Physical Quantities and Dimensional Analysis

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Physics explains the world around us by identifying *meaningful relationships* that characterize nature—relationships which are evaluated in terms of *physical quantities*. Generically, a "physical quantity" is simply anything to which we can assign a precise numerical value, via some sort of measurement. Note, though, that the particular *values* taken by physical quantities are not what matters most; in physics, we are interested in *how the quantities are related to one another*.

- **Every measurable quantity has a unique "physical dimension" associated with it.**

"Physical dimension" does not have anything to do with *spatial dimension*—directions like up/down, left/right or forwards/backwards; we are not talking about whether a quantity describes something in "1D" versus "2D" or "3D". Instead:

- **Physical dimension is a generic description of the kind of quantity being measured.**

Physical dimension is an inherent and unvarying property for a given quantity. A given quantity can only ever have *one* specific physical dimension. The converse, however, is *not* true: *different* physical quantities can have the *same* physical dimension.

Examples:

- The height of a building, the diameter of a proton, the distance from the Earth to the Moon, and the thickness of a piece of paper *all* involve a measurement of a ***length,*** **L**, of some kind. They are all measured in different ways, and might have their values assigned using different units, but they all share the common feature of being a kind of ***length***.

- Time intervals, timestamps, or any other measurement characterizing duration, all share the common feature of having a physical dimension of ***time,*** **T**. You will never be able to assign a value to a time coordinate using units of length—***length*** and ***time*** are fundamentally distinct and inequivalent physical dimensions.

When we talk about a quantity's *physical dimension*, using symbols such as **L** for ***length***, or **T** for ***time***, we are *not* using those symbols to represent an actual *variable*—we will not assign any particular *values* or use any specific *units* for those symbols. Instead, when we write, "this quantity has physical dimension **L**", the symbol **L** *really* just means "…having *a* length unit". To make this distinction explicit, we will henceforth set aside symbols in square brackets whenever making a statement about physical dimensions. So: all distances, lengths, heights, and widths have dimension of *length*, [L], all timestamps and time intervals have dimension of *time*, [T], and so forth.

The physical dimension of a quantity is determined by *how we measure* that quantity—and to do *that*, we need to define an appropriate unit for the measurement. We can make different choices for the units we use (e.g. the SI system versus the British Engineering system, BE), but *whatever* those alternative unit choices are, they *must* all have the same physical dimension. For example: meters, inches, furlongs, and nautical miles are all very different units, but the all share the common feature of being lengths, [L].

Systems of Units

A *system of units* is a complete set of units that can measure *any* quantity in the universe. Such a system must have a unit for every possible physical dimension imaginable! Fortunately, we have found that to describe motion, only three basic physical dimensions are necessary: length [L], mass [M], and time [T]—all other quantities we will need in mechanics can be defined as algebraic combinations of those fundamental unit types. Thus, we need only define the meter, kilogram, and second, in order to be able to measure *anything* of interest.(1) Some examples of physical quantities, their dimensions, and derived units are tabulated below.

Quantity	Dimension	SI unit	equivalent unit
speed or velocity	[L/T]	m/sec	—
acceleration	[L/T^2]	m/sec^2	—
momentum	[ML/T]	kg·m/sec	—
force	[ML/T^2]	kg·m/sec^2	newton, N
work or energy	[ML^2/T^2]	kg·m^2/sec^2	joule, J = N·m
power	[ML^2/T^3]	kg·m^2/sec^3	watt, W = J/sec

Keep in mind that the expressions above for physical dimension ***do not*** provide formulas for actually *calculating* the quantities in question; you do not find a *value* for acceleration by dividing a length by the square of a time. The physical dimensions above are simply a statement of the generic *kind* of thing being measured. For example, the phrase "momentum has dimension [ML/T]" literally translates as, "a *unit* of momentum is a mass *unit* multiplied by a length *unit*, divided by a time *unit*".

Dimensional Analysis

It is useful to consider physical dimension because it provides a way to track what symbols *mean* through long calculations—a process known as *dimensional analysis*. Learning to incorporate it into your skillset is *not* a waste of time. It helps you to understand the formulas that you are using, to *see* how different expressions are related to each other, and to commit the most important formulas to memory. You will also become more *efficient* at solving problems, spending *less* time per problem than if you hadn't this technique. Finally, and most importantly: **you will find that you will make fewer mistakes if you take the time to learn dimensional analysis**!

