Physics 2211A/B
Fall 2017
**Test 2**

Test form 722

Name Solutions

Recitation Section (see cover page):_________

**A**

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

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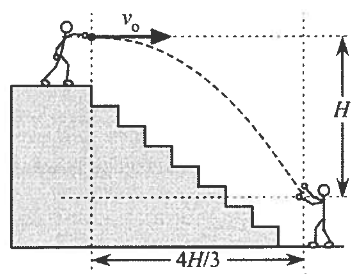
**[I]** *(20 points)* You are standing at the top of a stairwell of height $H$. Your friend is standing at the bottom of the stairwell, a horizontal distance $4H/3$ away from you. You throw a tennis ball horizontally, with just *exactly* the right speed to reach your friend.

Determine the final velocity of the tennis ball at the moment your friend catches it. Express the magnitude in terms of the symbols $g$ and $H$, and the direction as a numerical angle (to three-digit precision) relative to the horizontal.

*Hint: start by figuring out the correct throwing speed, in terms of H and g.*




$\vec{V}_i = \langle +V_0 \rangle \hat{i} + \langle 0 \rangle \hat{j}$

① Compare vertical and horizontal displacements

$\overrightarrow{\Delta x} = \vec{V}_x \Delta t \longrightarrow \boxed{\langle +\tfrac{4}{3}H \rangle = \langle +V_0 \rangle \Delta t_{TOTAL}}$ two equations for $V_0$, $\Delta t_{TOTAL}$

$\overrightarrow{\Delta y} = \cancel{\vec{V}_{0y}}^{\,0} \Delta t + \frac{1}{2}\langle -g \rangle \Delta t^2 \longrightarrow \boxed{\langle -H \rangle = \frac{1}{2}\langle -g \rangle \Delta t_{TOTAL}^2} \longrightarrow \Delta t_{TOTAL} = \sqrt{\frac{2H}{g}}$

② Since $\vec{V}_x$ = constant, we have:

$\boxed{\vec{V}_{xf} = \vec{V}_{xi} = \langle +\tfrac{2}{3}\sqrt{2gH} \rangle}$

so $V_0 = \frac{4}{3}H / \Delta t_{TOTAL} = \frac{4}{3}H \cdot \sqrt{\frac{g}{2H}}$

$\rightarrow V_0 = \frac{2}{3}\sqrt{2gH}$

③ Use "speed equation" to find $V_{yf}$: $V_{yf}^2 = \cancel{V_{yi}^2}^{\,0} + 2(-g)\Delta y = 2(-g)(-H)$

$V_{yf} = \sqrt{2gH}$

$\boxed{\vec{V}_{yf} = \langle -\sqrt{2gH} \rangle}$

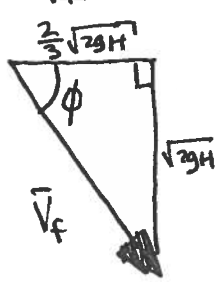
④ Find magnitude and direction of $\vec{V}_f$

$\tan\phi = \left| \frac{V_{yf}}{V_{xf}} \right| = \left| \frac{\sqrt{2gH}}{\frac{2}{3}\sqrt{2gH}} \right| = \frac{3}{2}$

$|\vec{V}_f| = \sqrt{V_{xf}^2 + V_{yf}^2}$

$= \sqrt{2gH}\left[\sqrt{(\tfrac{2}{3})^2 + (1)^2}\right]$

(common factor!)

