Physics 2211A/B Test form $\boxed{711}$ Name $\boxed{\text{Solutions}}$

Fall 2017

Test 1 Recitation Section (see cover page):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Test grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

[I] *(20 points)* A jogger runs a straight-line distance d at constant speed v. She then speeds up, and runs a further distance $2d$ (along the same straight line) at a speed $4v$. What is her average speed over the entire distance? Express your answer as a multiple of v.

① Average <u>speed</u> is $V_{av} = \dfrac{\text{total distance}}{\text{total time}} = \dfrac{3d}{\Delta t_1 + \Delta t_2}$

② elapsed time during each leg is found from $\vec{\Delta x} = \vec{V}\Delta t$

$0 \to 1 \quad \langle +d \rangle = \langle +V \rangle \Delta t_1$

$\boxed{\Delta t_1 = d/V}$

$1 \to 2 \quad \langle +2d \rangle = \langle +4V \rangle \Delta t_2$

$\boxed{\Delta t_2 = d/2V}$

③ substituting these times gives

$$V_{av} = \frac{3d}{\frac{d}{V} + \frac{d}{2V}} = \frac{3d}{\frac{2d}{2V} + \frac{d}{2V}} = \frac{3d}{\frac{3d}{2V}} = 3d \cdot \frac{2V}{3d}$$

$$\boxed{V_{av} = 2V}$$

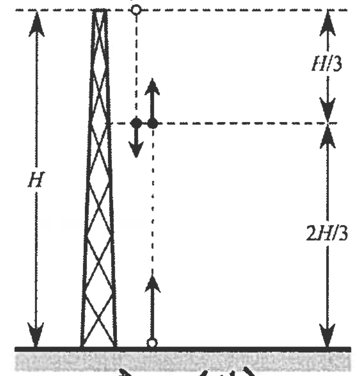
The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* Anne is standing at the top of a radio tower of height H when she drops an apple. Bill is standing at the bottom of the tower, and he throws a banana striaght up at unknown speed, at the the exact instant Anne releases the apple. The apple and banana are observed to cross paths at the moment the apple has fallen 1/3 of the way to the ground.

What is the time delay between the apple striking the ground and the banana striking the ground? Express your answer in terms of H and g. [DO NOT substitute the numerical <u>value</u> of g…just use the <u>symbol</u> "g".]

Hint: start by figuring out the banana's initial speed.

$\vec{V}_{Ai} = 0$



$\vec{V}_{Bi} = \langle +V_0 \rangle$

① A falls distance $\frac{H}{3}$ while B rises distance $\frac{2H}{3}$

$$\overrightarrow{\Delta Y_A} = \langle -H/3 \rangle = \cancel{\vec{V}_{Ai}}^{\,0}\,\Delta t_1 + \tfrac{1}{2}\langle -g \rangle \Delta t_1^2$$

$$\overrightarrow{\Delta Y_B} = \langle +\tfrac{2H}{3} \rangle = \langle +V_0 \rangle \Delta t_1 + \tfrac{1}{2}\langle -g \rangle \Delta t_1^2$$

→ these equations give two equations in the unknowns Δt_1 and V_0

→ so: $\frac{1}{2} g \Delta t_1^2 = H/3 \rightarrow \Delta t_1 = \sqrt{\frac{2H}{3g}}$ →

$$\frac{2H}{3} = V_0\left(\sqrt{\frac{2H}{3g}}\right) - \frac{1}{2} g \left(\sqrt{\frac{2H}{3g}}\right)^2 = V_0 \sqrt{\frac{2H}{3g}} - \frac{H}{3}$$

$$H = V_0 \sqrt{\frac{2H}{3g}} \rightarrow \boxed{V_0 = \sqrt{\tfrac{3}{2} g H}}$$

② Find total time for A to reach ground: $\overrightarrow{\Delta Y_A} = \langle -H \rangle = \frac{1}{2}\langle -g \rangle \Delta t_A^2$

$$\rightarrow \boxed{\Delta t_A = \sqrt{\frac{2H}{g}}}$$

③ Find total time for B to rise and fall back down: <u>net</u> displacement is $\overrightarrow{\Delta Y_{B,TOT}} = 0$

$$\overrightarrow{\Delta Y_{B,TOT}} = 0 = \langle +V_0 \rangle \Delta t_B + \tfrac{1}{2}\langle -g \rangle \Delta t_B^2 = \Delta t_B \left[V_0 - \tfrac{1}{2} g \Delta t_B \right]$$

so $V_0 - \frac{1}{2} g \Delta t_B = 0 \rightarrow \Delta t_B = \frac{2V_0}{g} = \frac{2}{g}\sqrt{\frac{3}{2} g H}$

$$\boxed{\Delta t_B = \sqrt{\frac{6H}{g}}}$$

④ Finally, time interval <u>between</u> impacts is

$$\Delta t_{B-A} = \Delta t_B - \Delta t_A = \sqrt{\frac{6H}{g}} - \sqrt{\frac{2H}{g}} = \boxed{\sqrt{\frac{H}{g}}\left(\sqrt{6} - \sqrt{2}\right) = 1.04\sqrt{\frac{H}{g}}}$$

