Physics 2211A/B Test form 733 Name Solutions

Fall 2017

Test 3 Recitation Section (see cover page):________

<table><tr><td>

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

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A

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3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

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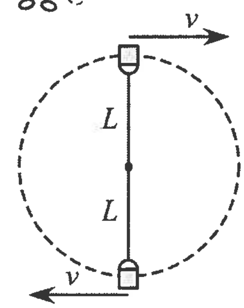
[1] *(20 points)* A water-filled bucket has *true* total weight W. A physics instructor then spins the bucket in a vertical circle at constant speed v, using a cord of length L. When the bucket is inverted at the top of its loop, the instructor feels that the apparent weight of the bucket is $2W$. What will be the apparent weight of the bucket when it is at the bottom of the loop? Express your answer as a numerical multiple of W.

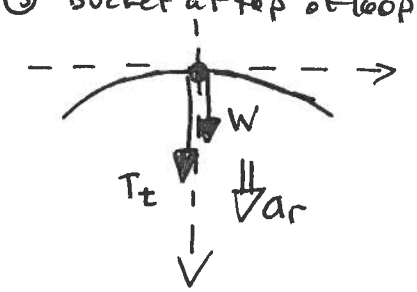
Instructor feels tension and perceives that as "weight of bucket"



① Note that perceived/apparent weight, in this case, is found as the magnitude of the tension force in the rope → given that $T_{top} = 2W$, what is T_{bottom}?

② Bucket rotates at constant speed V — so the magnitude of the radial acceleration $\boxed{a_r = V^2/L = \text{constant}}$

③ Bucket at top of loop



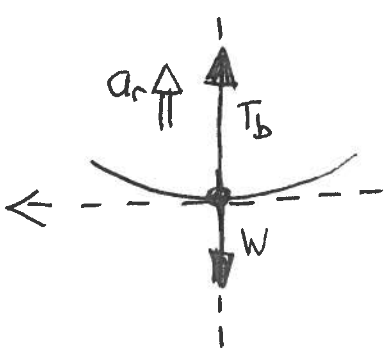
$\sum \vec{F}_r = m\vec{a}_r$ with down = positive

$$\boxed{+T_t + W = m(+a_r)}$$

④ Bucket at bottom of loop

$\sum \vec{F}_r = m\vec{a}_r$ with up = positive

$$\boxed{+T_b - W = m(+a_r)}$$



a_r is the same in both equations, so

$$T_t + W = ma_r = T_b - W$$

$$\Downarrow$$

$$T_b = T_t + 2W$$

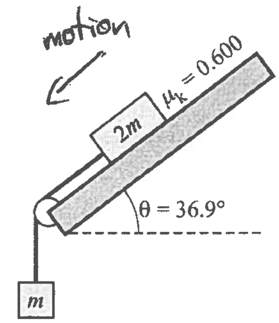
since $T_t = 2W$, we have

$$\boxed{T_b = 4W}$$

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* In the figure at right, the block $2m$ on the incline is given a tiny nudge to start it moving downslope. When it is moving freely, what will be the tension in the cord? Express your answer as a numerical multiple of the weight of the hanging block, mg.

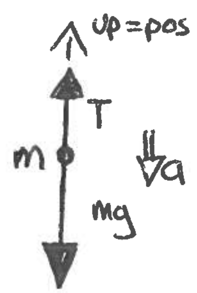
Your free body diagrams will be graded for accuracy and clarity!

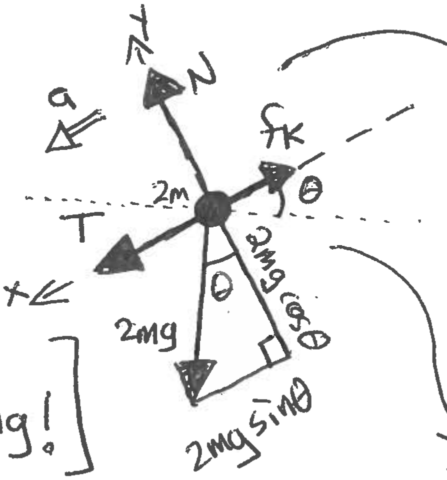


① Assume a downslope acceleration for 2m
→ since cord is ideal, mass m has same accel, a

② Tension in cord can be treated as a 3rd Law coupling force

motion downslope implies a kinetic friction force that is upslope





Note: since m is accelerating downward, we expect T < mg!