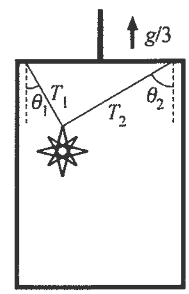



$\rightarrow \boxed{V_f = \sqrt{\frac{26}{9}gH}}$

$\boxed{\phi = 56.3^\circ \text{ below horizontal}}$

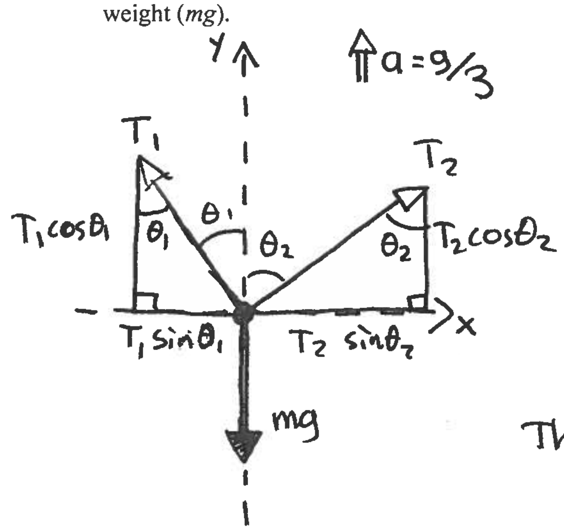
*The following problem will be hand-graded.* <u>*Show all your work for this problem*</u>*. Make no marks and leave no space on your answer card for it.*

**[II]** *(20 points)* You are riding in an executive express elevator. A decorative artwork hangs from the ceiling by two cords, making angles $\theta_1 = 30°$ and $\theta_2 = 60°$ from the vertical, as shown. You press the button for the penthouse, and the elevator begins a rapid upward ascent with acceleration $a = g/3$.




Draw a free body diagram for the artwork, decompose all vectors into components along your corrdinate axes, and write out 2nd law statements for each component direction.

Determine the tension in each cord. Express each answer as a multiple of the artwork's true weight ($mg$).




$$\Sigma \vec{F}_x = 0 \quad (\vec{a}_x = 0)$$

$$\boxed{\langle +T_2 \sin\theta_2 \rangle + \langle -T_1 \sin\theta_1 \rangle = 0} \quad (x)$$

$$\Sigma \vec{F}_y = m\vec{a}_y$$

$$\boxed{\langle +T_1 \cos\theta_1 \rangle + \langle +T_2 \cos\theta_2 \rangle + \langle -mg \rangle = m\langle +g/3 \rangle} \quad (y)$$

This gives us <u>two</u> equations in $T_1$ and $T_2$

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Equation for (x): $T_2 = T_1 \dfrac{\sin\theta_1}{\sin\theta_2}$ where $\sin\theta_1 = \sin 30° = \frac{1}{2}$, $\sin\theta_2 = \sin 60° = \sqrt{3}/2$

$$= T_1 \frac{(1/2)}{(\sqrt{3}/2)} \qquad \boxed{T_2 = \frac{1}{\sqrt{3}} T_1}$$

Equation for (y):

$$T_1 \cos\theta_1 + T_2 \cos\theta_2 = mg + mg/3 = 4/3\, mg$$

where $\cos\theta_1 = \frac{\sqrt{3}}{2}$ and $\cos\theta_2 = \frac{1}{2}$

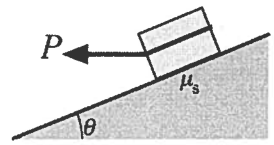
$$T_1\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{3}}T_1\right)\left(\frac{1}{2}\right) = 4/3\, mg$$

$$T_1\left[\frac{\sqrt{3}}{2}\left(1 + \frac{1}{3}\right)\right] = 4/3\, mg \longrightarrow \boxed{T_1 = \frac{2}{\sqrt{3}} mg = 1.15\, mg}$$

Then $T_2 = \frac{1}{\sqrt{3}} T_1 = \frac{1}{\sqrt{3}}\left(\frac{2}{\sqrt{3}} mg\right) \longrightarrow \boxed{T_2 = \frac{2}{3} mg = 0.667\, mg}$

*The following problem will be hand-graded.* *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