In practice, dimensional analysis involves systematically *keeping track* of the physical dimensions of every expression you write down.

- **A necessary precursor to using dimensional analysis is that you must work problems using *symbolic expressions* (i.e. "formulas"), rather than numerical values.**

Remember that physics is about *relationships*, not *values*. By jumping to numbers right away, you forfeit the ability to see how the relationships fit together, and thus forfeit potential learning. Make it a habit to work problems symbolically—**every problem, every time, no exceptions**.

(1) And as any physics student can attest, many things that are not at all interesting…

The use of dimensional analysis as a technique boils down to one very fundamental idea:

- **In all physically meaningful expressions, physical dimensions must be dealt with algebraically. Conversely, expressions that do not satisfy basic algebraic rules regarding physical dimensions are not physically meaningful.**

This simply means that dimensions will track through any mathematical operation that you make: adding, subtracting, multiplying, dividing, or equating. If the dimensions don't **all** match in an equation you've just written down, then whatever you've written is **not physics!**

In order to use this idea of "treating physical dimensions as algebraic quantities", we will set up some basic mathematical rules—nothing *new*, mind you. We are just emphasizing how *established* ideas about basic math operations can also be applied to generic dimensional relationships.

- **Products (and Quotients): The dimension of a product of several variables is the product of the individual dimensions of each of those variables. (Ditto for ratios.)**

Example: A sphere of radius R has a volume given by the expression $4/3\pi R^3$. The factors 4/3 and π are pure numbers (having *no* dimension), while radius is a type of *length*, [L]. Dimensional analysis simply involves the replacement of each variable with its generic *type:*

$$4/3\ \pi R^3 \rightarrow [\mathrm{L}]^3$$

We drop numerical factors because the physical dimension of any pure number is "[1]"—remember, we don't care about *values* when performing dimensional analysis, only the *kind* of thing being measured. Neither "4/3" nor "π" bring anything to the party, in that regard.

Dimensional analysis tells us that *generically*, volume measurements (liters, gallons, or cubic centimeters) will always evaluate as the product of three[(2)] length-type measurements. We would have reached this exact same conclusion if we had started with the formula for the volume of a box of length L, width W and height H: volume = $L\cdot W\cdot H$, so:

$$L\cdot W\cdot H \rightarrow [\mathrm{L}]\cdot[\mathrm{L}]\cdot[\mathrm{L}] = [\mathrm{L}]^3$$

Caution: don't confuse the symbol "L" representing the actual numerical value for the length of the box (i.e. a variable) with the generic symbol [L] *representing the fact that each of the box's three dimensions is individually a type of length unit (i.e. a physical dimension).*

A corollary of this result is that any expression that does *not* involve the product of three *lengths* (and nothing else[(3)]) cannot be an expression describing a volume!

- **Sums (and Differences): Different terms can only added together in a sum if each term in the sum has the same physical dimension. (Ditto for subtraction.)**

You cannot add a length to a volume, or subtract a mass from a velocity. Such operations are nonsense, and if your algebraic work in a problem leads to such malarkey, you *know* there is a mistake somewhere. Stop right there and track backwards to find and correct the error.

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(2) "Three shall be the number thou shall count, and the number of the counting shall be three."

(3) "Four shalt thou not count, neither count two, excepting that thou proceed to three. Five is right out."

- **Equalities: See the rule on sums. Recite it three times, twirl around twice, and tattoo it on the inside of your eyelids. You can NEVER equate two things of unequal dimension.**

Example: Consider a physics course that does not allow any formula sheets for tests, leaving the students with a hundred bazillion formulas to memorize. It is a safe bet that more than a few will get their mental wires crossed, and end up writing down something like the following for a kinematics problem:

$$v(t) = v_0 - \tfrac{1}{2} g\, t^2$$

This is just plain **wrong**—and needlessly so. It does not matter whether or not you can remember the "right" formula; any physics student with even a rudimentary understanding of what's going on should be able to immediately see that this equation is dimensionally wrong:

- The symbol $v(t)$ represents "velocity at the specific time t". The physical dimension of this quantity is clearly "velocity units", [L/T].
- The symbol v_0 represents "initial velocity at time zero". Obviously also of dimension [L/T].
- The term $\tfrac{1}{2} g\, t^2$ involves the factor "g", which represents Earth's gravitational acceleration (dimension $[\mathrm{L/T^2}]$), and the factor "t" representing elapsed time (dimension [T]). Thus, the final term evaluates overall as:

$$\tfrac{1}{2} g\, t^2 \rightarrow [\mathrm{L/T^2}]\cdot[\mathrm{T}]^2 = [\mathrm{L}]$$

Don't forget that the pure number ½ makes no contribution to the overall dimension of the final term.