$$\Sigma \vec{F}_y = m\vec{a}_y$$

$$\langle +N \rangle + \langle -(2m)g\cos\theta \rangle = 0$$

$$\boxed{N = 2mg\cos\theta}$$

so $\boxed{f_K = \mu_K 2mg\cos\theta}$

$$\Sigma \vec{F}_y = m\vec{a}_y$$

sign!

$$\langle +T \rangle + \langle -mg \rangle = m\langle -a \rangle$$

$$\boxed{T - mg = -ma}$$

two equations in T and a

$$\Sigma \vec{F}_x = m\vec{a}_x$$

$$\langle +T \rangle + \langle +2mg\sin\theta \rangle + \langle -f_K \rangle = (2m)\langle +a \rangle$$

$$\boxed{T + 2mg\sin\theta - (\mu_K 2mg\cos\theta) = 2ma}$$

$$2ma = 2mg - 2T$$

$$T + 2mg\sin\theta - \mu_K 2mg\cos\theta = 2mg - 2T$$

$$3T = 2mg\left[1 + \mu_K\cos\theta - \sin\theta\right]$$

$$T = \frac{2}{3}mg\left[1 + (0.6)(0.8) - (0.6)\right]$$

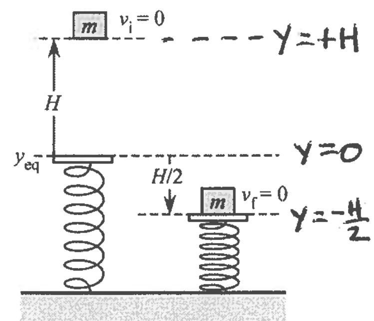
$$= \frac{2}{3}mg\left[0.88\right]$$

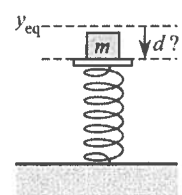
$$\boxed{T = 0.587\,mg}$$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* A block of mass m is held stationary at a height H above a vertical spring, and dropped. It is observed to compress the spring by a distance $H/2$ before momentarily coming to rest (top figure). If, instead, the block had simply been gently placed atop the spring (with the block in equilibrium), by what distance d would the spring compress (bottom figure)?

Hint: use the Energy Principle to analyze the drop, in order to learn about the spring constant.





① Drop from height H: $V_i = 0$ and $V_f = 0$

$\Rightarrow K_i = K_f = 0$ so net $\boxed{\Delta K = 0}$

② Work by gravity, during $\vec{\Delta y} = \vec{Y}_f - \vec{Y}_i$

$= (-H/2) - (+H)$

$\rightarrow \vec{\Delta y} = \langle -\frac{3}{2}H \rangle$

so $W_{grav} = \vec{F}_{grav} \cdot \vec{\Delta y} = \langle -mg \rangle \cdot \langle -\frac{3}{2}H \rangle$

$\boxed{W_g = +\frac{3}{2}mgH}$

③ work by spring:

$W_s = -\frac{1}{2}k s_f^2 + \frac{1}{2}k s_i^2$ with $s_f = \langle -H/2 \rangle$ and $s_i = 0$

$\rightarrow \boxed{W_s = -\frac{1}{2}k\left(\frac{H^2}{4}\right) = -\frac{kH^2}{8}}$

④ Energy Principle tells us $\Delta E_{sys} = W_{ext} \rightarrow \Delta K = W_g + W_s$

so $0 = \left(+\frac{3}{2}mgH\right) + \left(-\frac{kH^2}{8}\right) \rightarrow \frac{kH}{8} = \frac{3}{2}mg$

$kH = 12mg \rightarrow \boxed{k = \frac{12mg}{H}}$

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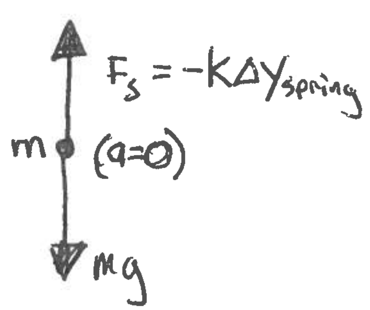
⑤ Now, place mass gently on spring — allow mass to attain equilibrium

$\Rightarrow \left(\sum \vec{F}\right)_{on\ m} = 0$ at some $\vec{Y}_f = \langle -d \rangle \rightarrow \Delta Y_{spring} = -d$



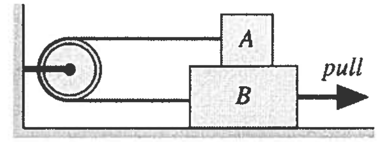
$\langle -k\Delta y \rangle + \langle -mg \rangle = 0$

$-k(-d) = mg$

$d = \frac{mg}{k} = \frac{mg}{12mg/H}$

$\boxed{d = \frac{H}{12}}$

The next two questions involve the following situation:
Wooden block A is stacked on wooden block B, and an ideal cord passes over a pulley to connect the blocks as shown at right. Block B is then pulled to the right with sufficient force to cause it to slip along the ground.