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

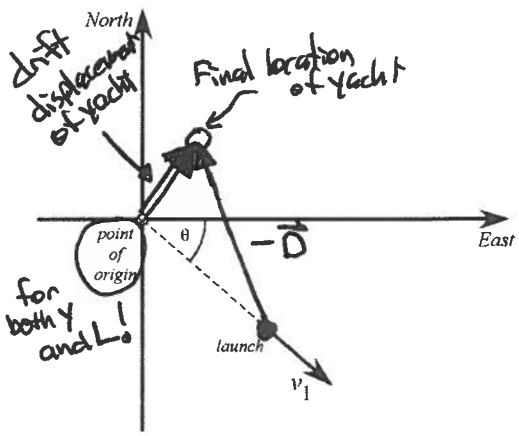
θ

[III] *(20 points)* A yacht is adrift with all engines dead. The first officer sets out for help in a motor launch, travelling 41.0° south of east at a speed of 22.0 kph while the captain remains behind and tracks the launch using the yacht's onboad radar. After 2.50 hours, radar indicates that the launch is 65.0 km from the yacht, on a bearing of 72.0° south of east.

V_1 ϕ D = distance from Y to L

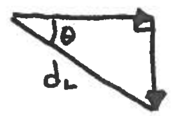
What was the average drift velocity of the yacht during this interval?

Hint: Use the radar data to describe the final position of the <u>*yacht*</u> *relative to the final position of the* <u>*launch*</u>.



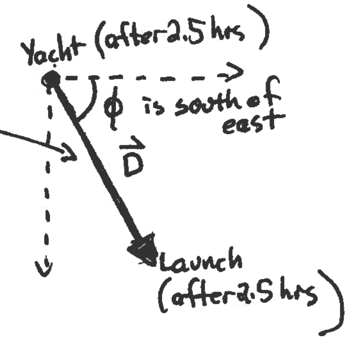
① location of launch after 2.5 hrs

$d_L = V_1 \Delta t = 55\text{ km}$



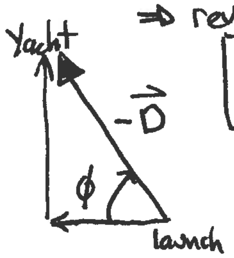
$$\boxed{\vec{r}_{Lf} = (+d_L \cos\theta)\hat{i} + (-d_L \sin\theta)\hat{j}}$$

② Radar gives displacement <u>from yacht</u> <u>to launch</u>: $\vec{D}$



⇒ reverse this to get displacement from launch to yacht

$$\boxed{\vec{r}_{LtoY} = -\vec{D} = (-D\cos\phi)\hat{i} + (+D\sin\phi)\hat{j}}$$



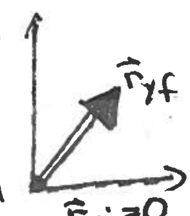
φ is now North of West

Position of yacht, after all this, is the vector sum of these:

$$\vec{r}_{yf} = \vec{r}_{Lf} + \vec{r}_{LtoY} = (+d_L\cos\theta - D\cos\phi)\hat{i} + (-d_L\sin\theta + D\sin\phi)\hat{j}$$

$$= (+41.5\text{ km} - 20.1\text{ km})\hat{i} + (-36.1\text{ km} + 61.8\text{ km})\hat{j}$$

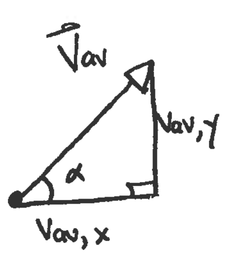
$$\vec{r}_{yf} = (+21.4\text{ km})\hat{i} + (+25.7\text{ km})\hat{j}$$



since $\vec{r}_{yi} = 0$, <u>displacement</u> of yacht is $\Delta\vec{r}_y = (+21.4\text{ km})\hat{i} + (+25.7\text{ km})\hat{j}$

hence average velocity is $\boxed{\vec{V}_{av,y} = \frac{\Delta\vec{r}_y}{\Delta t} = (+8.57\text{ kph})\hat{i} + (+10.3\text{ kph})\hat{j}}$