So here's why the formula is wrong: on the right-hand side, you can't subtract a "length" from a "velocity"—and even if you *could*, you couldn't equate the resulting "velengthity" on the right with a "velocity" on the left!

Thus, you can often catch mistakes before they happen, by making sure that the formulas you write down at the start of the problem are physically reasonable. In fact, it's much easier to *memorize formulas correctly in the first place*, if you consider the dimensions of all the quantities involved.

In addition, dimensional analysis can alert you to the presence of mistakes you've made during a problem—not because you've written a formula wrong, but because of a basic math error. Everyone makes these sorts of errors from time to time—and their cumulative impact on final grades can be costly. However, if you apply dimensional analysis along the way, you can often spot such an error before it propagates through your work, and avoid wasting time on a dead-end solution.

Example: a particle is at rest from t = 0.0 sec to t = 1.0 sec. From t = 1.0 sec to t = 3.0 sec, the particle experiences a non-uniform acceleration, given by the time-dependent formula:

$$a(t) = \frac{K}{t^2}$$

where K is an overall physical constant having the value K = 2.0 m. What is the velocity of the particle at time t = 3.0 sec? Maybe you don't know how to solve this problem (yet), but don't worry about that—you'll learn it soon enough.

Let's just accept that the correct way to solve the problem is to obtain the final velocity by computing the time-integral of the acceleration:

$$v_f = v_i + \int_{t_i}^{t_f} a(t)dt$$

We will solve the problem *twice*, using two different approaches to compute the integral above—first by using a quick numerical computation, and second by using a longer symbolic approach.

Numerical:
I will substitute values at the outset—usually not a good idea, but if I am careful to make all of my substitutions using SI units, then things *should* work out okay, to get a final answer in units of m/s.

$$v_f = 0 + \int_1^3 \frac{2}{t^2} dt = 2\int_1^3 \frac{dt}{t^2} = 2\left[\frac{-3}{t^3}\right]_1^3 = 2\left[\frac{-3}{27} - \frac{-3}{1}\right] = 5.8 \text{ m/s}$$

Symbolic:
Use the same operations as above, but without numerical substitutions. But first, note the dimension of the parameter K, based on the value assigned to it: K = 2.0 m → K is a length, [L]. This means that K/t^2 has dimension of $[L]/[T]^2$, matching the known dimension of acceleration. So far, so good:

$$v_f = 0 + \int_{t_i}^{t_f} \frac{K}{t^2} dt = K\int_{t_i}^{t_f} \frac{dt}{t^2} = K\left[\frac{-3}{t^3}\right]_{t_i}^{t_f} = K\left[\frac{-3}{t_f^3} - \frac{-3}{t_i^3}\right] = \frac{3K}{t_i^3} - \frac{3K}{t_f^3}$$

Whoa, Kimosabe! Both terms on the far right clearly have the same dimension (that's good), but they don't match the dimension on the far left of the equation (that's bad—REALLY bad). Remember: K has dimension of length [L], so $3K/t^3$ has dimension $[L]/[T]^3$—that does not match the [L/T] on the far left! Something has gone wrong in a big way.

Perhaps you saw my mistake in this example at the moment I made it. Now that you know about the existence of a mistake, you can probably go back and find it yourself. But finding the mistake isn't the point, here—**the real issue is in *knowing* that there WAS a mistake, in the first place!** In the numerically-based solution, there were no warning signs to indicate that something was wrong.

You cannot fix a mistake that you do not know about!

Not only can the symbolic approach spotlight the *existence* of a mistake, but also, by applying further dimensional analysis the error can be *tracked down* and *removed*:

When I left off, I had just written:

$$v_f = \frac{3K}{t_i^3} - \frac{3K}{t_f^3}$$

Dang it! There are two extra factors of time [T] in the denominator…where did those pesky buggers come from? Well, let's see…I started out with a $(1/t^2)$ in the integrand, and I then integrated that to get a $(1/t^3)$—hey, wait just a darn minute…that seems fishy. Integration should *raise* the power of t, but increasing a power in the denominator actually *reduces* the power of the expression. Guess I need to look *that* part of my solution over more carefully, and review my calculus:

$$\int t^n \, dt = \frac{t^{n+1}}{n+1} + C \quad \text{(as long as } n \neq -1\text{)}$$