Question value 4 points

(1) According to the Third Law, What force is paired with the upward normal force by the ground on the bottom of block B?

(a) The gravitational force down on block B.

(b) The weight of *both* blocks A and B, down on block B.

(c) The downward normal force by block A on block B

(d) The downward normal force by block B on block A.

(e) A downward normal force by block B on the ground.

Third Law Partner is...
... downward normal force
... by the bottom of block B
... on the ground

Question value 4 points

(2) What pair of horizontal "Third Law" forces act on the two blocks?

(a) Kinetic friction to the right on block B and to the left on block A.

(b) ~~Tension~~ to the left on block B and Tension to the right on block A.

(c) ~~Static friction~~ to the right on block B and to the left on block A.

(d) Kinetic friction to the left on block B and to the right on block A.

(e) ~~Tension~~ to the left on block B and also to the left on block A.

① Tension is not a 3rd Law pair here — it acts to the left on both blocks

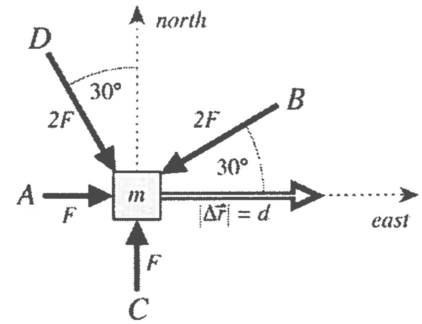
② If B moves to left, A necessarily moves to right — their surfaces are in relative motion, so friction force cannot be static

⇒ 3rd Law force acting horizontally is kinetic friction

- Motion of A relative to B is leftward, so $\vec{f}_K$ on A is rightward
- Motion of B relative to A is rightward, so $\vec{f}_K$ on B is leftward

Question value 8 points

(3) The figure at right shows a top-down view of a block resting on a frictionless horizontal surface. Four forces (A through D, all horizontal) push the block, with the magnitudes (F or $2F$) and directions indicated. The block is observed to displace a distance d in a direction due east (as shown). Rank, from greatest (i.e. most positive) to least (i.e. most negative), the work done by each force.



(a) $W_D = W_B > W_A > W_C$

(b) $\boxed{W_A = W_D > W_C > W_B}$

(c) $W_B > W_D = W_A > W_C$

(d) $W_A > W_D > W_B > W_C$

(e) $W_D > W_A > W_C > W_B$

Work by a constant force is

$W = \vec{F} \cdot \Delta\vec{s} = Fd\cos\beta$ where β = angle between tail-to-tail vectors

$\boxed{A: \vec{F} \quad \Delta\vec{r} \quad \beta = 0°, \; W_A \stackrel{+}{=} Fd}$

$\boxed{B: \; 30° \; \beta \; 2\vec{F} \quad \Delta\vec{r} \quad \beta = 150°, \; W_B = -\frac{\sqrt{3}}{2}(2F)d}$

$\boxed{C: \; \vec{F} \quad \Delta r \quad \beta = 90°, \; W_C = 0}$

$\boxed{D: \; \Delta\vec{r} \quad 30° \quad \beta = 60° \quad 2F \quad W_D = (2F)(d)(\tfrac{1}{2}) = +Fd}$

$$\boxed{\underbrace{W_A = W_D}_{+Fd} > \underbrace{W_C}_{0} > \underbrace{W_B}_{-\sqrt{3}Fd}}$$

Question value 8 points

(4) A worker lowers a crate from a shelf at height H to the ground, with the crate beginning at rest on the shelf, and ending at rest on the ground. If the "system of interest" is defined to be just the crate itself, what work is being done by each of the forces acting on the crate?

(a) Gravity and the worker are both doing positive work—with the total work ~~being positive.~~

(b) Gravity is doing positive work while the worker is doing negative work—with the total work ~~being negative~~.

(c) Gravity and the worker are both doing negative work—with the total work ~~being negative.~~

(d) $\boxed{\text{Gravity is doing positive work while the worker is doing negative work—with the total work being } \textit{zero}.}$

(e) Gravity is doing negative work while the worker is doing positive work—with the total work being *zero*.

