV_0^2\right) = mV_0^2$

$$K_f = 2\left(\tfrac{1}{2}mV_f^2\right) = 2\left(\tfrac{1}{2}m\left[\sqrt{\tfrac{2}{3}}V_0\right]^2\right)$$

$$\to K_f = \frac{2}{3}mV_0^2$$

$\Rightarrow$ 2/3 of original KE remains, so $\boxed{\Delta K_{lost} = \frac{1}{3}K_i}$

The next two questions involve the following situation:
A particle moves in 2D while subject to only a single conservative force, with a potential energy given by the expression

$$U(x, y) = Ax^3y^2 - 2Ax^2y^3$$

Question value 4 points
(1) What is the x-component of the force on the particle when it is at position $\langle x, y\rangle = \langle -d, +d\rangle$?

(a) $\vec{F}_x = \langle +3Ad^4\rangle$
(b) $\vec{F}_x = \langle +7Ad^4\rangle$
(c) $\vec{F}_x = \langle 0\rangle$
(d) $\vec{F}_x = \langle -3Ad^4\rangle$
(e) $\boxed{\vec{F}_x = \langle -7Ad^4\rangle}$

$$\vec{F}_x = \left\langle -\frac{dU}{dx}\right\rangle_{y \text{ held constant}}$$ Note the minus sign!

$$= \left\langle -Ay^2\frac{d}{dx}[x^3] + 2Ay^3\frac{d}{dx}[x^2]\right\rangle$$

$$= \langle -3Ay^2x^2 + 4Ay^3x\rangle$$

→ evaluate at $x = -d$, $y = +d$

$$\vec{F}_x = \langle -3A\,d^2d^2 + 4Ad^3(-d)^2\rangle$$

$$\boxed{\vec{F}_x = \langle -7Ad^4\rangle}$$

Question value 4 points
(2) What is the y-component of the force on the particle when it is at position $\langle x, y\rangle = \langle -d, +d\rangle$?

(a) $\vec{F}_y = \langle +3Ad^4\rangle$
(b) $\boxed{\vec{F}_y = \langle +8Ad^4\rangle}$
(c) $\vec{F}_y = \langle -3Ad^4\rangle$
(d) $\vec{F}_y = \langle -8Ad^4\rangle$
(e) $\vec{F}_y = \langle 0\rangle$

$$\vec{F}_y = \left\langle -\frac{dU}{dy}\right\rangle_{x \text{ held constant}}$$

$$= \left\langle -Ax^3\frac{d}{dy}[y^2] + 2Ax^2\frac{d}{dy}[y^3]\right\rangle$$

$$= \langle -2Ax^3y + 6Ax^2y^2\rangle$$

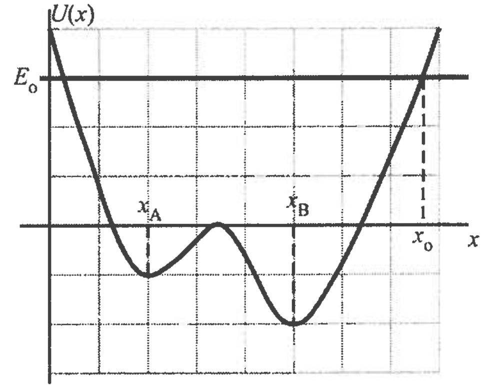
→ evaluate at $x = -d$, $y = +d$

$$\vec{F}_y = \langle -2A(-d)^3d + 6A(-d)^2d^2\rangle$$

$$\boxed{\vec{F}_y = \langle +8Ad^4\rangle}$$

Question value 8 points

(3) Georgia Tech researcher George P. Burdell has designed the "Asymmetric Bistable Sprang". It acts very much lik a regular spring, but has two stable equilibrium lengths, x_A and x_B. The potential energy curve for the sprang is shown at right. A mass m is attached to the sprang, and then stretched out to position x_o, where the total energy of the system is E_o. If the mass is released from rest, what will be its maximum kinetic energy as it oscillates back and forth?



(a) $-\frac{2}{5}E_o$

(b) E_o

(c) $-\frac{2}{3}E_o$

(d) $\boxed{\frac{5}{3}E_o}$

(e) $\frac{4}{3}E_o$

$K + U = E_o = \text{constant}$

$\rightarrow$ at x_o $U_o = E_o$ and $K_o \equiv 0$

$\rightarrow$ at x_B, $U_B = -\frac{2}{3}E_o$

(note grid scale)

so $K_B + U_B = E_o$

$K_B + (-\frac{2}{3}E_o) = E_o \longrightarrow \boxed{K_B = +\frac{5}{3}E_o}$

[Answers (a) and (c) above deserve negative credit! It is impossible to have negative kinetic energy!]

Question value 8 points

(4) You are on the interstate, driving due south at speed v. You enter a broad, uphill left turn, and emerge traveling due east at at speed ¾ v. What was the direction θ of the impulse delivered to the car during the turn?