Using $n = -2$ for a $(1/t^2)$ integrand gives us:

$$\int t^{-2}\,dt = \frac{t^{-2+1}}{-2+1} + C = \frac{t^{-1}}{-1} + C = \frac{-1}{t} + C$$

Doh! I <u>should</u> have written:

$$v_f = \int_{t_i}^{t_f} \frac{K}{t^2}dt = K\left[\frac{-1}{t}\right]_{t_i}^{t_f} = K\left[\frac{-1}{t_f} - \frac{-1}{t_i}\right] = \frac{K}{t_i} - \frac{K}{t_f}$$

Before we substitute numerical values, notice that <u>now</u> the dimension on the far right matches that of the far left; the two extra factors of [T] in the denominator have vanished. The terms containing *K*/*t* have dimension [L]/[T], as required. We can thus perform the calculation step with confidence, to find:

$$v_f = \frac{2.0\ m}{1.0\ s} - \frac{2.0\ m}{3.0\ s} = 2.0\text{ m/s} - 0.67\text{ m/s} = 1.3\text{ m/s}$$

We'll look at another example of dimensional analysis in a few moments, but before moving along, there is one last gold nugget to be pried out of the last example. As you recall, it was stated that the final velocity was found as the definite time integral of the acceleration:

$$v_f = v_i + \int_{t_i}^{t_f} a(t)dt$$

This is a *calculus* equation rather than a simple *algebraic* equation, but it is still an equation describing <u>real physics</u>—if it is "meaningful physics", it must be dimensionally valid. The initial and final velocities obviously have dimension [L/T], but what about that final term with all that calculus? Does that it *also* work out to have dimension [L/T]?

First of all, the "$\int$" symbol itself simply denotes continuous summation. It isn't a *quantity*—it is an *operand telling you what to do*. It contributes nothing to the physical dimension of the integral.

Next, we have the acceleration $a(t)$. It is expressed as a function of time, but we've already seen how that function evaluates to proper acceleration units; its dimension is [L/T^2].

Finally, there is the differential dt. Does it *have* any dimension? After all—it is *infinitesimal*: that is, $dt \rightarrow 0$. Can we therefore set its *dimension* to "0"? HECK NO! If we assume that dt has zero dimension, then the entire integral would have zero dimension: [L/T^2]·[0] = [0]. Nor can we say dt is "dimensionless", like a pure number. If that were true, dt would not contribute to dimension, and the integral would evaluate to [L/T^2] overall—but that **won't** match the other velocity terms!

The key here is that the symbol "dt" represents a very small *time interval*. Sure, it is considered to be infinitesimal, but it still <u>is</u> a measure of *elapsed time*. It thus contributes a dimension of [T] to the integral, and the physical dimension of the integral evaluates properly as something with velocity units:

$$\int a(t)dt \rightarrow [L/T^2]\cdot[T] = [L/T]$$

- **Differentials: The *differential* of any physical quantity has the same physical dimension as the quantity itself.**

  This holds true because a differential simply represents a *small change* in that physical quantity. A length can only change by small bits of length, a mass can only change by small bit of mass, and a time can only change by small bits of time.

There is one more benefit of learning to dimensional analysis techniques. They can often help you to quickly re-derive an important formula that you have forgotten.

Example: You are taking a test on circular motion, when you come upon a problem involving the "centripetal force" acting on a body.(4) Oh noes! You have forgotten the formula for centripetal force—all you can recall is that it involves the mass and speed of the object, and the radius of the circular path. Are you toast?

Not if you can draw upon the phenomenal cosmic powers of dimensional analysis! Mass has dimension [M], speed has dimension [L/T], and radius has dimension [L]. All you have to do is to figure out a way to put the variables m, v, and r together in a way that makes their combined dimension come out to be that of **force**. What? You don't *remember* the dimension of force? But you **do** remember Newton's 2nd Law, right? You know, "force equals mass times acceleration". Well then: force ***units*** equal mass ***units*** times acceleration ***units***: so [force] = [M]·[L/T²]. It's that dadgum simple.