(either form is acceptable)



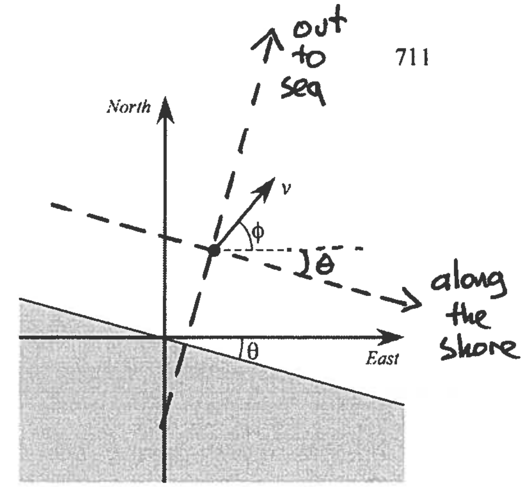
or, in magnitude/direction

$V_{av} = \sqrt{V_{avx}^2 + V_{avy}^2} = \boxed{13.4\text{ kph}}$

at angle $\alpha = \tan^{-1}\left(\frac{V_{avy}}{V_{avx}}\right) = \boxed{50.2^\circ \text{ North of East}}$

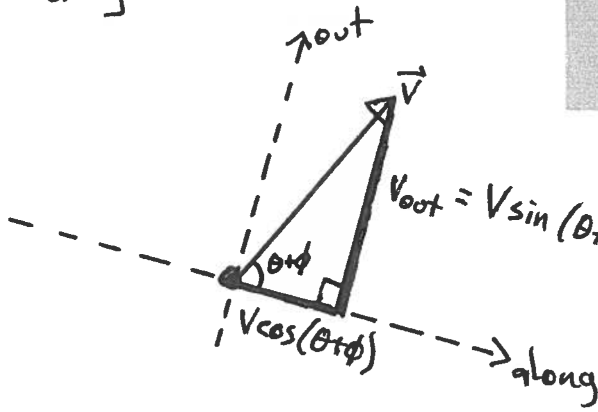
Question value 8 points

(1) A boat is travelling $\phi = 50°$ north of east at a speed v. An observer on the shore, watches the boat as it sails away. The shoreline is straight, oriented at $\theta = 15°$ south of east. At what rate is the boat moving directly out to sea, away from the shore?



- $\boxed{\text{(a)}\quad 0.906\, v}$
- (b) $0.574\, v$
- (c) $0.766\, v$
- (d) $0.423\, v$
- (e) $0.819\, v$

Note that boat's velocity vector makes angle $\theta+\phi$ relative to the "along the shore" axis



$V_{out} = V\sin(\theta+\phi) = V\sin(65°) = \boxed{0.906\,V}$

Question value 8 points

(2) A particle moves along the x-axis, with its velocity given by the expression $\vec{v}(t) = \vec{A}t^3 - \vec{B}$, where $\vec{A}$ and $\vec{B}$ are inherently positive vector constants. What is the average acceleration for the particle during the interval $t_1 = T$ to $t_2 = 2T$?

- ~~(a) $\vec{a}_{av} = 7\vec{A}T^2/2 - \vec{B}$~~ wrong units
- $\boxed{\text{(b)}\quad \vec{a}_{av} = 7\vec{A}T^2}$
- (c) $\vec{a}_{av} = 9\vec{A}T^2$
- ~~(d) $\vec{a}_{av} = 3\vec{A}T^2 - \vec{B}$~~ wrong units
- (e) $\vec{a}_{av} = 15\vec{A}T^2/2$

definition of average acceleration

$$\vec{a}_{av} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t} = \frac{\vec{V}(2T) - \vec{V}(T)}{T}$$

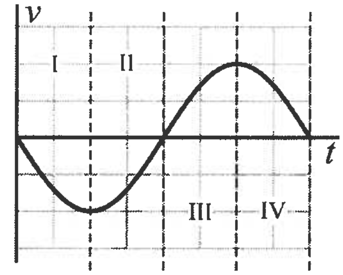
$$= \frac{\left(\vec{A}(2T)^3 - \vec{B}\right) - \left(\vec{A}(T)^3 - \vec{B}\right)}{T}$$

$$= \frac{8\vec{A}T^3 - \cancel{\vec{B}} - \vec{A}T^3 + \cancel{\vec{B}}}{T}$$

$$\boxed{\vec{a}_{av} = 7\vec{A}T^2}$$

Note from formula $\vec{V} = \vec{A}t^3 - \vec{B}$, we recognize that "$\vec{B}$" must have the same units as $\vec{V}$... "length over time" units. Hence, $\vec{a}_{av}$ cannot have a term with $\vec{B}$ by itself, because the units for $\vec{B}$ do not match units for accel... "length over time squared"

The next two questions involve the following situation:
The graph at right depicts a velocity-versus-time for a particle that starts at the origin and moves in one dimension.