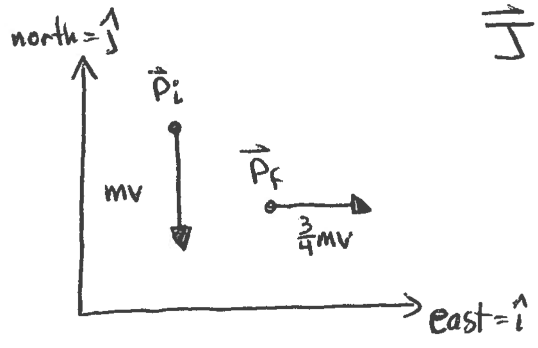
(a) θ = due east

(b) θ = 53° north of east

(c) θ = 45° north of east

(d) The impulse cannot be determined because the elapsed time Δt is unknown.

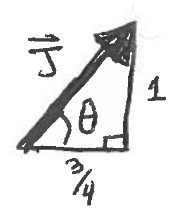
(e) θ = 37° south of west



$\vec{J} = \vec{\Delta p} = \vec{p}_f - \vec{p}_i$

$= \langle +\frac{3}{4}mv \rangle \hat{i} - \langle -mv \rangle \hat{j}$

$= mv \left[\langle +\frac{3}{4} \rangle \hat{i} + \langle 1 \rangle \hat{j} \right]$



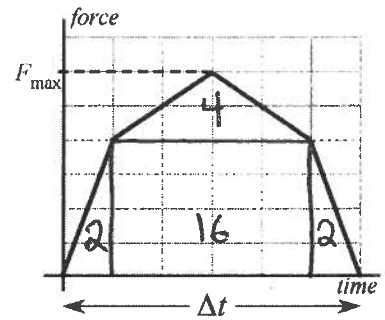
$\theta = \tan^{-1}\left(\frac{1}{3/4}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$\boxed{\theta = 53.1^\circ \text{ North of East}}$

superfluous information

Question value 8 points

(5) A rubber ball of mass m is thrown against a wall. It strikes the wall perpendicularly moving with speed v, and rebounds perpendicularly with speed $v/2$. The force by the wall on the tennis ball is graphed as a function of time at right. The total time spent in contact with the wall is Δt. What is the magnitude of the average force exerted by the wall on the ball?



(a) $F_{av} = \frac{mv}{2\,\Delta t}$

(b) $\boxed{F_{av} = \frac{2}{3} F_{max}}$

(c) $F_{av} = \frac{3}{2} F_{max}$

(d) $F_{av} = \frac{3mv}{\Delta t}$

(e) $F_{av} = \frac{1}{2} F_{max}$

$$\vec{F}_{av} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t}$$

→ impulse $\vec{J}$ is found as area under force curve

from graph, we see 24 squares of area,

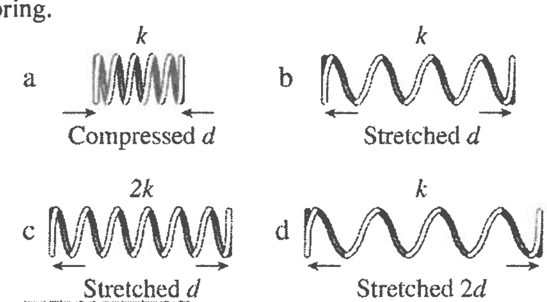
and each square has "height" $F_{max}/6$, "width" $\Delta t/6$

so $|\vec{J}| = 24\left(\frac{F_{max}}{6}\right)\left(\frac{\Delta t}{6}\right) = \frac{24}{36} F_{max}\,\Delta t = \frac{2}{3} F_{max}\,\Delta t$

Thus, $\boxed{F_{av} = \frac{J}{\Delta t} = \frac{2}{3} F_{max}}$

Question value 8 points

(6) Consider the four springs, a–d, at right. Rank, from greatest to least (where large positive > small positive > zero > small negative > large negative), the elastic potential energy stored in each spring.



(a) $U_d = U_c > U_b > \cancel{0 > U_a}$

(b) $U_c > U_d = U_b > \cancel{0 > U_A}$

(c) $U_d = U_c > U_b = U_a > 0$

(d) $\boxed{U_d > U_c > U_a = U_b > 0}$

(e) $U_a > 0 > \cancel{U_b > U_c > U_d}$

$U_s = \frac{1}{2}k(\Delta s)^2$ where Δs = displacement away from equilibrium

→ U_s is always non-negative — one can immediately eliminate three bogus answers in the list above!

$U_a = \frac{1}{2}k(-d)^2 = \frac{1}{2}kd^2$

$U_b = \frac{1}{2}k(+d)^2 = \frac{1}{2}kd^2$

$\Big] \; U_a = U_b$

$U_c = \frac{1}{2}(2k)(+d)^2 = 2\left(\frac{1}{2}kd^2\right) = 2U_{a/b}$

$U_d = \frac{1}{2}k(2d)^2 = 4\left(\frac{1}{2}kd^2\right) = 2U_c = 4U_{a/b}$

$\boxed{U_d > U_c > U_a = U_b > 0}$,