So, write a very generic "guess" for the formula:

$$F_{cent} = m^a \cdot v^b \cdot r^c$$

here, *a*, *b*, and *c* are unknown integers. Your quest, young Padawan, is to deduce what they *must* be. They are found by requiring the *dimensions* of both sides of the equation to match:

$$\left[\frac{\mathrm{ML}}{\mathrm{T}^2}\right] = [\mathrm{M}]^a \cdot \left[\frac{\mathrm{L}}{\mathrm{T}}\right]^b \cdot [\mathrm{L}]^c$$

- If we match mass units [M] on each side, we see that the integer *a* must equal 1.
- If we match time units [T] on each side, we see that the integer *b* must equal 2.
- If we match length units [L] on each side, we see that the sum $b + c$ must equal 1.
  Since $b = 2$, c must equal –1.

We have deduced that the only possible set of values for *a*, *b*, and *c* that will result in an expression with dimension of **force** are: (a, b, c) = (1, 2, –1). So, we have:

$$F_{cent} = m^1 v^2 r^{-1} \quad \text{or} \quad F_{cent} = m\frac{v^2}{r}$$

Thus, we have managed to reproduce the required formula from our incomplete memory, just by making sure that the variables combine together in a way to give units of force.

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(4) Physics nit-picking: the formula that we're considering here is not actually for "centripetal *force*", but for for "*mass* times centripetal *acceleration*"—and it goes on the acceleration-side of Newton's 2nd Law, not the force-side.

Having repeatedly sung the praises of the awesomeness of dimensional analysis, now would be a good time to point out some of the dirty laundry associated with that last technique. Dimensional analysis is really good at tracking physical dimensions. It stinks at analyzing pure number relationships. Particularly, it is utterly incapable of tracking pure numbers, or even predicting the possible *need* for them in some situations.

> Example: watch with awe as I use dimensional analysis to remember the formulas for the circumference and area of a circle:
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> Well, a circle is ultimately defined by just one thing: its radius, R[(5)]—and that clearly has dimension of length [L]. I therefore seek to create a formula for circumference, which is a measure of the distance around the rim of the circle…but a *distance* is generically a dimension of length [L], as well. So let's start with the trial formula:
>
> $$C = R^a,$$
>
> where the integer a is to be found via dimensional analysis. Clearly, I want to match length [L] on the left with $(\text{length [L]})^a$ on the right. The only possible way to do that is if $a = 1$. Thus the final formula for the circumference of a circle is:
>
> $$C_{circle} = R$$
>
> Similarly, the *area* of a circle must have dimension of $[L]^2$. We know this because a rectangle has its area computed as *length·width*, which is obviously of dimension $[L]^2$—and since *all* areas (no matter how they are computed) are the same *kind* of quantity, they must have the *same* physical dimension, no matter how different their actual formulas might be. So, we make the following trial guess for the are formula:
>
> $$A = R^b,$$
>
> where this time, it's the integer b that we will find via dimensional analysis. We have dimension $[L]^2$ on the left, and dimension $[L]^b$ on the right, so clearly $b = 2$. Our final formula is:
>
> $$A_{circle} = R^2,$$
>
> As you can see, dimensional analysis knows nothing of this "π" thingamajob, that was (and is) such a big deal to the ancient Greeks (and modern geeks).

In a similar vein, another physics formula widely known to students is the expression for kinetic energy, $K = ½\, mv^2$. It very is easy to show that the *only* possible combination of mass [M] and speed [L/T] to yield energy $[ML^2/T^2]$ is to use a formula involving m^1v^2. That "½", though, cannot be predicted using dimensional analysis techniques. It might seem that that inability to detect or track pure numerical factors is a downside—a *huge* downside. But ultimately, this is a "cup half-empty versus cup half-full" issue. You can't get *everything* with dimensional analysis, but you can get a lot more with it that you can without it. If fact, one can often learn a surprisingly large amount of information in situations where you *know for a fact* that dimensional analysis is hiding some important numerical factors from you!

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(5) Although technically, those clever Greeks who inflicted basic geometry on us used *diameter, D*, as the basic parameter defining the size of a circle.

A good example of this is the so-called "square-cube" law which explains why those crazy sci-fi movies, where scientist shrink people down to the size of bugs and radioactive waste grows bugs up to the size of elephants, are…well…just absolutely impossible according to the physical nature of the universe. It all hinges on understanding how certain quantities scale as you change the size of the creature—and that's dimensional analysis!

Consider growing an ant (proportionally) by a factor of 1000 in each *spatial* dimension. That means that any property of the ant that scales linearly with size (in dimensional-analysis-lingo, any quantity *proportional* to $[L]^1$) changes by a factor of 1000—its length, its height, and its width, surely…but also the circumferences of each of its legs, the diameter of its abdomen, and so on.

