PHYS 2211 Recitation Week 10

Team Score:
(see reverse)

Wagon Wheel — Team Formulation

While writing a team term paper about settlers crossing the Great Plains in wagons, you get into an argument with your co-author regarding the moment of inertia of a replica wooden wagon wheel. The 75-kg wheel is 120-cm in diameter and has heavy spokes connecting the rim to the axle. Your friend claims that you can approximate using $I = MR^2$ (as for a hoop) but you believe that because of the mass in the spokes, I will be more like a solid disk ($I = MR^2/2$). To see who is closer, you decide to find I experimentally. You mount the wheel on a horizontal low-friction axle, then wrap a light cord around the outside of the rim to which you attach a 25-kg bag of sand. When the bag is released from rest, it drops 3.77 m in 1.52 seconds, during which time the wheel rotates through one full revolution as the cord unwinds from it. (Hint: Use energy considerations.)

Team Members

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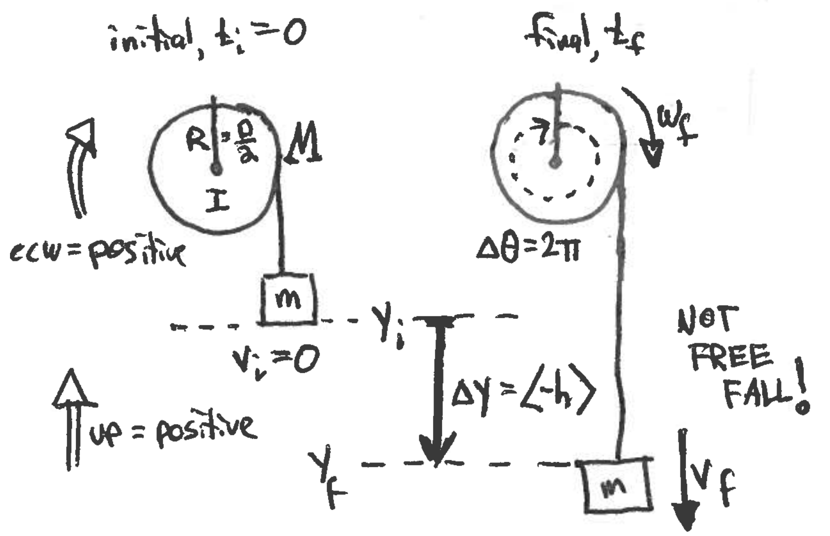
Part One:
Working as a team, formulate a specific physics problem that will answer the question posed. Your formulation should include: a brief written statement of the problem you will solve; a visual representation of the situation that includes all essential physics information; and an explanation of the essential physics approach for solving the problem—i.e. the necessary concepts or principles, any special problem-solving techniques, and/or any key assumptions that must be made in order to set up or solve the problem.

Formulation: Use the experimental data given to determine the wheel's moment of inertia, and compare that value to the two approximations



Assumptions
- drag can be neglected, friction too
- cord is massless and unstretchable
- cord unwinds from wheel without slipping:
  $|\Delta y_{bag}| = \Delta S_{cord} = R\Delta\theta_{wheel}$
  $v_{bag} = v_{cord} = R\omega_{wheel}$
- acceleration is constant while unwinding (but is NOT g!)

Procedure
1. Use kinematics to determine final speeds, from time-and-distance data
2. Apply conservation of energy, including wheel's rotational KE, AND using the no-slip condition to relate the two speeds
3. Find the wheel's moment of inertia, compare to the two approximations mentioned

PHYS 2211 Recitation Week 10 YOUR Name

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Your Team Members

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Wagon Wheel — Your Solution

While writing a team term paper about settlers crossing the Great Plains in wagons, you get into an argument with your co-author regarding the moment of inertia of a replica wooden wagon wheel. The 75-kg wheel is 120-cm in diameter and has heavy spokes connecting the rim to the axle. Your friend claims that you can approximate using $I = MR^2$ (as for a hoop) but you believe that because of the mass in the spokes, I will be more like a solid disk ($I = MR^2/2$). To see who is closer, you decide to find I experimentally. You mount the wheel on a horizontal low-friction axle, then wrap a light cord around the outside of the rim to which you attach a 25-kg bag of sand. When the bag is released from rest, it drops 3.77 m in 1.52 seconds, during which time the wheel rotates through one full revolution as the cord unwinds from it. (Hint: Use energy considerations.)

Team Score: (from Part One)	**YOUR Score:** (see reverse)

Part Two:

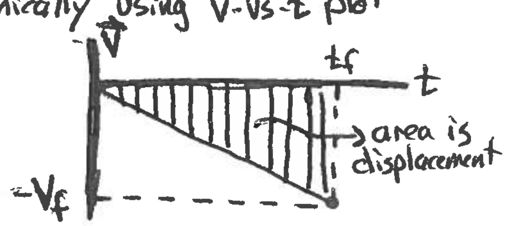
Solve the problem that you formulated in Part One, using the physics approach you have identified. You may work with your team members, but each of you should individually write up your own solution.

① Constant Accel for bag's linear motion — analyze graphically using $\vec{v}$-vs-t plot

displacement = area under graph

$\langle -h \rangle = \frac{1}{2}(-v_f)(\Delta t)$ (area of triangle)

$\rightarrow v_f = \frac{2h}{\Delta t}$ = final speed of bag



BUT:

$h = \Delta s_{cord} = 2\pi R$

and $\omega_f(\text{wheel}) = v_f/R$

$\longrightarrow$ bag $\boxed{v_f = \frac{4\pi R}{\Delta t}}$ and $\boxed{\omega_f = \frac{4\pi}{\Delta t}}$ wheel

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② Conservation of energy: let $U_{gi} = 0$ Then $U_{gf} = -mgh$ for sandbag (wheel's GPE does not change)

$K_i + U_i = K_f + U_f$

$0 + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I_w\omega_f^2\right) + (-mg2\pi R)$

$\frac{1}{2}I_w\omega_f^2 = 2\pi mgR - \frac{1}{2}mv_f^2$

$$\boxed{I_w = \frac{4\pi mgR - m\left(\frac{4\pi R}{\Delta t}\right)^2}{(4\pi/\Delta t)^2}}$$

substitution gives $\boxed{I_w = 18\ \text{kg-m}^2}$ (two digit precision)

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③ Comparisons

hoop: $I = MR^2 = 27\ \text{kg-m}^2$

disk: $I = \frac{1}{2}MR^2 = \frac{1}{2}(27\ \text{kg-m}^2) = 13.5\ \text{kg-m}^2$ (rounds to $14\ \text{kg-m}^2$)

actual wheel: $I_w = 18\ \text{kg-m}^2 = \frac{2}{3}I_{hoop}$ or $\frac{4}{3}I_{disk}$

Which is "better"?

$\rightarrow$ Find % error, ε

hoop: $\varepsilon = \frac{I_{hoop} - I_w}{I_w} = 50\%$ (high)

disk: $\varepsilon = \frac{I_{disk} - I_w}{I_w} = \boxed{-25\%}$ (low)