Crate begins and ends at rest, so $\Delta K = 0$

while worker is lowering crate: only gravity (by earth) and lifting force (by worker) act

displacement is down, grav force is down $\rightarrow$ $\boxed{\text{gravity does positive work}}$

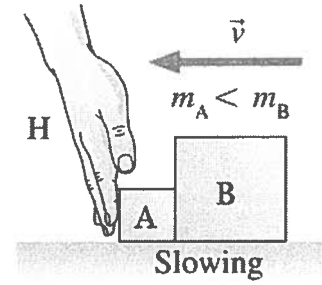
$\Delta K = 0$, so $\boxed{W_{TOT} = 0}$ by energy principle

$W_{TOT} = 0 = W_g + W_w \rightarrow W_w = -W_g$ where gravity does positive work

$\boxed{\text{Worker does negative work}}$

Question value 8 points

(5) Boxes A and B are sliding to the left across a frictionless table. The hand H is slowing them down. The mass of A is less than the mass of B. Rank the magnitudes of the horizontal forces on A, B, and H, from greatest to least.



$\boxed{\text{(a)} \quad F_{\text{H on A}} = F_{\text{A on H}} > F_{\text{B on A}} = F_{\text{A on B}}}$

(b) $F_{\text{H on A}} > F_{\text{A on B}} = F_{\text{A on H}} > F_{\text{B on A}}$

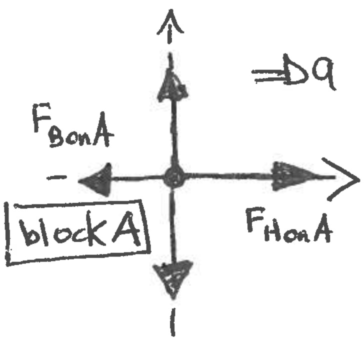
(c) $F_{\text{B on A}} = F_{\text{A on B}} = F_{\text{H on A}} = F_{\text{A on H}}$

(d) $F_{\text{A on B}} = F_{\text{B on A}} > F_{\text{H on A}} = F_{\text{A on H}}$

(e) $F_{\text{H on A}} > F_{\text{A on H}} > F_{\text{A on B}} > F_{\text{B on A}}$

$\vec{V}$ = leftward, and slowing

$\Rightarrow$ rightward acceleration



$\sum \vec{F}_x = m\vec{a}_x$ for block A

$\langle + F_{HonA} \rangle + \langle - F_{BonA} \rangle = m_A \langle +a \rangle$

$F_{HonA} = F_{BonA} + m_A a$

$\Rightarrow \boxed{F_{HonA} > F_{BonA}}$

$\rightarrow$ Now invoke third Law

$$\boxed{\begin{aligned} F_{HonA} &= F_{AonH} \\ F_{BonA} &= F_{AonB} \end{aligned}}$$

Question value 8 points

(6) Two identical coins are placed on a turntable, at distance R and $2R$ from the central axis. Starting from rest, the turntable is gradually spun up with a constant (but small) angular acceleration α. At some angular speed ω_A, coin A is "flung off" the turntable, and at some speed ω_B, coin B is "flung off" the turntable. Compare these two speeds.

(a) $\omega_B = 0.500\ \omega_A$

$\boxed{\text{(b)} \quad \omega_B = 0.707\ \omega_A}$

(c) $\omega_B = 1.414\ \omega_A$

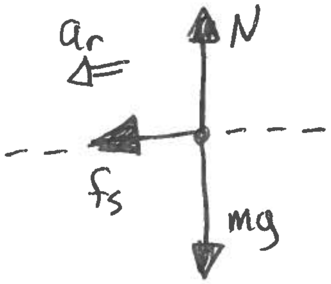
(d) $\omega_B = 0.250\ \omega_A$

(e) $\omega_B = 2.000\ \omega_A$

in terms of angular speed, radial/centripetal accel is:

$a_r = V^2/r \rightarrow a_r = \omega^2 r$

Consider either coin at limit where it is about to be thrown off $\rightarrow$ static friction is at its maximum limit $f_s = \max = \mu_s N$



$\langle +N \rangle + \langle -mg \rangle = 0$

$N = mg$

$\langle +f_s \rangle = m \langle +a_r \rangle$

$\mu_s N = m(\omega^2 r)$

$\mu_s \cancel{m} g = \cancel{m} \omega^2 r$

$\boxed{\omega = \sqrt{\frac{\mu_s g}{r}}}$

$\rightarrow$ compare critical speeds for A/B by comparing radii

$\omega_A = \sqrt{\mu_s g / R}$

$\omega_B = \sqrt{\mu_s g / 2R} = \frac{\sqrt{\mu_s g / R}}{\sqrt{2}}$

$\boxed{\omega_B = \frac{1}{\sqrt{2}} \omega_A}$