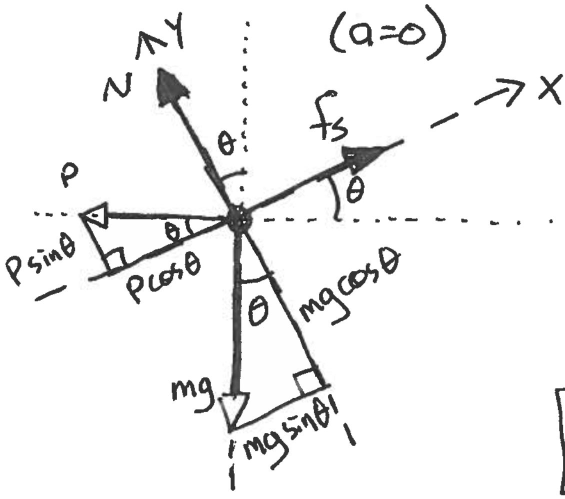
**[III]** *(20 points)* A crate of mass $m$ is stuck on a rough ramp inclined at an angle $\theta = 22°$ above the horizontal. The coefficient of static friction between the crate and ramp is $\mu_s = 0.64$. You decide to get the crate moving by attaching a rope, and pulling horizontally away from the ramp, with some force $P$.




(i) Draw a free body diagram for the crate. Select appropriate coordinate axes and decompose all forces into their proper components.

(ii) Write out 2nd law statements for each component direction.

(iii) Determine the *minimum* pulling force $P$ that will cause the crate to start sliding down the ramp. Express your answer as a numerical multiple of the crate's weight, $mg$.




Assume crate is not yet slipping, but "on the verge" of doing so

$$\sum \vec{F}_x = m\vec{a}_x = 0 \quad \text{(up incline = positive)}$$

$$\boxed{\langle +f_s \rangle + \langle -P\cos\theta \rangle + \langle -mg\sin\theta \rangle = 0}$$

$$\sum \vec{F}_y = m\vec{a}_y = 0 \quad \text{(off ramp = positive)}$$

$$\boxed{\langle +N \rangle + \langle +P\sin\theta \rangle + \langle -mg\cos\theta \rangle = 0}$$

"Minimum pulling force that will move crate" is synonymous with "Maximum pulling force that will not move crate"

→ use the latter POV, so that we can use $f_s = \text{max} = \mu_s N$

Solve for N: $N = mg\cos\theta - P\sin\theta$

so $f_{s,max} = \mu_s mg\cos\theta - \mu_s P\sin\theta$

→ plug result into equation for x-axis:

$$(+\mu_s mg\cos\theta - \mu_s P\sin\theta) - P\cos\theta - mg\sin\theta = 0$$

$$\mu_s mg\cos\theta - mg\sin\theta = \mu_s P\sin\theta + P\cos\theta$$

$$P = mg\left[\frac{\mu_s\cos\theta - \sin\theta}{\mu_s\sin\theta + \cos\theta}\right] = mg\left[\frac{0.2188}{1.167}\right]$$

$$\boxed{P = 0.187\, mg}$$

*Question value 8 points*

(1) The driver of a powerboat wishes to travel upriver, from dock A to dock B to drop off a passenger, and then return from dock B to dock A. He travels both directions at full throttle, resulting in a speed $v$ through the water. He notes that the downstream trip requires only two-thirds the time of the upstream trip. What is the speed of the current?

(a) $v/2$
(b) $v/3$
(c) $v/4$
(d) $2v/3$
(e) $v/5$

let "upstream" = positive : $\vec{V}_{current} = \langle -V_c \rangle$

① while aiming upstream : $\vec{V}_{Boat\text{-}to\text{-}shore} = \langle +V \rangle + \langle -V_c \rangle = \langle +V - V_c \rangle$

distance travelled is $\boxed{D = |\Delta \vec{r}_u| = (V - V_c)\Delta t_{up}}$

② while travelling downstream

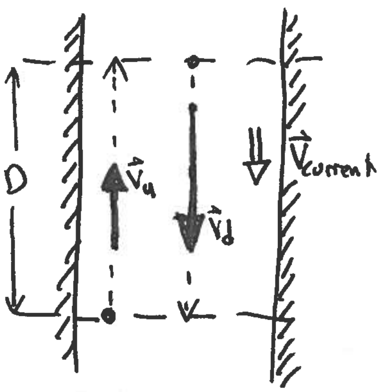
$\vec{V}_{Boat\text{-}to\text{-}shore} = \langle -V \rangle + \langle -V_c \rangle = \langle -(V+V_c) \rangle$

distance travelled is $D = |\Delta \vec{r}_d| = (V+V_c)\Delta t_{down}$

③ Equate these expressions, and use $\Delta t_{down} = \frac{2}{3}\Delta t_{up}$

$(V+V_c)\Delta t_{down} = (V - V_c)\Delta t_{up} \rightarrow (V+V_c)(\tfrac{2}{3})\cancel{\Delta t_{up}} = (V-V_c)\cancel{\Delta t_{up}}$