Similarly, there are properties that scale as $[L]^2$ for the ant—in particular, the total body surface area of the ant. In other words, our gigantic ant now has a surface area a *million* times greater than a regular ant. It does not matter that we do not know a precise formula for calculating the surface area of an ant—its an ***area***, and if all types of *lengths* for the ant are 1000 times bigger, then all types of *areas* for the ant are 1000x1000 times larger. That *also* means that the cross-sectional areas (there's that word "area" again!) of the ant's legs (and muscles) are 1,000,000 times larger, as well—we deduce that our mutant ant is a million times stronger than an ordinary ant. That's **awesome**, right?

Well, not if you're the ant. Your volume would scale as $[L]^3$. Assuming that only *linear size* has been altered—but not body composition, physiology, or metabolism—your mass would be a *billion* times greater (because *mass* = *density* times *volume*, and density doesn't change). Unfortunately, the muscles supporting you are only a million times stronger, because strength scales according to cross-sectional area. It is 1000 times harder to stand, walk, clack your mandibles, wave your antennae around, or do any of the other things necessary to terrorize the puny humans.

Not only *that*, but your metabolism generates waste heat, which must be radiated away via the surface of your exoskeleton. The rate of heat generation scales with body mass—the formula is complicated, and certainly involves other *physical* quantities, as well as pure *numerical* factors (the final formula for "rate of heat generation" would have to end up having units of *power* $[ML^2/T^3]$)—but for sure, a mass one billion times greater means a rate of waste heat generation that is one billion times greater. Remember, though—the heat has to get out through your *surface*, which is only one million times larger. Without a drastic change in physiology, you can only radiate away heat through that surface at a rate that is a thousandth of what is needed: 99.9% of your body heat is trapped inside your body, and your brain boils to mush before you even have time to discover that your mighty mandibles are not strong enough to pick up and crush a single puny human.

Similarly, looking at the flip-side of this situation, shrinking a human down by a factor of 1000, you should now be able to reason out a good explanation as to why having that done to you would ultimately lead to a nasty case of hypothermia.

We have managed to deduce a significant amount of rather *specific* knowledge about this situation, just by understanding how various physical quantities scale with length—even though we had almost no details about the *full* formulas for many of those quantities. It all boiled down to tracking physical dimensions, and worrying only about how changes in the key parameter (in this case, length) would necessarily effect other quantities derived from that parameter, based on those dimensionality relationships. This is often referred to as "ratio reasoning", "scaling arguments" or "proportionality arguments", and it would not be possible without knowledge about the physical dimensions of the quantities involved.

Summary:

Just to have it all in one place, let's review the important ideas that were developed in these notes:

- The physical dimension of a variable is a generic description of the kind of quantity that variable measures. Dimension is represented by symbols [L], [M], and [T], that do **not** represent the variables themselves, or take any specific values.

- Dimensional expressions are not formulas and are not used for computation. Their purpose is solely to characterize the relationships between the types of quantities in an expression.

- In all physically meaningful expressions, physical dimensions and units must be dealt with algebraically. Expressions that do not satisfy basic algebraic rules regarding physical dimensions and units are not valid physics.

- In order to use dimensional analysis as a problem-solving tool, you **must** work problems using symbolic expressions, rather than numerical values.

- The dimension of a product of several variables is the product of the individual dimensions of each of those variables.

- Two expressions of unequal physical dimensions **cannot** be added together, subtracted from one another, or equated to one another.

- Paying heed to the physical dimension of intermediate expressions in the *middle* of a problem can help catch errors as soon as they occur, and suggest a possible remedy.

- In calculus, the *differential* of a physical quantity has the same physical dimension as the quantity itself.

Incorporating these ideas into one's toolbox of everyday problem-solving techniques may require a significant investment of time, for most students. Dimensional analysis is not a particularly *hard* concept, but it does require consistent practice before one is able to "get it". However, those who make the commitment will find that it is worth the investment. A student who can think in dimensional terms will begin to recognize not only how all the math "makes sense", but also how it *means something about physics*—and "finding meaning" is what this class is all about.

If you learn nothing else from this course, learn dimensional analysis. Students commonly express the opinion that they'll never use 90% of what we cover in this course. If you adhere to that viewpoint, you should consider dimensional analysis to be part of the 10%.