Question value 4 points

(3) During what time interval(s) is the particle experiencing *negatively-directed* acceleration?

(a) During intervals I and IV.

(b) During intervals ~~II~~ and ~~III~~.

(c) During intervals I and ~~II~~.

(d) During none of the intervals shown.

(e) During intervals ~~II~~I and IV.

"acceleration" is realized as slope of velocity curve

⇒ looking at graph, slopes are: I: neg →
II: pos
III: pos
IV: neg →

acceleration is negative during intervals I and IV

Question value 4 points

(4) During what time interval(s) is the particle *slowing down*?

(a) During intervals ~~I~~ and IV.

(b) During interval ~~I~~ only.

(c) During intervals II and IV.

(d) During interval ~~III~~ only

(e) During intervals II and ~~II~~I.

"slowing down": $|\vec{V}|$ is getting smaller

I: negative velocity, magnitude increasing: NO

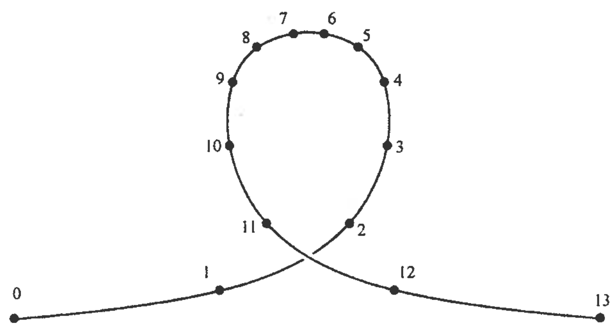
II: negative velocity, magnitude decreasing: YES

III: positive velocity, magnitude increasing: NO

IV: positive velocity, magnitude decreasing: YES

$|\vec{V}|$ decreasing / particle slowing down during intervals II and IV

The next two questions involve the following situation:
The figure at right displays the motion diagram for a rolloer-coaster car performing a loop-the-loop. Successive frames of the diagram are indexed by the integers 0 through 13.



Question value 4 points

(5) Which of the arrows below *best* depicts the direction of the average velocity for the car, between frames 9 and 12?

(a) ↗

(b) (zero vector)

(c) ↘

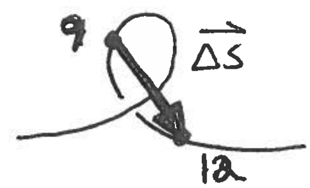
(d) ↖

(e) ↙

$$\vec{V}_{av} = \frac{\vec{S}_f - \vec{S}_i}{\Delta t} = \frac{\vec{\Delta S}}{\Delta t}$$

where, in this case, $\vec{\Delta S}$ = displacement from frame 9 to frame 12

[since Δt is a <u>positive</u> scalar, direction of $\vec{V}_{av}$ is the same as $\vec{\Delta S}$]



Question value 4 points

(6) Which of the arrows below *best* depicts the direction of the average accelration for the car, during frame 4?

(a) ↗

(b) ↘

(c) (zero vector)

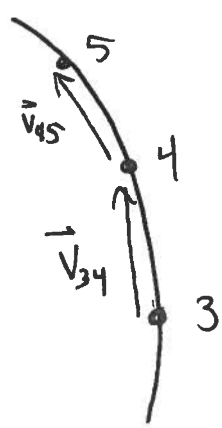
(d) ↙

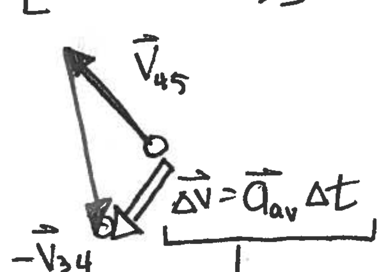
(e) ↖

$$\vec{a}_{av} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t}$$

→ use $\vec{V}_{3\to4}$ for $\vec{V}_i$ and $\vec{V}_{4-5}$ for $\vec{V}_f$

$$\vec{a}_{av} = \frac{1}{\Delta t}\left[\vec{V}_{45} + (-\vec{V}_{34})\right]$$





Δt is a positive scalar, so direction of $\vec{a}$ is the same as direction of $\Delta\vec{V}$