$2(V+V_c) = 3(V-V_c) \rightarrow 5V_c = V$ $\quad \boxed{V_c = V/5}$




*Question value 8 points*

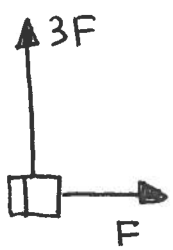
(2) When block A experiences a force of magnitude $F$, its resulting acceleration vector has a magnitude $a$. When block B experiences a force of magnitude $3F$, its resulting acceleration vector also has a magnitude $a$. Blocks A and B are then glued together. If the combined blocks are subject to a force of magnitude $F$ along the x-axis plus a force of magnitude $3F$ along the y-axis, what will be the magnitude of the resulting acceleration?

(a) 0.35 $a$
(b) 1.00 $a$
(c) 0.79 $a$
(d) 1.41 $a$
(e) 0.50 $a$

$F = M_a a \rightarrow a = F/M_a$

$3F = M_b a \rightarrow M_b = 3M_a \rightarrow$ when combined, their mass is $M = M_a + M_b = 4M_a$

Apply force $\vec{F} = \langle +F \rangle \hat{\imath} + \langle 3F \rangle \hat{\jmath}$




$$\bar{a} = \frac{\Sigma \vec{F}}{M} = \frac{\langle F \rangle}{4M_a}\hat{\imath} + \frac{\langle 3F \rangle}{4M_a}\hat{\jmath}$$

$$= \frac{F/M_a}{4}\hat{\imath} + \frac{3(F/M_a)}{4}\hat{\jmath}$$

$$= \frac{a}{4}\hat{\imath} + \frac{3a}{4}\hat{\jmath}$$

$$|\vec{a}| = a\sqrt{(\tfrac{1}{4})^2 + (\tfrac{3}{4})^2} = \boxed{\frac{\sqrt{10}}{4}a = 0.79a}$$

*Question value 6 points*

(3) A heavy cable can support a maximum weight of 2500 lb. A crate weighing 2200 lb is suspended vertically from the cable. In which (if any) of the following situations is there a possibility that the cable might break?

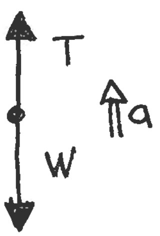
(a) While the crate is being raised at constant speed. $(a=0)$

(b) There is no danger of the cable breaking under *any* of the other listed circumstances.

$\boxed{\text{(c) While the crate is being lowered with a decreasing speed. } (a = \text{upward})}$

(d) While the crate is being lowered at constant speed. $(a=0)$

(e) While the crate is being raised with a decreasing speed. $(a=\text{downward})$




although $W < T_{max}$, we must recognize that actual tension T <u>can</u> be <u>considerably greater</u> than W, and thus, greater than $T_{max}$ $\boxed{\text{IF}}$ there is <u>acceleration</u> that is <u>directed</u> <u>upward</u>:

that could be:
- rising and gaining speed
- $\boxed{\text{descending and losing speed}}$

$$\langle +T\rangle + \langle -W\rangle = m\langle +a\rangle$$

$$T = W + ma \rightarrow T > W$$

*Question value 8 points*

(4) The figure at right displays three solid disks that are spinning with <u>constant</u> angular speeds. Rank, from greatest to least, the magnitudes of the acceleration vectors for a point on the rim of each disk.

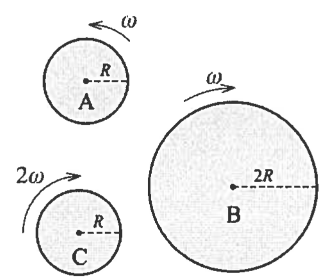
$\boxed{\text{(a) } a_C > a_B > a_A}$

(b) $a_A > a_B > a_C$

(c) $a_C > a_A > a_B$

(d) $a_C = a_B = a_A$

(e) $a_B > a_C > a_A$




Constant speed so all $\vec{a}_{tangential} = 0$

<u>BUT</u> circular motion requires $\vec{a}_{radial} \neq 0$

where $|\vec{a}_{rad}| = \frac{V^2}{R}$ or, using $V = \omega R$, $|\vec{a}_{rad}| = \frac{(\omega R)^2}{R} = \omega^2 R$

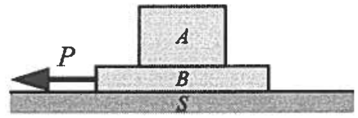
$$A: a_A = (\omega)^2(R) = \omega^2 R$$

$$B: a_B = (\omega)^2(2R) = 2\omega^2 R = 2a_A$$

$$C: a_C = (2\omega)^2(R) = 4\omega^2 R = 4a_A = 2a_B$$

so $\boxed{a_C > a_B > a_A}$

*The next two questions involve the following situation:*
Block $A$ rests atop board $B$, which in turn rests upon a frictionless horizontal surface $S$. The surface between blocks $A$ and $B$ is rough. Board $B$ is pulled very strongly to the left by a force $P$, as shown at right. The pull is strong enough that board $B$ is actually pulled out from under block $A$, with the block slipping along the board.




*Question value 4 points*
(5) What will be the nature of block $A$'s motion relative to board $B$, and also relative to surface $S$?

(a) Block A will move to the right relative to $B$, and to the left relative to $S$.
(b) Block A will move to the left relative to $B$, and to the left relative to $S$.
(c) Block A will move to the left relative to $B$, and will not move relative to $S$.
(d) Block A will move to the right relative to $B$, and to the right relative to $S$.
(e) Block A will move to the left relative to $B$, and to the right relative to $S$.

if B slips out under A, to left relative to A,
then A is moving to right relative to B

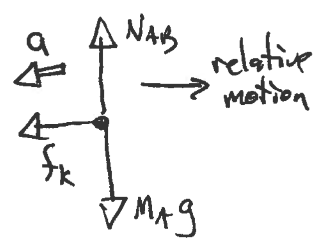
Now, if A moves to right relative to B, then there is a leftward kinetic friction force on A (see below)
result is a leftward acceleration for A, relative to the fixed surface

A moves to left relative to S

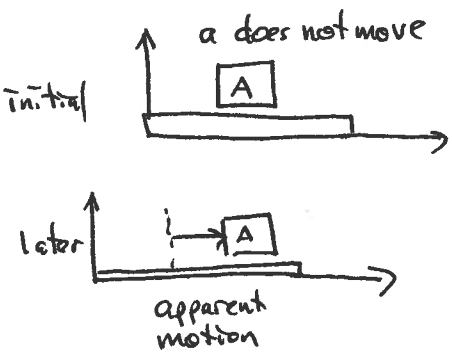
*Question value 4 points*
(6) What will be the nature of the of the friction force (if any) on block $A$?

(a) There will be a static friction force to the left.
(b) There will be a kinetic friction force to the left
(c) There will be a kinetic friction force to the right.
(d) There will be no friction force on block $A$.
(e) There will be a static friction force to the right.

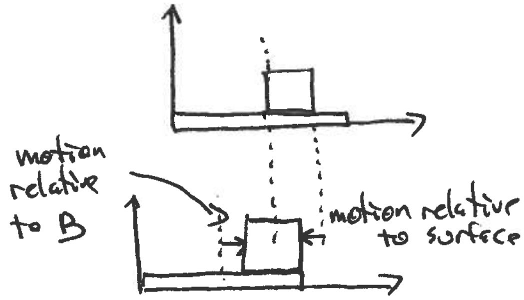
To determine friction by B on A, look at motion of A relative to B




in absence of friction:




In presence of friction




A moves some, but not as much as B